# A study of the Effects of Mounting Supports, and Dissipation on a Piezoelectric Quartz Double –Ended Tuning Fork

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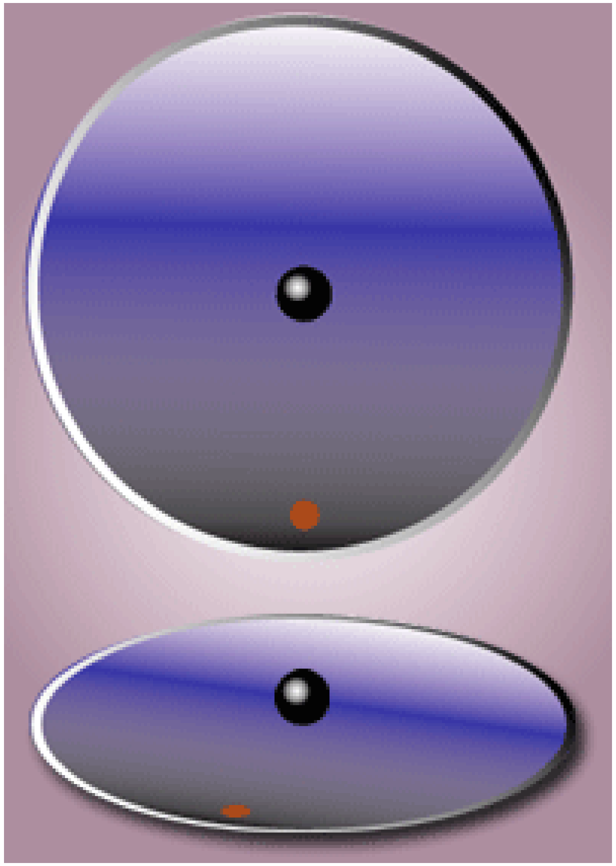
# Gyroscope

The gyroscope is a device for measuring the changes in angular momentum, and it can detect angular velocity from the Coriolis force applied to the sensor.

- *Mechanical Gyroscope*
- *MEMS Gyroscope*
- *Optical Gyroscope*

# Coriolis force



The Coriolis force $F_c$ is a fictitious force that only appears in a rotating non-inertial reference frame.

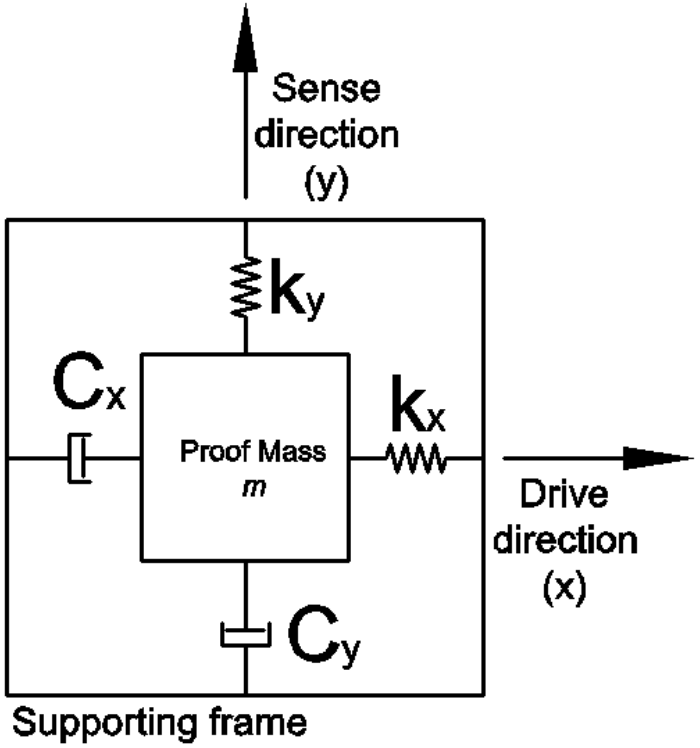
$$\mathbf{F}_c = m * \mathbf{a}_c = 2^* m^* \boldsymbol{\Omega} \times \mathbf{v}$$

$m$ : Proof mass
$a_c$ : Coriolis acceleration
$\Omega$ : Angular velocity vector of the rotating frame
$v$ : Velocity vector of the mass in the reference frame.

Due to the cross product of $v$ and $\Omega$, $\mathbf{F}_c = 0$ when the velocity vector $\mathbf{v}$ is parallel to the angular velocity vector $\boldsymbol{\Omega}$.

Ref. [1]:http://upload.wikimedia.org/wikipedia/commons/b/b6/Corioliskraftanimation.gif

# *Equation of the motion of simple z-axis vibratory gyroscope*



Simplified two-degree of freedom (2-d.o.f) equations of motion for z-axis gyroscope.

$$m\, \ddot{u} + C_x\, \acute{u} + k_x u = \text{Ʈ}_x$$

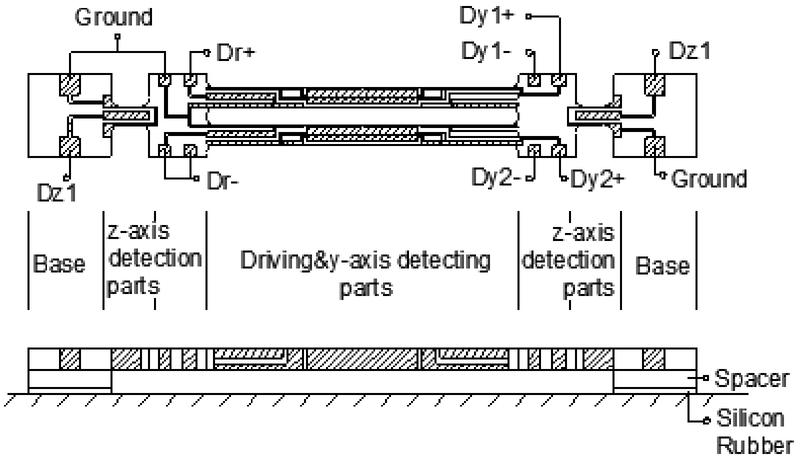
$$m\ddot{u} + C_y\, \acute{u} + k_y v = \text{Ʈ}_y - 2m\Omega_z\, \acute{u}$$

$\text{Ʈ}_x$ : External harmonic excitation force in the driving direction

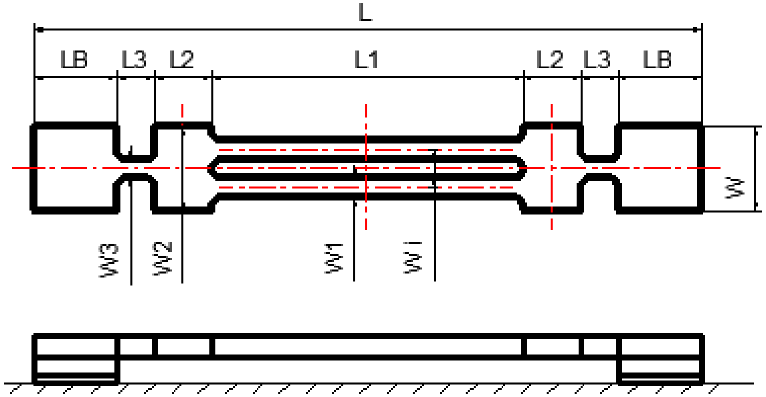
$\text{Ʈ}_{y:}$ External harmonic excitation force in the detecting direction

$2m\Omega_z\, \acute{u}$ : Coriolis force induced by the cross project of the angular velocity $\mathbf{\Omega}_z$ rotation about the z-axis and momentum $m\ddot{u}$ in the x-direction

# Structure of the Piezoelectric Double-Ended gyroscope



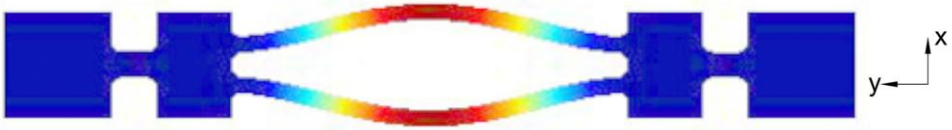
a) Electrode arrangements of gyroscope



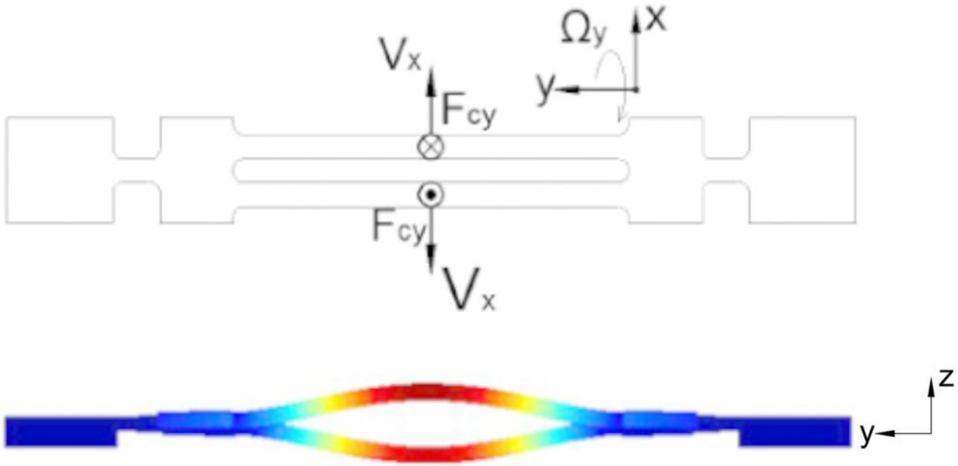
b) Dimension of the gyroscope

| Items | Dimension (*mm*) |
|---|---|
| L | 16.0 |
| $L_1$ | 7.46 |
| $L_2$ | 1.38 |
| $L_3$ | 0.89 |
| $L_B$ | 2.00 |
| t | 0.30 |
| W | 2.00 |
| $W_1$ | 0.31 |
| $W_2$ | 1.60 |
| $W_3$ | 0.43 |
| $W_i$ | 0.75 |
| Spacer, $H_1$ | 0.28 |
| S. Rubber, $H_2$ | 0.03 |

# *Operation principle of the gyroscope 1*



(a) Driving Mode



(b) y-axis detection mode

The driving arms are vibrating in the horizontal x-y plane.

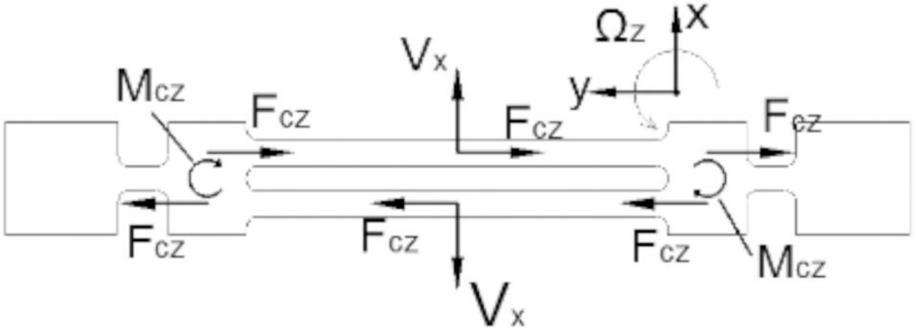
When the angular velocity $\Omega_y$ of the y-axis rotation is applied to the resonator, the Coriolis forces $F_y$ are generated in the direction of the z-axis.

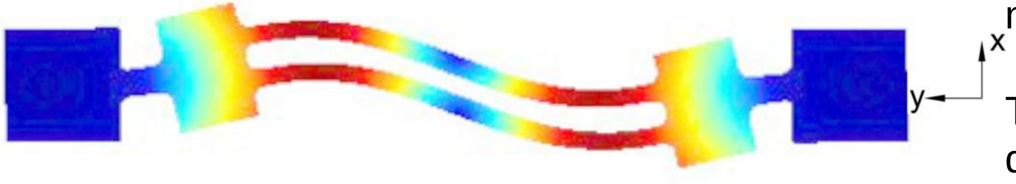
Therefore, the each arm of the resonator is displaced in opposite direction.

The Coriolis force $F_{cy}(y)$ is detected by the electrodes at the driving-detecting part

# Operation principle of the gyroscope 2

(a) Driving Mode





(c) z−axis detection mode

The driving arms are vibrating in the horizontal x-y plane.

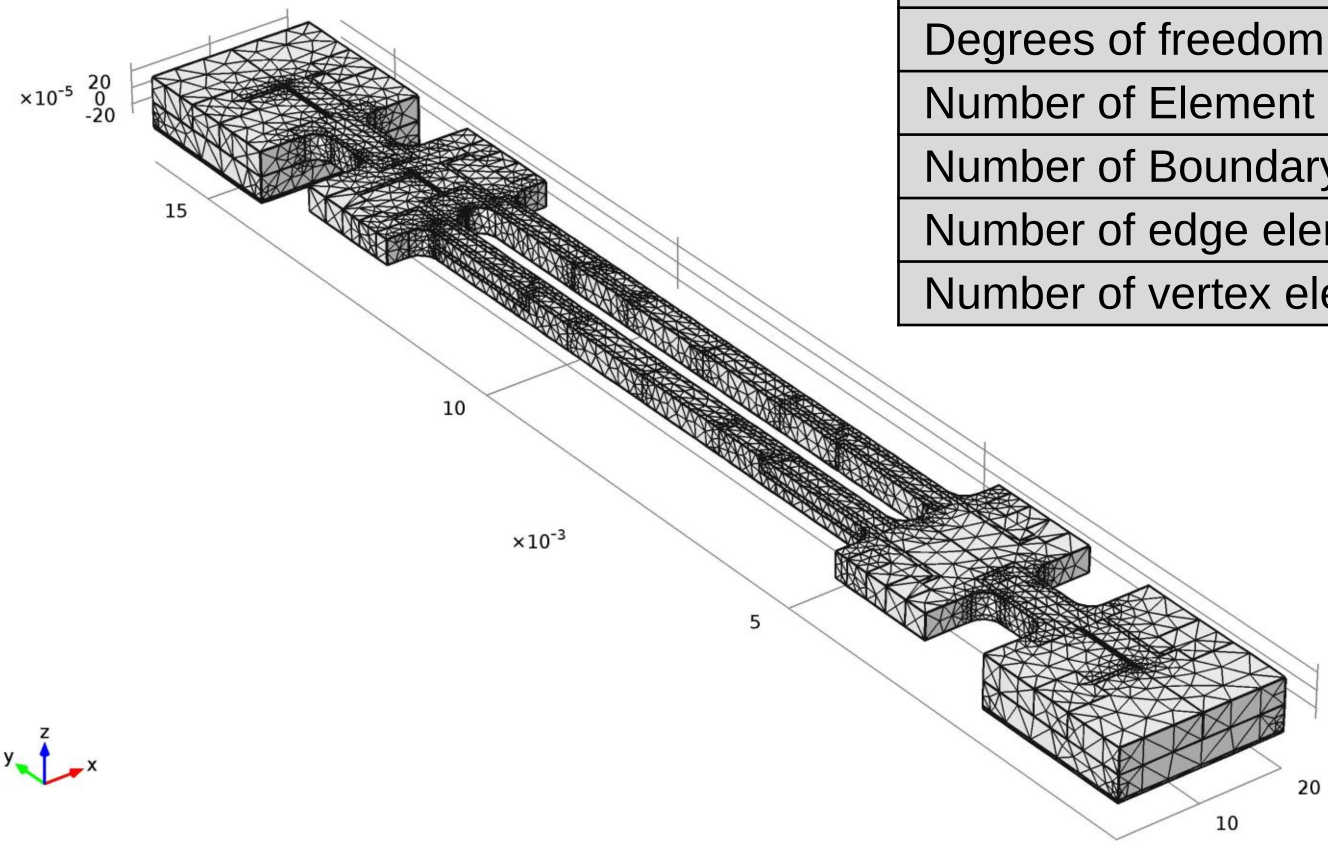
When the z-axis angular velocity $\Omega_z$ is applied to the gyroscope, a pair of Coriolis forces $F_{cz}$ is produced.

The pair of Coriolis forces $F_{cz}$ act on the center arms in the opposite direction and generate a moment $M_{cz}$ at the base of the gyroscope.

The moment $M_{cz}$ generates the asymmetric mode in the horizontal x-y plane.

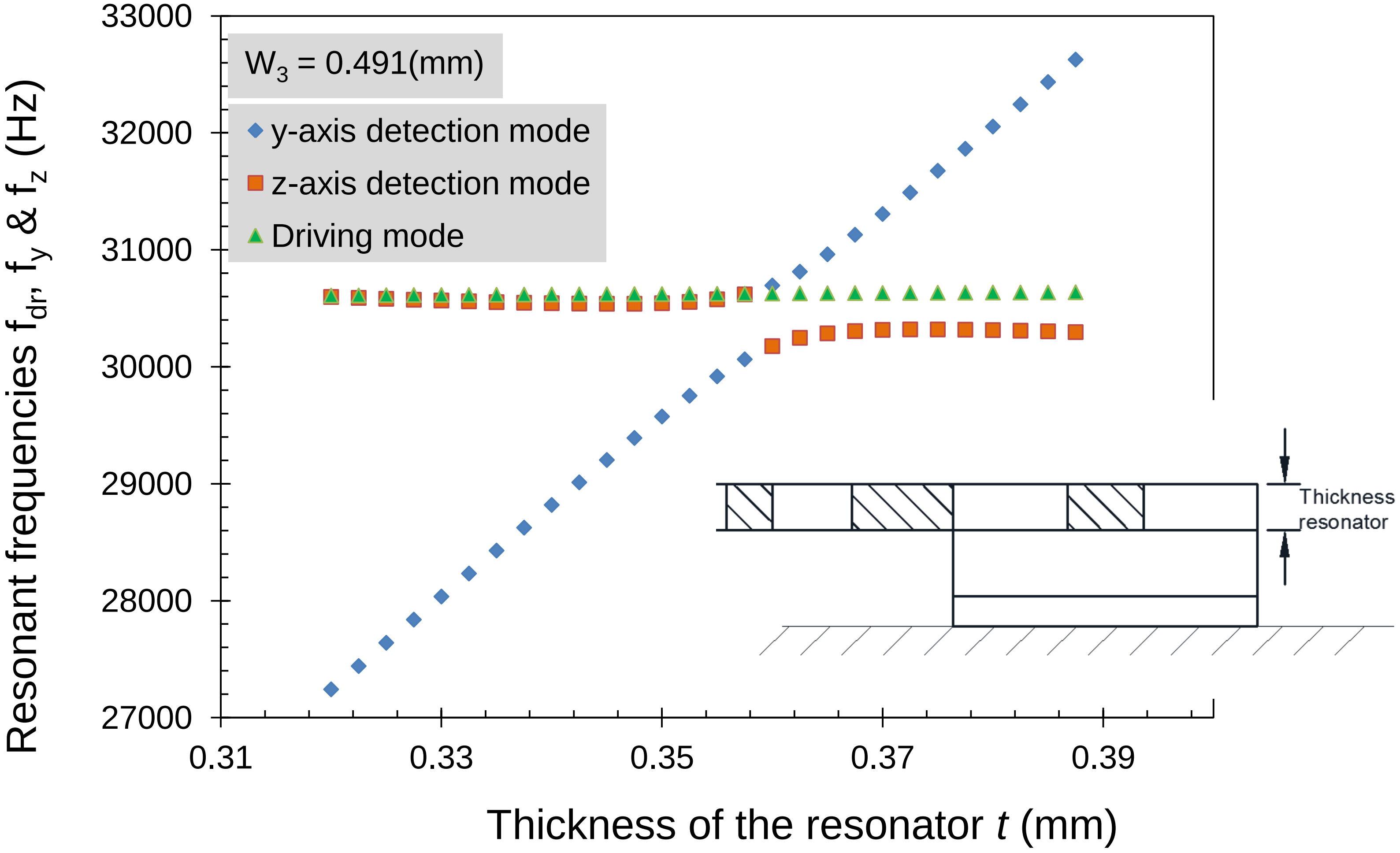
The z-axis detecting electrodes on the arm L3 detect the Coriolis force $F_{cz}$.

# *Elements of the gyroscope for COMSOL simulation*

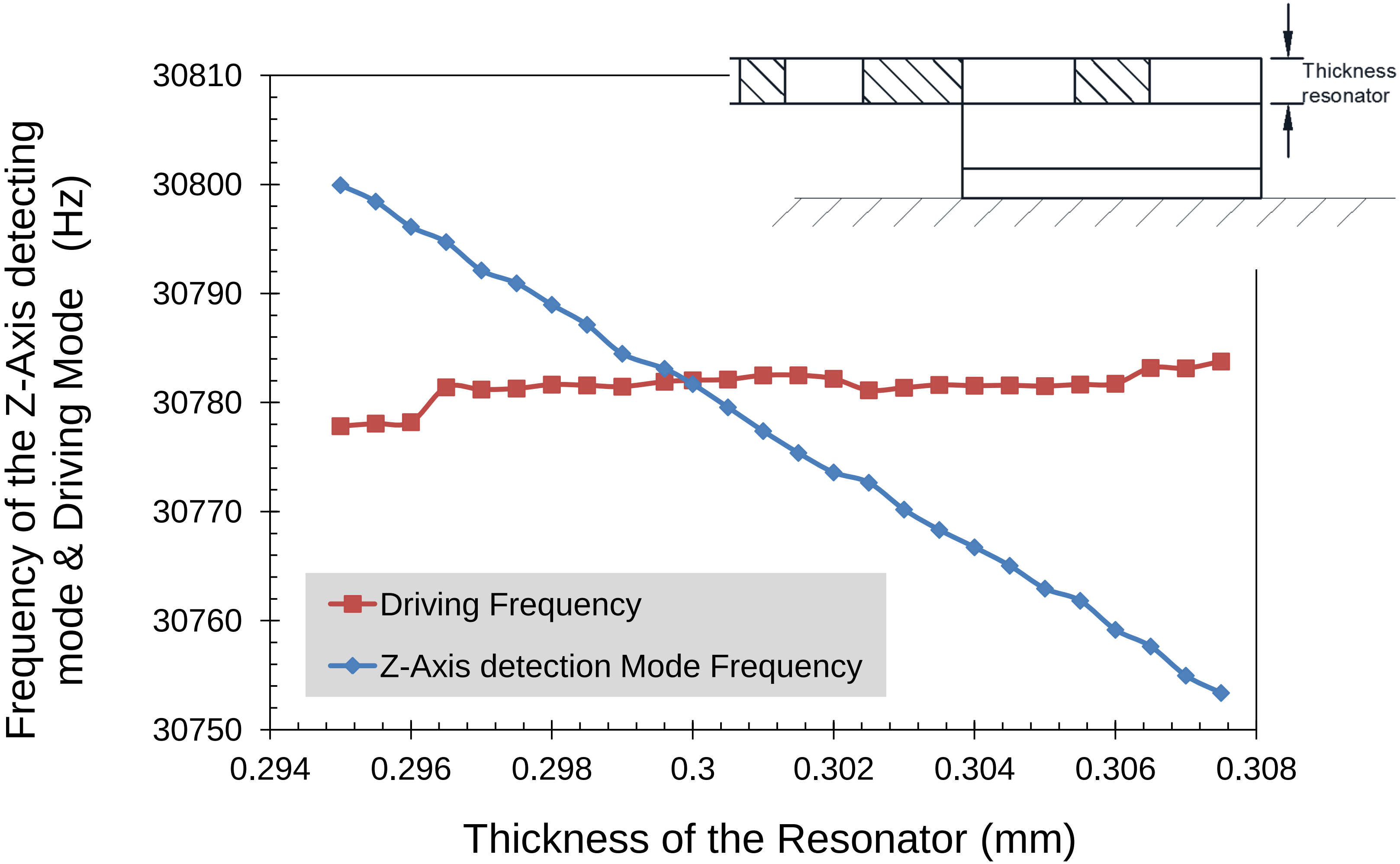


| | Value |
|---|---:|
| Degrees of freedom | 321757 |
| Number of Element | 50594 |
| Number of Boundary element | 15149 |
| Number of edge element | 4158 |
| Number of vertex elements | 656 |

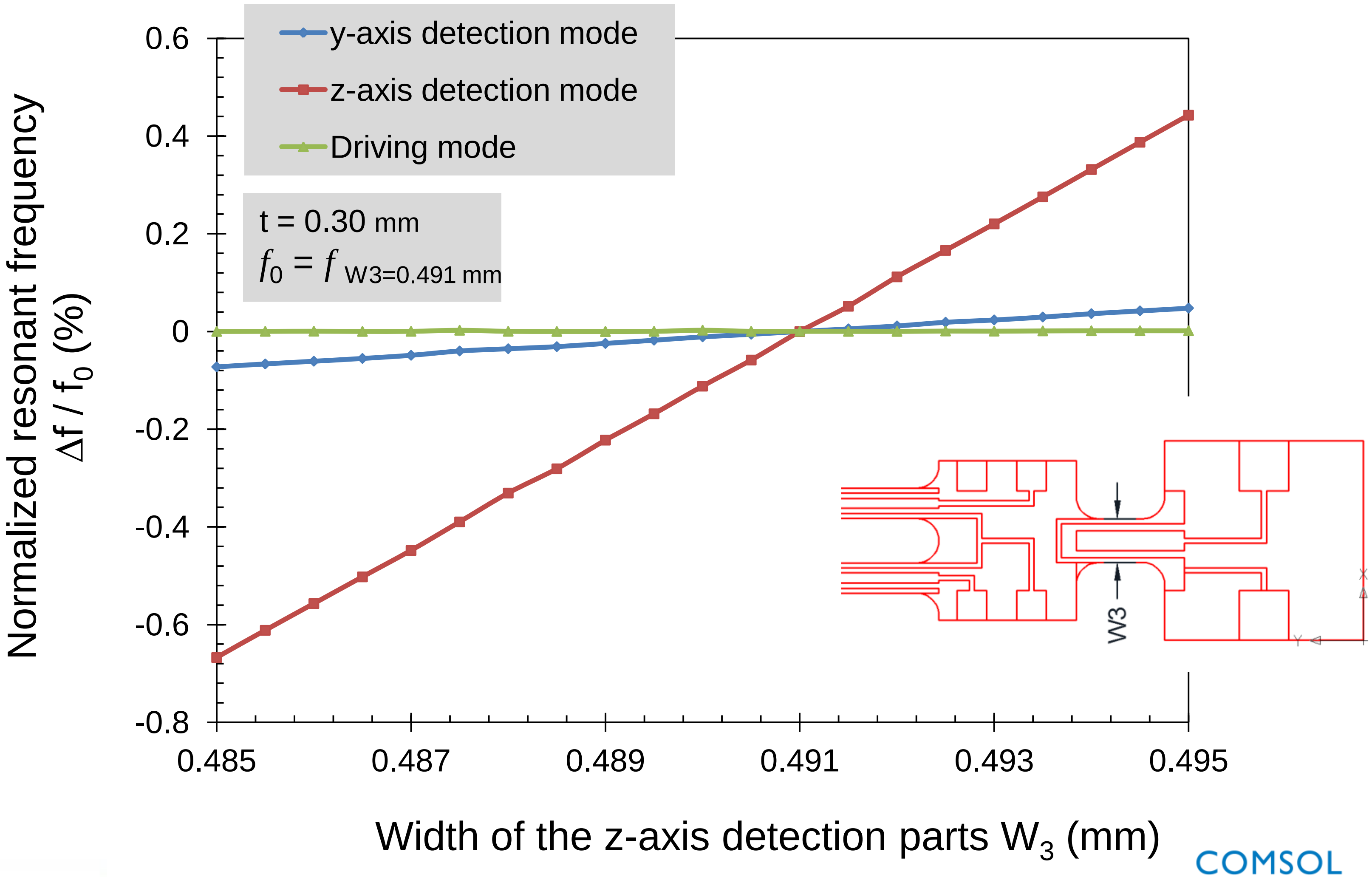
# Change in resonant frequencies for thickness t of the resonator



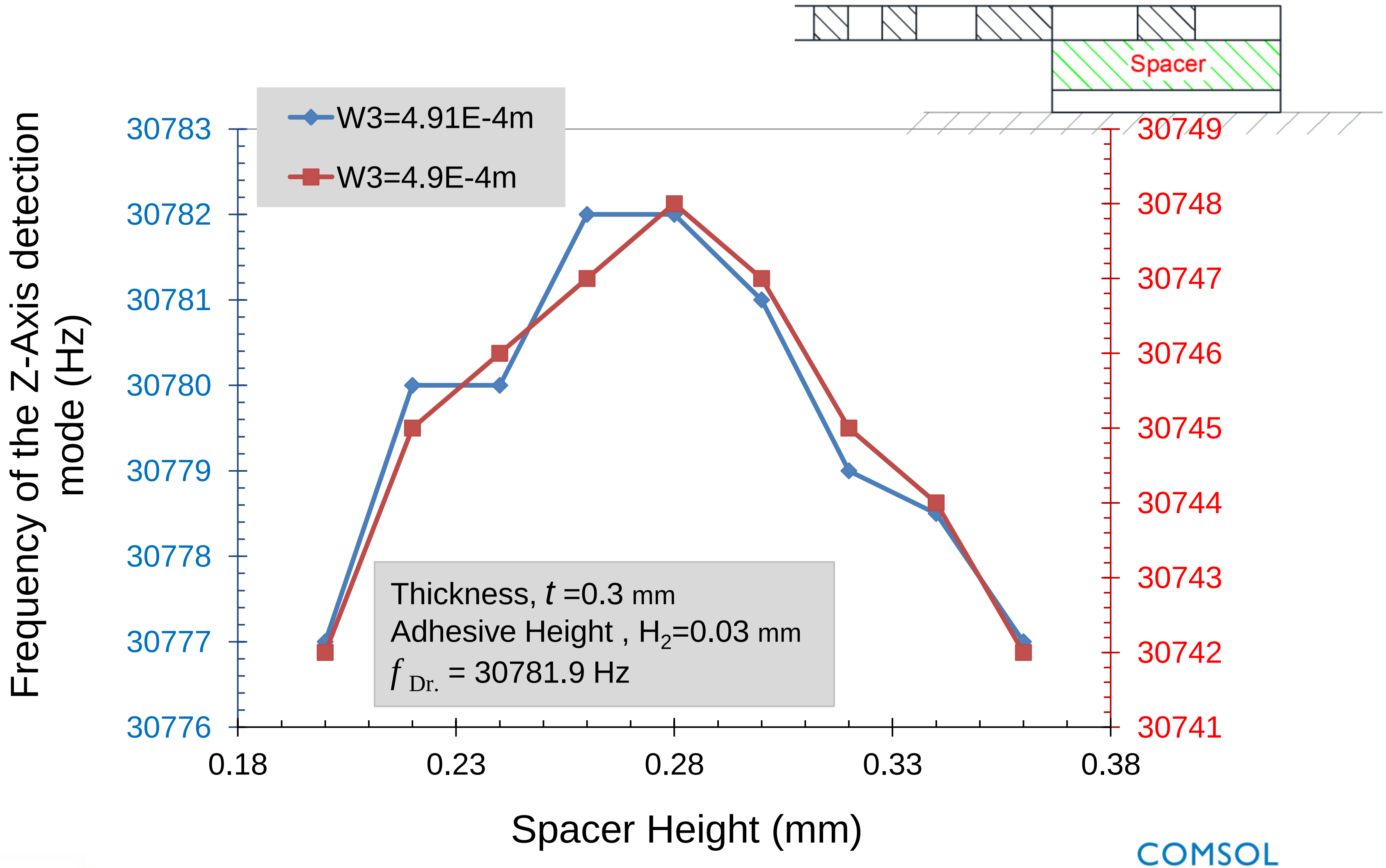
# *Change in resonant frequencies of the z-axis detection mode for thickness of the gyroscope*



# *Change in resonant frequencies for width $W_3$ of the resonator*

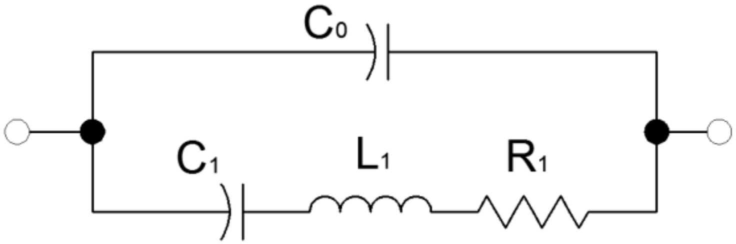


# Change in resonant frequency of the z-axis detection mode as function of height of spacer



Thickness, $t$ =0.3 mm
Adhesive Height , $H_2$=0.03 mm
$f_{\mathrm{Dr.}}$ = 30781.9 Hz

# Equivalent circuit parameter of the Butterworth van Dyke Electrical resonator



Equivalent parameters C1,L1, and R1 by Eigenvalue analysis [2]

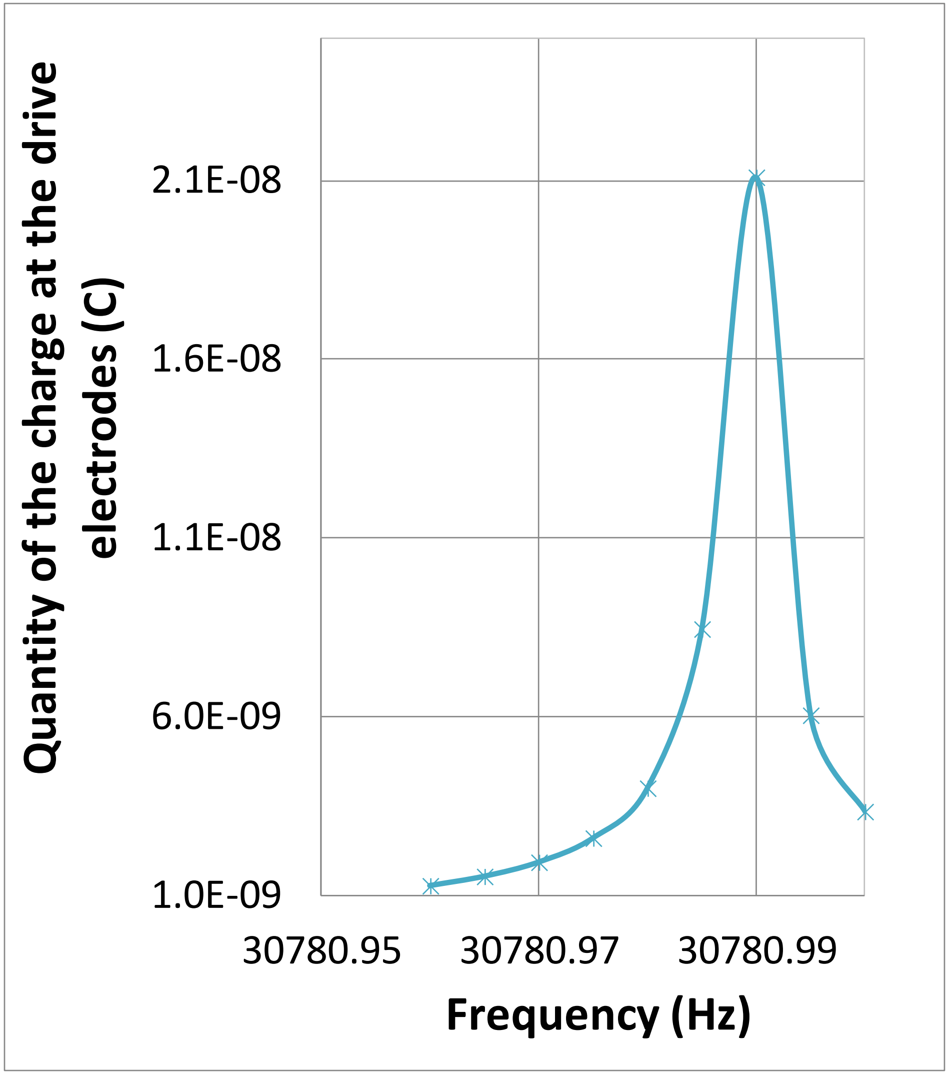
| Frequency (Hz) | 30781 |
|---|---|
| $C_1$ (fF) | 1.04 |
| $R_1$ (Ω) | 3332 |
| $L_1$ (H) | 25838 |
| $C_0$ (pF) | 184 |

- $C_1 = q^2 / E$
- $L_1 = 1 / [C_1 * f_r^2]$
- $R_1 = 1 / [Q * f_r * C_1]$
- $C_0 = q / V$

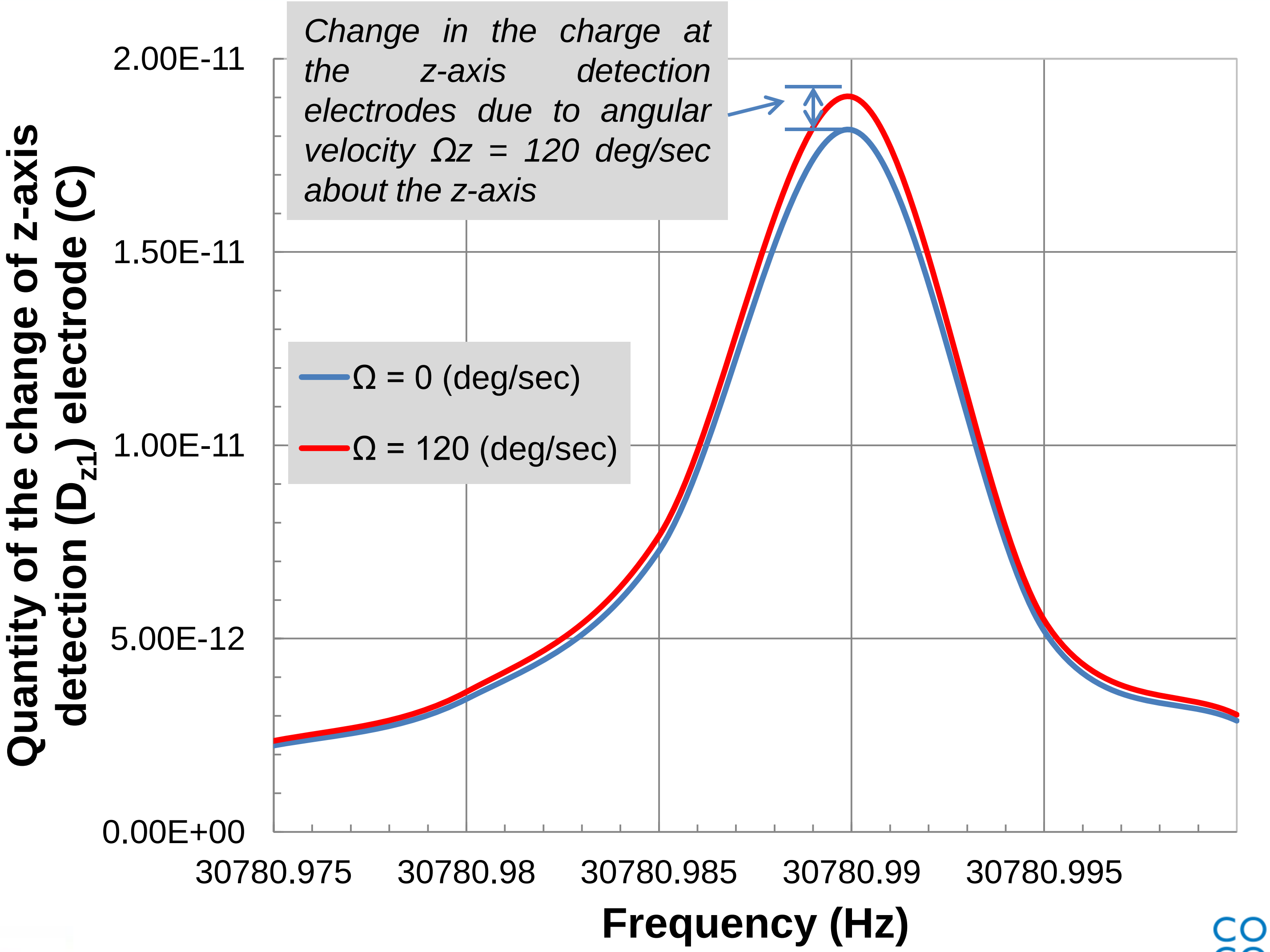
- $f_r$ (Eigenfrequency in rad/s), and Q
- $E = f_r^2 \int_V (\rho * |\underline{u}|^2)\, dV$
- $q = \mathrm{abs}\left( \int_{electrode} D_2\, dA \right)$

Ref. [2]: Motional Capacitance of Layered Piezoelectric thickness-Mode resonators”, M.Schmid, E. Benes, W.Burger and V.Kravchenko, IEEE Transactions on Ultrasonics Ferroelectrics, and Frequency Control, Vol. 38, No.2, May 1991, pp 199-206

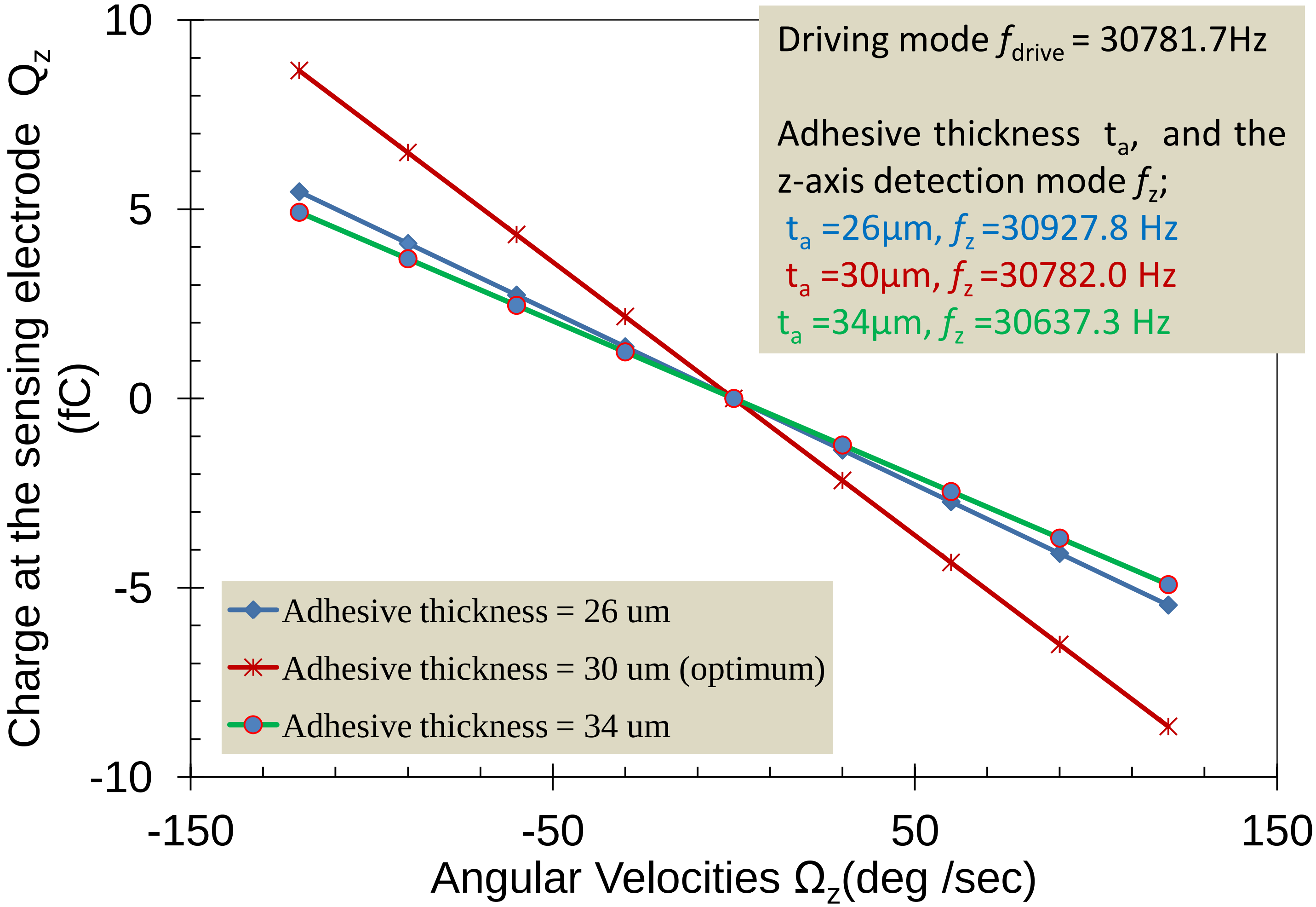
# *Frequency response of the driving mode with 1 V at the drive electrodes*



# The z-axis detection sensitivity of the gyroscope as a function adhesive thickness



# *The z-axis detection sensitivity of the gyroscope as a function of adhesive thickness*



Driving mode $f_{drive}$ = 30781.7Hz

Adhesive thickness $t_a$, and the z-axis detection mode $f_z$;

$t_a$ =26µm, $f_z$ =30927.8 Hz

$t_a$ =30µm, $f_z$ =30782.0 Hz

$t_a$ =34µm, $f_z$ =30637.3 Hz

# Conclusion

- *We can design a piezoelectric quartz double-ended tuning fork gyroscope to detect angular velocity in two axes.*

- *Mechanical dissipation in the model is needed in the COMSOL in order to calculate the electrical parameters.*

- *An optimal geometry of the gyroscope is necessary to match the frequencies for better sensitivity.*

- *Driving mode is not affected by the spacer or adhesive.*

- *Height of the spacer can be used for the fine tuning of detecting mode frequencies to match the driving mode frequency.*

- *Adhesive height (silicon rubber) is a important factor in the matching of the z-axis detection mode to driving mode. It would affect the production yields of the gyroscope.*