Analytical, COMSOL Simulation Optimization of Vibration-Based Energy Harvesters for EV Applications

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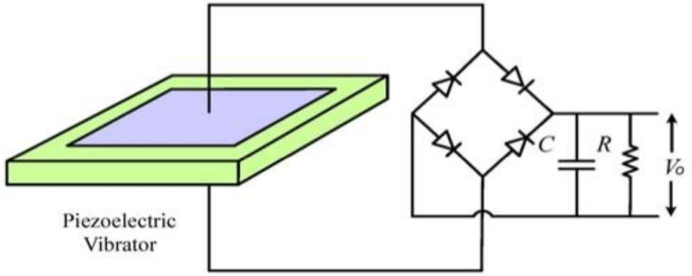
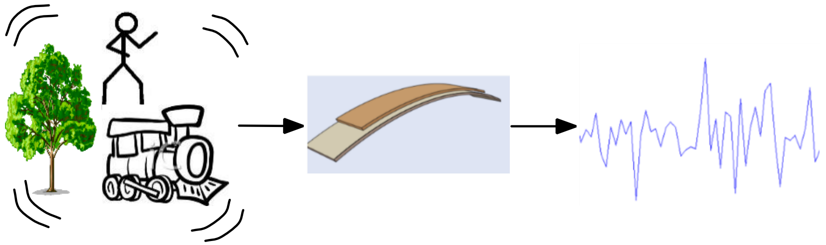
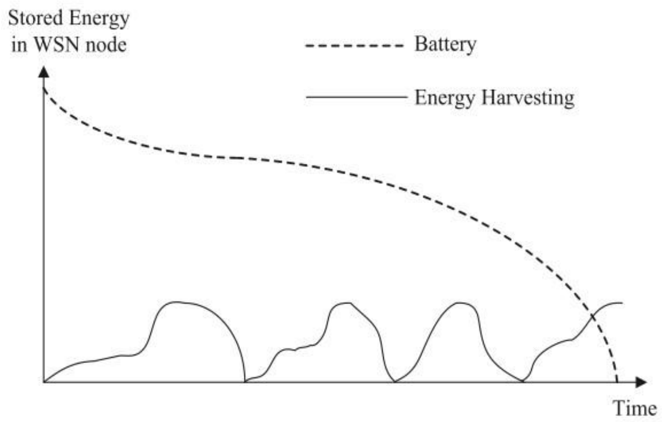
Definition- converts Ambient vibration Energy into useful electrical energy to charge small electronic devices when used at the micro or Nanoscale and charge Electric vehicles if used at the Macro Scale.

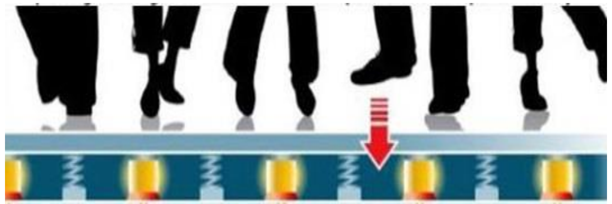
Application-wireless sensors, IOT, Electric vehicle range improvement, small Medical and electronic devices, soldiers' boots, piezoelectric dance floor etc.

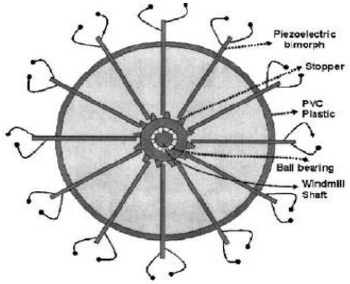


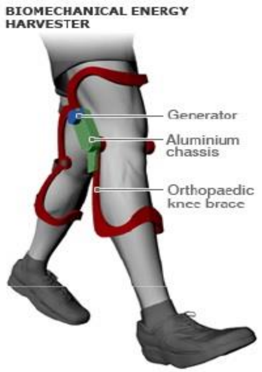


EH Technology	Setting	Output $\frac{\mu W}{cm^2}$
Solar (15%)	Outdoor	15000
	Indoor	10
Thermal (5k)	Human	20
	Machine	5000
Vibration (Walking)	Human	4
	Machine	200









Objective of Work

- Development of an analytical model of energy harvesters with different geometric shapes
- Using COMSOL simulation for these geometric Shape for validation
- Optimize the power and voltage of Harvesters

Mathematical Modelling

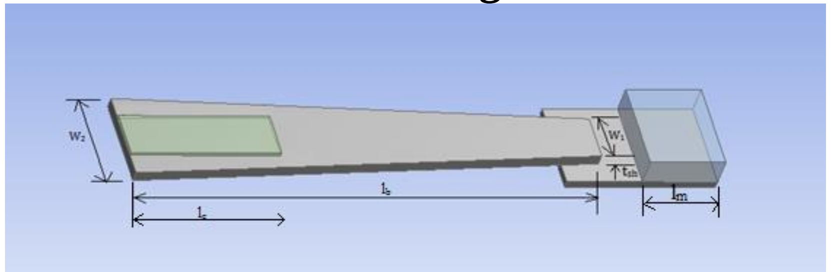


Figure 1. Tapered beam with unimorph harvesters.

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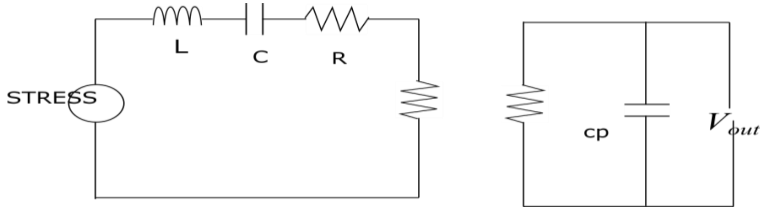


Figure 2. Circuit representation of the piezoelectric generator.

The moment of inertia of this beam can be written as

$$I(x) = \left[\frac{wt_c^3}{12} + wt_p b^2\right] + \eta_s\left(\left(\frac{w_1 - w_2}{L}\right)x + w_2\right)\frac{t_{sh}^3}{12} \qquad (4)$$

The piezoelectric equations (1) and (2).

$$\{S\} = [s^E]\{T\} + [d^T]\{E\} \qquad (1)$$

$$\{D\} = [d]\{T\} + [\varepsilon^T]\{E\} \qquad (2)$$

{S}=6-D strain vector,{T}=stress,{D}=3-D electric displacement

{E}=electric field, $[s^E]$ = (6×6) compliance matrix

[d] = (3×6) matrix of piezoelectric strain components

$[\varepsilon^T]$ = (3×3) dielectric constant matrix

The system equations are derived through the application of Kirchhoff's voltage law. (KVL) and and Kirchhoff's current law (KCL).

$$\sigma_{in} = L_m\ddot{S} + R_b\dot{S} + \frac{S}{C_k} + \mathrm{nV}\,,\, i = C_b\dot{\mathrm{V}} \qquad (3)$$

The stress value σ can be determined from

$$\sigma = \frac{1}{l_e}\int_0^{l_e}\frac{\mathrm{M(x)b}}{\mathrm{I}}\mathrm{dx} \qquad (5)$$

Mathematical Modelling

The moment equations can be written as

$$M(x) = m(\ddot{y} + \ddot{z})\left(l_b + l_e + \frac{1}{2}l_m - x\right) \qquad (6)$$

After substituting the equation (6) into (5) we get

$$\sigma = F\left(\frac{b}{l_e}\left[\frac{B'}{O'} + \frac{N'}{O'^2}\right]\left[ln\left(\frac{O'l_e + N'}{N'}\right)\right] - \frac{b}{O'}\right) \qquad (7)$$

Here,

$$k_1 = \frac{b}{l_e}\left[\frac{B'}{O'} + \frac{N'}{O'^2}\right]\left[ln\left(\frac{O'l_e + N'}{N'}\right)\right] - \frac{b}{O'} \qquad (8)$$

Where

$$B' = \left(l_b + l_e + \frac{l_m}{2}\right), O' = \left(\left(\frac{w_1 - w_2}{L}\right)\left(\frac{t_c^3}{6} + 2t_p b^2 + \eta_s \frac{t_{sh}^3}{12}\right)\right),$$

$$N' = w_2\left(\frac{t_c^3}{6} + 2t_p b^2 + \frac{\eta_s t_{sh}^3}{12}\right)$$

Consider the standard beam equation for k_2.

$$\frac{d^2 z}{dx^2} = \frac{1}{c_p I}\, m(\ddot{y} + \ddot{z})\left(l_b + l_e + \frac{1}{2}l_m - x\right) \qquad (9)$$

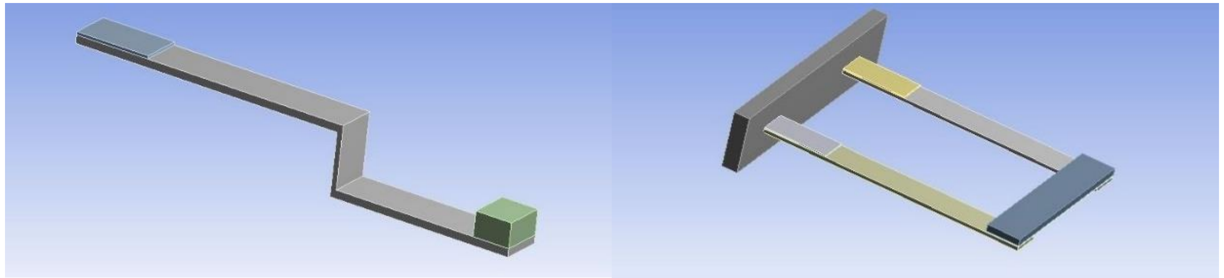
Z= $k_2 S$

$$k_2 = k_1\left[\left(\frac{B'}{O'^2} + \frac{N'}{O'^3}\right)\left((O'(l_b+l_e) + N')(ln(O'(l_b+l_e) + N') - 1)\right) - \frac{N'}{O'^2} - \frac{l_b^2}{2O'} + 0.783(l_b+l_e) + 13.34\right] \qquad (10)$$

Expression for Voltage and power can be written as per [15]

$$V = \frac{j\omega \frac{2c_p d_{31} t_c}{a\varepsilon}}{j\omega\left(\omega^2 k_{31}^2 + \frac{2\zeta\omega}{RC_b}\right) - 2\zeta\omega^3}\frac{A_{in}}{k_2},$$

$$P = \frac{1}{2\omega^2}\frac{C_b^2\left(\frac{2c_p d_{31} t_c}{k_2 a\varepsilon}\right)^2 A_{in}^2}{(4\zeta^2 + k_{31}^4)(RC_b\omega)^2 + 4\zeta k_{31}^2(RC_b\omega) + 4\zeta^2}$$



$$k_1 = \frac{b}{2I}[3l_b + l_m], k_2 = \frac{\left(19l_b^3 - 6l_m l_b^2 - \frac{3l_b l_m^2}{2}\right)}{9b(l_m + 4l_b - l_e)} \quad k_1 = \frac{b(2l_b + l_m - l_e)}{I}, k_2 = \frac{l_b^2}{3b}\frac{\left(2l_b + \frac{3}{2}l_m\right)}{(2l_b + l_m - l_e)}$$

COMSOL Simulation

During analytical and COMSOL Simulation, the Table parameters taken into consideration

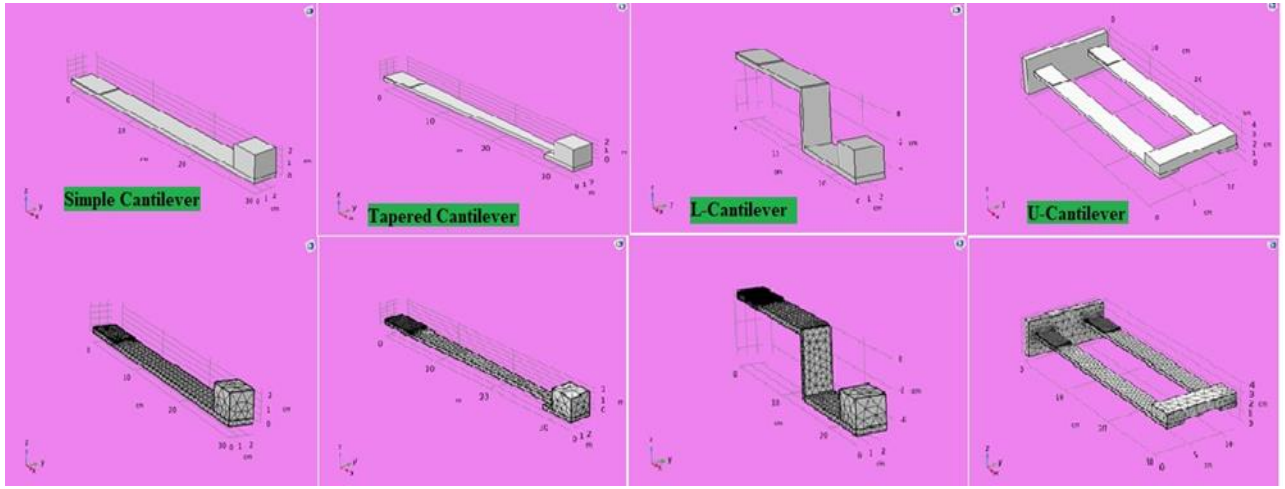


Figure 3. Energy harvesters with different geometric shapes.

t_{sh}(mm)	t_c(mm)	E_s(Gpa)	E_p(Gpa)	mass(gm)	d_{31}(m/V)	$\mathcal{E}_{33}$(F/m)
4	1	69	66	60	180 e^{-12}	1800 $\mathcal{E}_0$

Mode	Mode 1		Mode 2		Mode 3	
	Analytical	COMSOL	Analytical	COMSOL	Analytical	COMSOL
Simple Cantilever	43.074	42.75	230.05	230.95	323.31	321.75
Tapered Cantilever	41.226	34.686	246.14	243.22	302.01	307.43
L-shape	39.116	37.358	123.65	128.67	246.0	241.37
U-shape	34.48	29.68	201.12	199.24	433.6	431.13

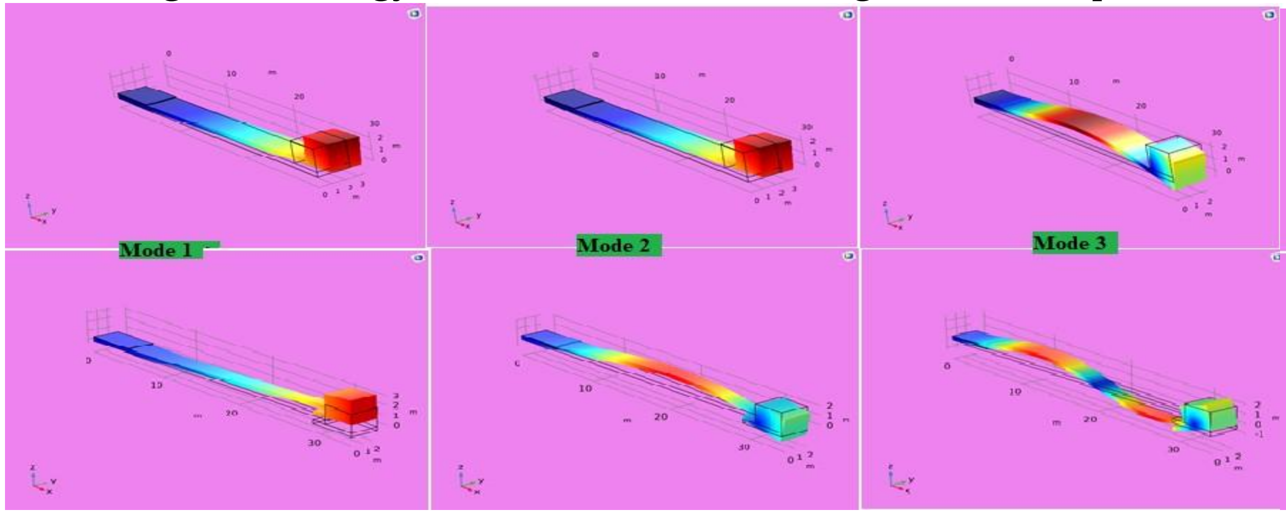


Figure 4. Mode shape for simple and tapered harvesters.

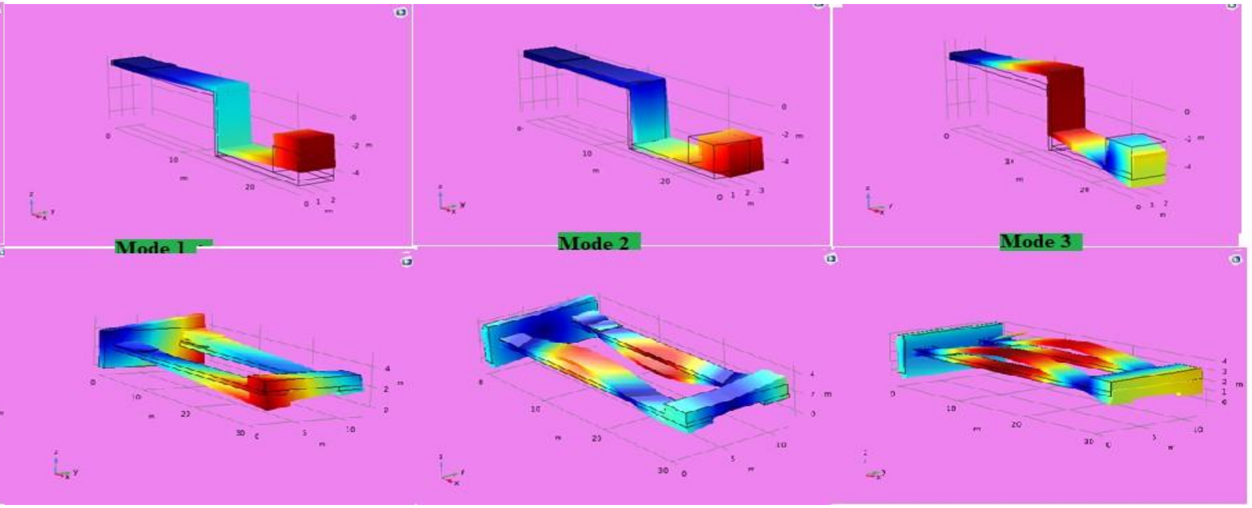


Figure 5. Mode shape for L-shape and U-shape harvesters.

Results and Discussion

Configuration	Simple Cantilever		Tapered Cantilever		L-shape Cantilever		U-shape Cantilever	
	V	P (mW)	(V)	(mW)	Voltage	P (mW)	Voltage	P (mW)
Analytical	8.4	4.2	12.6	12.5	14.6	15.5	20.4	23
COMSOL	8.6	4.21	13.46	11.2	14.8	13.8	22.4	23.2
Relative Error	2.3 %	0.238%	6.825%	10.4 %	1.36%	10.96	9.80	0.869%

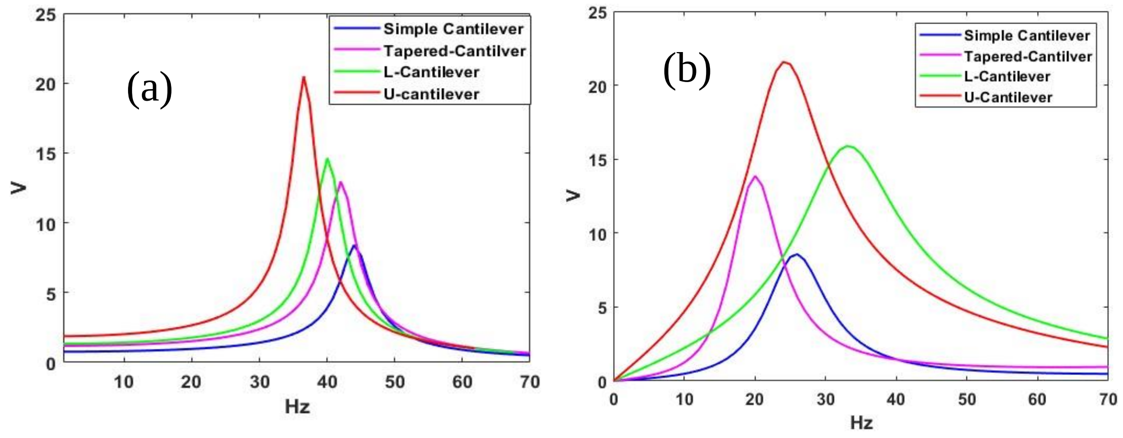


Figure 6. Voltage variation with frequency (a) Analytical (b) COMSOL

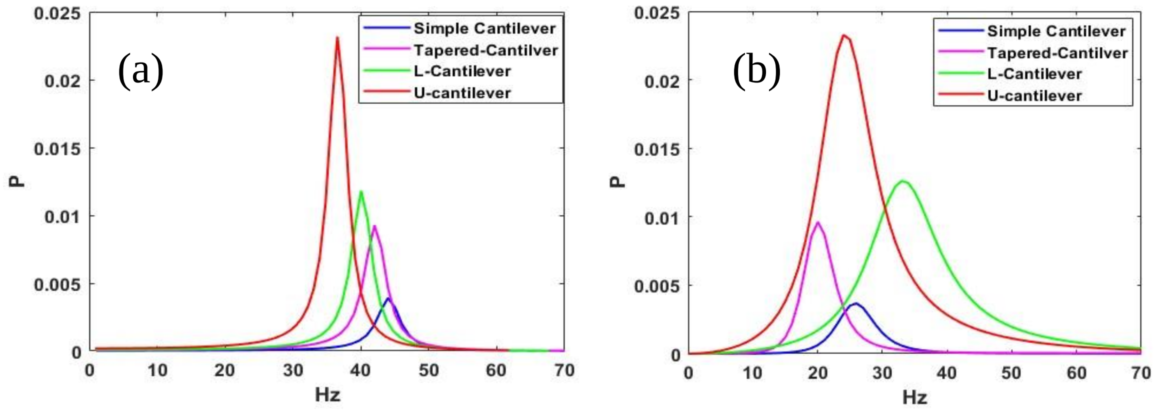


Figure 7. Power Variation with frequency (a) Analytical (b) COMSOL

- ❖ Voltage and power response for the analytical method are minimal for The simple shape cantilever 8.4 V and 4.2 mW, respectively.
- ❖ U-shape Harvesters generate more voltage and power, about 22.4 V using COMSOL but 20.4 V in analytical.
- ❖ Voltage response gives less relative error for the L-shape while more for the U-shape when compared with COMSOL.
- ❖ Power response gives less relative error for simple shape and U-shape Harvesters, while it gives more relative error for Tapered/L-shape with their COMSOL simulation.
- ❖ Mode shape using analytical and COMSOL simulations is validated with each other.

Conclusion

- ❖ Analytical models of Energy harvesters with Different geometric shapes are derived.
- ❖ COMSOL simulation of simple, Tapered, L-shape, and U-shape harvesters is built.
- ❖ The first three mode shapes of harvesters for all shapes are validated with each other.
- ❖ The power (23.2 mW) and voltage (22.4 V) response by the U-shape harvesters is maximum among all the harvesters.
- ❖ Analytically obtained results are well validated with COMSOL Results with a maximum of 10.96 relative error.

Thank You