Appendix -B

Numerical Scheme For Dynamic Behaviour of Three-Fluid Crossflow Heat Exchanger

Conservation of energy for the three fluid streams considering longitudinal conduction in separating sheet and the axial dispersion in fluids can be expressed in non-dimensional form as given below,

$$\frac{V_a}{R_{ab}}\frac{\partial T_a}{\partial \theta} = T_{w1} - T_a - \frac{E_{ab}}{R_{ab}}\frac{\partial T_a}{\partial Y} + \frac{N_a}{Pe_a}\frac{E_{ab}}{R_{ab}}\frac{\partial^2 T_a}{\partial Y^2} \tag{B.1}$$

$$V_b\frac{\partial T_b}{\partial \theta} = \frac{1}{\phi}(T_{w1} - T_b) + \frac{1}{(1-\phi)}(T_{w2} - T_b) - \frac{\partial T_b}{\partial X} + \frac{N_a}{Pe_b}\frac{\partial^2 T_b}{\partial X^2} \tag{B.2}$$

When fluid c is moving in x-direction

$$\frac{V_c}{R_{cb}}\frac{\partial T_c}{\partial \theta} = T_{w2} - T_c - \frac{E_{cb}}{R_{cb}}\frac{\partial T_c}{\partial X} + \frac{N_a}{Pe_c}\frac{E_{cb}}{R_{cb}}\frac{\partial^2 T_c}{\partial X^2} \tag{B.3a}$$

When fluid c is moving in y-direction

$$\frac{V_c}{R_{cb}}\frac{\partial T_c}{\partial \theta} = T_{w2} - T_c - \frac{E_{cb}}{R_{cb}}\frac{\partial T_c}{\partial Y} + \frac{N_a}{Pe_c}\frac{E_{cb}}{R_{cb}}\frac{\partial^2 T_c}{\partial Y^2} \tag{B.3b}$$

Similarly the equations for the two separating sheets can be given by

$$\psi\frac{\partial T_{w1}}{\partial \theta} = R_{ab}(T_a - T_{w1}) + \frac{1}{\phi}(T_b - T_{w1}) + \lambda_x N_a\frac{\partial^2 T_{w1}}{\partial X^2} + \lambda_y N_a\frac{\partial^2 T_{w1}}{\partial Y^2} \tag{B.4}$$

$$(1-\psi)\frac{\partial T_{w2}}{\partial \theta} = \frac{1}{(1-\phi)}(T_b - T_{w2}) + R_{cb}(T_c - T_{w2}) + \lambda_x N_a\frac{\partial^2 T_{w2}}{\partial X^2} + \lambda_y N_a\frac{\partial^2 T_{w2}}{\partial Y^2} \tag{B.5}$$

The equations (B1-B5) are subjected to following initial and boundary conditions

$T_a(X,Y,0) = T_b(X,Y,0) = T_c(X,Y,0) = T_{w1}(X,Y,0) = T_{w2}(X,Y,0) = 0,$ (B.6)

$$\left.\frac{\partial T_{w1}(X,Y,\theta)}{\partial X}\right|_{X=0} = \left.\frac{\partial T_{w1}(X,Y,\theta)}{\partial X}\right|_{X=N_a} = \left.\frac{\partial T_{w1}(X,Y,\theta)}{\partial Y}\right|_{Y=0} = \left.\frac{\partial T_{w1}(X,Y,\theta)}{\partial Y}\right|_{Y=N_a} = 0 \tag{B.7}$$

$$\left.\frac{\partial T_{w2}(X,Y,\theta)}{\partial X}\right|_{X=0} = \left.\frac{\partial T_{w2}(X,Y,\theta)}{\partial X}\right|_{X=N_a} = \left.\frac{\partial T_{w2}(X,Y,\theta)}{\partial Y}\right|_{Y=0} = \left.\frac{\partial T_{w2}(X,Y,\theta)}{\partial Y}\right|_{Y=N_a} = 0 \tag{B-8}$$

$$T_a(X,0,\theta) = 0 \tag{B.9}$$

$$\left.\frac{\partial T_a(X,Y,\theta)}{\partial Y}\right|_{Y=N_a} = 0 \tag{B.10}$$

$$T_b(0,Y,\theta) = \phi(\theta) \tag{B.11}$$

$$\left.\frac{\partial T_b(X,Y,\theta)}{\partial X}\right|_{X=N_b} = 0 \tag{B.12}$$

$$T_{c,in} = T_{c,in} \tag{B.13}$$

Where $T_{c,in}$ is $T_c(0,Y,\theta)$, $T_c(N_a,Y,\theta)$, $T_c(X,N_b,\theta)$ and $T_c(X,0,\theta)$ for the arrangements C1, C2, C3 and C4 respectively.

When fluid c moves in x-direction

$$\left.\frac{\partial T_c(X,Y,\theta)}{\partial X}\right|_{X=Z} = 0\,, \tag{B.14}$$

where Z=Na and 0 for the arrangements C1 and C2 respectively.

When fluid c moves in y-direction

$$\left.\frac{\partial T_c(X,Y,\theta)}{\partial Y}\right|_{Y=Z} = 0\,, \tag{B.15}$$

where Z=0 and Na for the arrangements C3 and C4 respectively.

The functional form of $\phi(\theta)$ is expressed as follows

$$\phi(\theta) = \begin{cases} 1; \quad \text{for step input} \\ \begin{bmatrix} \alpha\theta, \ \theta \le 1 \\ 1, \ \theta > 1 \end{bmatrix} ; \text{for ramp input} \\ 1 - e^{-\alpha\theta}; \ \text{for exponential input} \\ \sin(\alpha\theta); \text{for sinusoidal input} \end{cases} \tag{B.16}$$

Figure B.1 shows the schematic representation of the heat exchanger space, flow arrangement, grids and temperature distribution for the numerical scheme. In three-fluid case $T_{b,in}$ is the hot fluid inlet temperature where perturbations are given. It is assumed that, for an element, fluid a, b and c have got the unidirectional temperature variation in x or y direction.

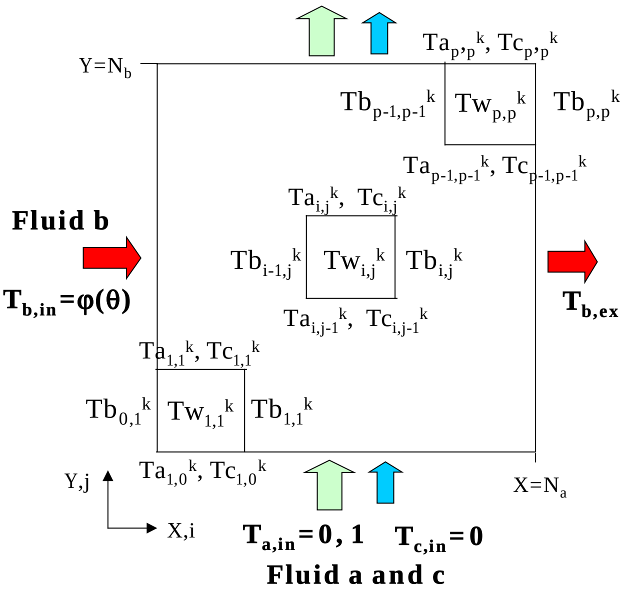

Figure B.1 Arrangement of grids for the numerical scheme.

The conservation equations are discretised using the implicit finite difference technique (Ozisic, 1994). Forward difference scheme is used for time derivatives, while upwind scheme and central difference scheme are used for the first and second order space derivatives respectively. The discretisation of the energy equations for the two walls and the three fluids are given as under,

$$\frac{\psi}{\Delta\theta}[(T_{w1})_{i,j}^{k+1} - (T_{w1})_{i,j}^{k}] = \frac{R_{a-b}}{2}[(T_a)_{i,j}^{k+1} + (T_a)_{i,j-1}^{k+1}] + \frac{1}{2\phi}[(T_b)_{i,j}^{k+1} + (T_b)_{i-1,j}^{k+1}] - (\frac{1}{\phi} + R_{a-b}).(T_{w1})_{i,j}^{k+1} + \frac{\lambda_x N_a}{(\Delta X)^2}[(T_{w1})_{i+1,j}^{k+1} - 2.(T_{w1})_{i,j}^{k+1} + (T_{w1})_{i-1,j}^{k+1}] + \frac{\lambda_y N_a}{(\Delta Y)^2}[(T_{w1})_{i,j+1}^{k+1} - 2.(T_{w1})_{i,j}^{k+1} + (T_{w1})_{i,j-1}^{k+1}] \quad \text{(B.17)}$$

$$\frac{1-\psi}{\Delta\theta}[(T_{w2})_{i,j}^{k+1} - (T_{w2})_{i,j}^{k}] = \frac{1}{2(1-\phi)}[(T_b)_{i,j}^{k+1} + (T_b)_{i-1,j}^{k+1}] + \frac{R_{c-b}}{2}[(T_c)_{i,j}^{k+1} + (T_c)_{i,j-1}^{k+1}] - (\frac{1}{(1-\phi)} + R_{c-b}).(T_{w2})_{i,j}^{k+1} + \frac{\lambda_x N_a}{(\Delta X)^2}[(T_{w2})_{i+1,j}^{k+1} - 2.(T_{w2})_{i,j}^{k+1} + (T_{w2})_{i-1,j}^{k+1}] + \frac{\lambda_y N_a}{(\Delta Y)^2}[(T_{w2})_{i,j+1}^{k+1} - 2.(T_{w2})_{i,j}^{k+1} + (T_{w2})_{i,j-1}^{k+1}] \quad \text{(B.18)}$$

$$\frac{V_a}{R_{a-b}.\Delta\theta}[(T_a)_{i,j}^{k+1} - (T_a)_{i,j}^{k}] = (T_{w1})_{i,j}^{k+1} - \frac{1}{2}[(T_a)_{i,j}^{k+1} + (T_a)_{i,j-1}^{k+1}] - \frac{E_{a-b}}{R_{a-b}}\frac{1}{\Delta Y}[(T_a)_{i,j}^{k+1} - (T_a)_{i,j-1}^{k+1}] + \frac{N_a}{Pe_a(\Delta Y)^2}\frac{E_{a-b}}{R_{a-b}}[(T_a)_{i,j+1}^{k+1} - 2.(T_a)_{i,j}^{k+1} + (T_a)_{i,j-1}^{k+1}] \quad \text{(B.19)}$$

$$\frac{V_b}{\Delta\theta}[(T_b)_{i,j}^{k+1}-(T_b)_{i,j}^{k}]=\frac{1}{\phi}\left[(T_{w1})_{i,j}^{k+1}-\frac{1}{2}[(T_b)_{i,j}^{k+1}+(T_b)_{i-1,j}^{k+1}]\right]+$$
$$\frac{1}{(1-\phi)}\left[(T_{w2})_{i,j}^{k+1}-\frac{1}{2}[(T_b)_{i,j}^{k+1}+(T_b)_{i-1,j}^{k+1}]\right]-\frac{1}{\Delta X}[(T_b)_{i,j}^{k+1}-(T_b)_{i-1,j}^{k+1}]+$$
$$\frac{N_a}{Pe_b(\Delta X)^2}[(T_b)_{i+1,j}^{k+1}-2.(T_b)_{i,j}^{k+1}+(T_b)_{i-1,j}^{k+1}] \qquad \text{(B.20)}$$

When fluid c moves in x-direction,

$$\frac{V_c}{R_{c-b}.\Delta\theta}[(T_c)_{i,j}^{k+1}-(T_c)_{i,j}^{k}]=(T_{w2})_{i,j}^{k+1}-\frac{1}{2}[(T_c)_{i,j}^{k+1}+(T_c)_{i-1,j}^{k+1}]\mp$$
$$\frac{E_{c-b}}{R_{c-b}}\frac{1}{\Delta X}[(T_c)_{i,j}^{k+1}-(T_c)_{i-1,j}^{k+1}]+\frac{N_a}{Pe_c(\Delta X)^2}[(T_c)_{i+1,j}^{k+1}-2.(T_c)_{i,j}^{k+1}+(T_c)_{i-1,j}^{k+1}], \qquad \text{(B.21a)}$$

where (-) and (+) sign indicates the rightward and leftward direction respectively (Fig. B.1).

When fluid c moves in y-direction,

$$\frac{V_c}{R_{c-b}.\Delta\theta}[(T_c)_{i,j}^{k+1}-(T_c)_{i,j}^{k}]=(T_{w2})_{i,j}^{k+1}-\frac{1}{2}[(T_c)_{i,j}^{k+1}+(T_c)_{i,j-1}^{k+1}]\mp$$
$$\frac{E_{c-b}}{R_{c-b}}\frac{1}{\Delta Y}[(T_c)_{i,j}^{k+1}-(T_c)_{i,j-1}^{k+1}]+\frac{N_a}{Pe_c(\Delta Y)^2}[(T_c)_{i,j+1}^{k+1}-2.(T_c)_{i,j}^{k+1}+(T_c)_{i,j-1}^{k+1}], \qquad \text{(B.21b)}$$

where (-) and (+) sign indicates the upward and downward direction respectively (Fig. B.1).

The difference equations along with the boundary conditions are solved using Gauss-Seidel iterative technique. Following steps are taken for getting the solution.

1. Initially some values are assumed for $T_a(X,Y,\theta)$, $T_b(X,Y,\theta)$, $T_c(X,Y,\theta)$, $T_{w1}(X,Y,\theta)$ and $T_{w2}(X,Y,\theta)$ i.e. for $(T_{w1})_{i,j}^{k+1}$, $(T_{w2})_{i,j}^{k+1}$, $(T_a)_{i,j}^{k+1}$, $(T_b)_{i,j}^{k+1}$ and $(T_c)_{i,j}^{k+1}$ for all the grids.
2. Latest values of $(T_{w1})_{i,j}^{k+1}$ and $(T_{w2})_{i,j}^{k+1}$ are calculated using eq. (B.17) and (B.18).
3. Using the latest values of $(T_{w1})_{i,j}^{k+1}$ and $(T_{w2})_{i,j}^{k+1}$, new values of $(T_a)_{i,j}^{k+1}$, $(T_b)_{i,j}^{k+1}$ and $(T_c)_{i,j}^{k+1}$ for the whole exchanger space are calculated using eq. (B.19), (B.20) and (B.21) respectively.
4. The steps (2-3) are repeated till all the values of $(T_{w1})_{i,j}^{k+1}$, $(T_{w2})_{i,j}^{k+1}$, $(T_a)_{i,j}^{k+1}$, $(T_b)_{i,j}^{k+1}$ and $(T_c)_{i,j}^{k+1}$ are obtained up to the desired accuracy.
5. The whole procedure is repeated by increasing the time step.

The convergence and the grid independence of the solution have been checked by varying the number of space grids and size of the time steps. It has been observed that the space grids 50x50 along with time steps 50 give the grid independence for the solution.