

$$\text{let } \mathbf{u} = \begin{bmatrix} u & v & w \end{bmatrix}$$

expanding the expression that appear in the user guide

$$c\nabla\mathbf{u} = \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{bmatrix} \begin{bmatrix} \nabla u \\ \nabla v \\ \nabla w \end{bmatrix}$$

$$\begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{bmatrix} \begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} [c_{11}\nabla u + c_{12}\nabla v + c_{13}\nabla w] \\ [c_{21}\nabla u + c_{22}\nabla v + c_{23}\nabla w] \\ [c_{31}\nabla u + c_{32}\nabla v + c_{33}\nabla w] \end{bmatrix}$$

expanding one row i

$$[c_{i1}\nabla u + c_{i2}\nabla v + c_{i3}\nabla w]$$

$$\begin{bmatrix} c_{i11} & c_{i12} & c_{i13} \\ c_{i21} & c_{i22} & c_{i23} \\ c_{i31} & c_{i32} & c_{i33} \end{bmatrix} \begin{bmatrix} \frac{\partial u}{\partial x} \\ \frac{\partial u}{\partial y} \\ \frac{\partial u}{\partial z} \end{bmatrix} + \begin{bmatrix} c_{i21} & c_{i22} & c_{i23} \\ c_{i231} & c_{i232} & c_{i233} \end{bmatrix} \begin{bmatrix} \frac{\partial v}{\partial x} \\ \frac{\partial v}{\partial y} \\ \frac{\partial v}{\partial z} \end{bmatrix} + \begin{bmatrix} c_{i31} & c_{i32} & c_{i33} \\ c_{i321} & c_{i322} & c_{i323} \\ c_{i331} & c_{i332} & c_{i333} \end{bmatrix}$$

$$\begin{bmatrix} [c_{i11} \frac{\partial u}{\partial x} + c_{i12} \frac{\partial u}{\partial y} + c_{i13} \frac{\partial u}{\partial z}] \\ [c_{i21} \frac{\partial u}{\partial x} + c_{i22} \frac{\partial u}{\partial y} + c_{i23} \frac{\partial u}{\partial z}] \\ [c_{i31} \frac{\partial u}{\partial x} + c_{i32} \frac{\partial u}{\partial y} + c_{i33} \frac{\partial u}{\partial z}] \end{bmatrix} + \begin{bmatrix} [c_{i11} \frac{\partial v}{\partial x} + c_{i12} \frac{\partial v}{\partial y} + c_{i13} \frac{\partial v}{\partial z}] \\ [c_{i21} \frac{\partial v}{\partial x} + c_{i22} \frac{\partial v}{\partial y} + c_{i23} \frac{\partial v}{\partial z}] \\ [c_{i31} \frac{\partial v}{\partial x} + c_{i32} \frac{\partial v}{\partial y} + c_{i33} \frac{\partial v}{\partial z}] \end{bmatrix} + \begin{bmatrix} [c_{i11} \frac{\partial w}{\partial x} + c_{i12} \frac{\partial w}{\partial y} + c_{i13} \frac{\partial w}{\partial z}] \\ [c_{i21} \frac{\partial w}{\partial x} + c_{i22} \frac{\partial w}{\partial y} + c_{i23} \frac{\partial w}{\partial z}] \\ [c_{i31} \frac{\partial w}{\partial x} + c_{i32} \frac{\partial w}{\partial y} + c_{i33} \frac{\partial w}{\partial z}] \end{bmatrix}$$

evaluating $\nabla \cdot [c_{i1}\nabla u + c_{i2}\nabla v + c_{i3}\nabla w]$

$$\begin{bmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{bmatrix} \cdot [c_{i1}\nabla u + c_{i2}\nabla v + c_{i3}\nabla w] =$$

$$\begin{bmatrix} [c_{i11} \frac{\partial^2 u}{\partial x \partial x} + c_{i12} \frac{\partial^2 u}{\partial y \partial x} + c_{i13} \frac{\partial^2 u}{\partial z \partial x}] \\ [c_{i21} \frac{\partial^2 u}{\partial x \partial y} + c_{i22} \frac{\partial^2 u}{\partial y \partial y} + c_{i23} \frac{\partial^2 u}{\partial z \partial y}] \\ [c_{i31} \frac{\partial^2 u}{\partial x \partial z} + c_{i32} \frac{\partial^2 u}{\partial y \partial z} + c_{i33} \frac{\partial^2 u}{\partial z \partial z}] \end{bmatrix} + \begin{bmatrix} [c_{i11} \frac{\partial^2 v}{\partial x \partial x} + c_{i12} \frac{\partial^2 v}{\partial y \partial x} + c_{i13} \frac{\partial^2 v}{\partial z \partial x}] \\ [c_{i21} \frac{\partial^2 v}{\partial x \partial y} + c_{i22} \frac{\partial^2 v}{\partial y \partial y} + c_{i23} \frac{\partial^2 v}{\partial z \partial y}] \\ [c_{i31} \frac{\partial^2 v}{\partial x \partial z} + c_{i32} \frac{\partial^2 v}{\partial y \partial z} + c_{i33} \frac{\partial^2 v}{\partial z \partial z}] \end{bmatrix} + \begin{bmatrix} [c_{i11} \frac{\partial^2 w}{\partial x \partial x} + c_{i12} \frac{\partial^2 w}{\partial y \partial x} + c_{i13} \frac{\partial^2 w}{\partial z \partial x}] \\ [c_{i21} \frac{\partial^2 w}{\partial x \partial y} + c_{i22} \frac{\partial^2 w}{\partial y \partial y} + c_{i23} \frac{\partial^2 w}{\partial z \partial y}] \\ [c_{i31} \frac{\partial^2 w}{\partial x \partial z} + c_{i32} \frac{\partial^2 w}{\partial y \partial z} + c_{i33} \frac{\partial^2 w}{\partial z \partial z}] \end{bmatrix}$$