Bayesian Estimation of Tumours in Breasts Using Microwave Imaging

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***Abstract*— According to Canadian Cancer Society, breast cancer is the most frequently diagnosed cancer in women. Our ability to potentially detect breast cancer in an early stage has potential of significantly decreasing mortality and hence is a very important issue in healthcare. Currently, mammography has been used widely for screening women over 50 who are statistically more vulnerable. However it suffers from some limitations such as false negative and positive results, using ionizing radiation and patient's discomfort. Microwave imaging has been introduced recently to overcome drawbacks of this method. In breast microwave imaging the imaging system consists of an array of antennas which can serve both as transmitting and receiving antennas. Therefore it is possible to illuminate object of interest (breast) from multiple directions thus obtaining a full three dimensional scan whose resolution depends only on the number of antennas. One of the most difficult parts when detecting potential tumours in breasts is presence of modelling noise due to large amount of scattering, which severely deteriorates performance of estimation and detection algorithms. In this paper, we propose a parametric 3D model of breast microwave propagation, i.e., signal measured on antennas with respect to tumours' and breast parameters including electromagnetic properties and geometry (our model includes multiple tumours with arbitrary shapes.) We illustrate the applicability of our results through numerical examples.**

I. INTRODUCTION

According to Canadian Cancer Society, breast cancer is the most frequently diagnosed cancer in women with over 23,400 new cases expected in 2011 [1]. Due to the progressive nature of the disease early detection is extremely important and can significantly improve survival rates. Currently all of the clinical procedures are based on mammography which is routinely prescribed for older women who tend to be more susceptible to the disease [2]. Although mammography is extremely important diagnostic technique, it suffers from some limitations such as false negative and positive results, using ionizing radiation and patients discomfort [2], [3]. The number of false positives is rather significant in the case of so called dense breasts in which healthy tissue may be mistaken for malignant and as a consequence unnecessary biopsies are prescribed. Furthermore, complicating the matter is the fact that mammography is a two-dimensional technique in which three-dimensional images are obtained through image reconstruction from 2D projections which can also lead to false positives.

This work has been supported by National Sciences and Engineering Research Council of Canada.

Microwave imaging has been recently proposed as an additional medical imaging technique which can potentially overcome some of the shortcomings of the mammography. Essentially the technique is based on illuminating breast with electromagnetic-wave(s) in microwave range. From the physical point of view this can be represented as a wave propagation in medium (breast) that contains scatterers (both healthy and malignant tissue). Due to the fact that malignant tissue has larger conductivity the measurements obtained by receiving array of antennas will be different if the scatterers are present. Therefore once the wave propagates through the breast the received signal is analyzed in order to obtain permittivity map using appropriately selected image reconstruction technique [4]. Most of the image reconstruction techniques minimize a particular cost function (e.g. mean-square error). In most cases the number of unknowns (e.g. number of pixels in the map) is much larger than the number of available measurements which requires an additional constraint.

In this paper we propose a simplified parametric inverse 3D model which enables us detection of tumour presence and estimation of its size and/or position. Most of the existing solutions [7] employ non-parametric image reconstruction techniques. We believe that accuracy can potentially be improved by considering parametric models. In general parametric models can increase modelling noise. However we believe that appropriately defined parametric model can be useful as long as an appropriate clinical decision can be made based on reduced number of parameters. To this purpose we first develop a three-dimensional finite element model of electromagnetic wave propagation through the breast tissue. We define our model with respect to tumour size and location and assume that the permittivity of tumour can be modelled by Gaussian probability density function (pdf). We then derive probability density function of the measured data (power received by receiving antennas) and the corresponding likelihood function. We then maximize the likelihood with respect to the unknown parameters. The outline of the paper is as follows. In Section 2 we present computational models of electromagnetic wave propagation in breast when tumours are absent and present. In Section 3 we discuss how the aforementioned models were implemented using COMSOL. In Section 4 we present the statical models and present estimation algorithm. In Section 5 we present simulation results and discuss their potential use for

inverse problems. Finally, Section 6 concludes the paper.

II. PHYSICAL MODEL

In this Section we develop mathematical model describing the measured signals. The imaging system consists of several antennas which can both serve as transmitting and receiving antennas. These antennas in principle can be distributed over the breast surface thus allowing for a three-dimensional scan whose resolution depends only on number of antennas. Obviously the number of antennas is determined by their size which may be constrained by technical requirements such as antenna-to-antenna noise (interference). Once the microwave is generated it propagates through the volume of the breast according to Maxwell's equations which in this particular case can be reduced to the phasor form since microwave antennas operate in a single-frequency mode.

It should be observed that any non-homogeneity in the object can be modelled as a scatterer and thus in the presence of multiple scatterers the resulting electromagnetic field becomes very complex superposition of reflected and refracted waves. Since malignant tissue has significantly larger permittivity than healthy tissue it can also be modelled as a scatterer (see Figure 1). The reflection/refraction from scatterers can then be modelled as described in Figure 2.

In the remainder of the paper we will assume that in the absence of cancer the scattering in the breast is due only to small non-homogeneities which will be included as the modelling noise. Of course if a particular patient is submitted to continuous monitoring in a regular intervals the previous images can be used a reference signal and thus "healthy scattering" can be properly recorded and modelled.

In the absence of scatterers the propagation can be described as:

$$\nabla^2 E + k^2 E = 0 \quad (1)$$

$$\nabla^2 H + k^2 H = 0 \quad (2)$$

where we implicitly assume that the medium (breast) is source-free, E and H are intensities of electric and magnetic fields respectively, $k = \omega\sqrt{\mu\varepsilon}$ where ω is frequency, μ magnetic permittivity, and ε complex electric permittivity.

In this paper we assume that the tumours can be modelled as spheres and therefore are uniquely defined by location vector and radius. In general, arbitrarily shaped tumours can be represented by spatial Fourier transform and corresponding spatial frequency amplitudes and phases. As we stated before, the electromagnetic properties of the malignant tissue is significantly different than breast tissue and thus proper boundary conditions must be considered in order to ensure continuity (see Figure 2):

$$\vec{n} \times (\vec{E}_1 - \vec{E}_2) = 0 \quad (3)$$

$$\vec{n} \times (\vec{H}_1 - \vec{H}_2) = 0 \quad (4)$$

$$\vec{n} \times (\varepsilon_1 \vec{E}_1 - \varepsilon_2 \vec{E}_2) = 0 \quad (5)$$

$$\vec{n} \times (\mu_1 \vec{H}_1 - \mu_2 \vec{H}_2) = 0 \quad (6)$$

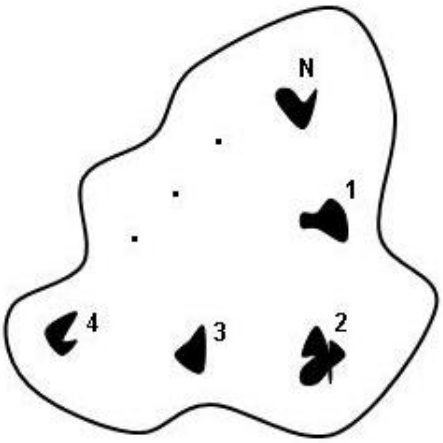

Fig. 1. Medium with multiple scatterers

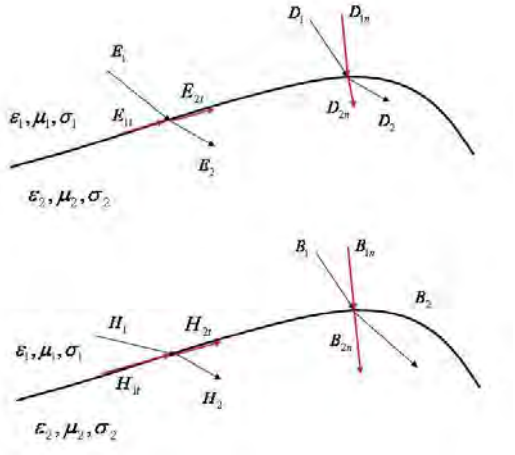

Fig. 2. Boundary conditions

III. FINITE-ELEMENT MODEL

In order to solve the above equations we utilize finite-element method by developing three-dimensional model using RF module in COMSOL Multiphysics software. In this paper we model the breast as a sphere with radius of 100mm as shown in Figure 3. One antenna which acts as a transmitter is located on one side and nine receiving antennas are distributed on the other side of the sphere. These antennas are modelled as slim cubes which are centred on the surface of the sphere. Three boundary conditions are used to send waves in the medium. Electric field is applied to transmitter antenna. Perfect electric conductor boundary condition is applied to sides of antennas in order to guide wave through them, and scattering boundary condition is applied to the rest of surfaces to let waves propagate freely. Afterward, for different studies one or multiple tumours with arbitrary shapes can be modelled. In this study tumour is considered as a sphere inside the breast with an arbitrary size and in arbitrary position. Three different studies are done using this model: effect of tumour location, tumour size

and tumour permittivity. As an example of wave propagation in simulation, in Figure 4 shows the distribution of nine receiving antennas with their indices.

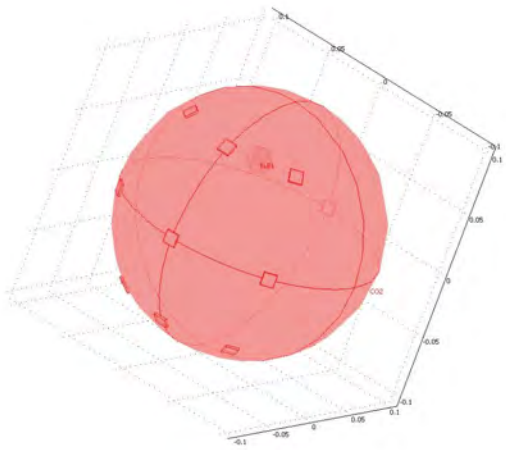

Fig. 3. Geometry of the breast

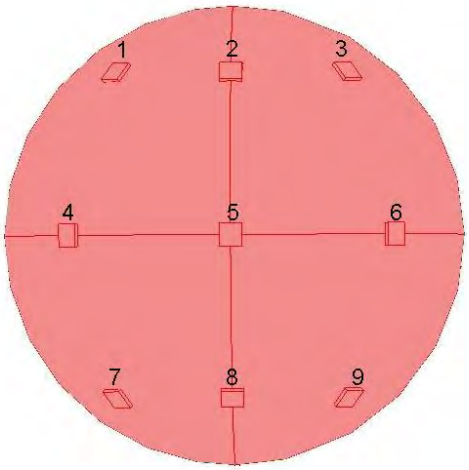

Fig. 4. Antenna positions

IV. STATISTICAL MODEL

We assume that the measured signal is given by average power and normalized to the transmitting antenna power. The measured signal when the tumour is absent $\vec{y}$ is then given by

$$\vec{y} = \vec{f}_0 + \vec{e} \tag{7}$$

where $\vec{f}_0$ is model predicted vector of measurements (power) calculated using finite-element model and $\vec{e}$ is the measurement/modelling noise.

In the presence of tumour the measured signal becomes

$$\vec{y} = \vec{f}(R_t, \vec{R}, \sigma, \varepsilon_r) + \vec{e} \tag{8}$$

where R_t is the radius of tumour, $\vec{R}$ is the position of tumour, σ and ε_r are conductivity and electric permittivities respectively. Note that in the above model we assumed a single tumour (scatterer) in order to simplify the computational cost but can be extended to account for multiple scatterers at the expense of computational time.

We assume that the conductivity and permittivity are given by Gaussian random variables with $\sigma = 4$S/m and $\varepsilon_r = 50$ means respectively and standard deviations of 10% of their nominal values [4]. The corresponding probability density function of the measured signal in the absence of tumours is multivariate Gaussian with mean $\vec{y}_0$ and covariance matrix $\sigma_e^2 I$ where we assumed that the measurement noise is uncorrelated in space and time. Note that the multiple measurements can be obtained in order to remove the measurement noise in statistically optimal way.

In presence of scattering the probability density function is given by

$$p_\sigma(\sigma) \cdot p_{\varepsilon_r}(\varepsilon_r) \cdot p(\vec{y}|\sigma, \varepsilon_r, \vec{R}) \tag{9}$$

Note that for a particular patient conductivity and permittivity can be treated as deterministic variables. In this paper we assume that sufficient set of previously measured properties is available and hence certain a priori knowledge on physiological values is available. The parameters can then be obtained by minimizing the cost function

$$c(\vec{R_t}, R, \sigma_r, \varepsilon_r) = \log\left[p_{\sigma_r}(\sigma_r) \cdot p_{\varepsilon_r}(\varepsilon_r) \cdot p(\vec{y}|\mu_r, \varepsilon_r, \vec{R})\right] \tag{10}$$

which is commonly know as maximum a posteriori estimate of unknown parameters. Note that because of nonlinear dependence of measured signals on physical parameters the above estimations hast to be performed using numerical optimization methods and Monte-Carlo simulations (in order to obtain posterior distribution).

V. NUMERICAL RESULTS

In order to evaluate the performance of our inverse model we simulate the measurement data using COMSOL and then add Gaussian noise in order to simulate measurement noise. In this preliminary approach we ignore several issues such as: skin reflection, calibration problems, antenna-to-antenna crosstalk, etc.

Although these issues are important our preliminary goal in this paper is to demonstrate ability of the model to accurately estimate tumour parameters using simulated data. The results of this analysis can then be useful in properly designing antennas for realistic measurements. We simulate measurement data for 100 patients for which the conductivity and permittivity are generated using aforementioned Gaussian distributions. For each of the patients we then generate posterior distribution $p(\vec{y}|\sigma, \varepsilon_r, \vec{R}, R_t)$ in order to calculate the parameter estimates. In Figures 5 and 6 we illustrate posterior distribution for arbitrarily chosen antenna for two different tumour sizes.

In Figure 7 we show the y component of the electric field in the presence of scatterer. As we can see the presence of tumour causes irregular patterns in the electric field which can be used for inverse modelling and consequently detection and estimation. In Figure 8 we show the mean-square error

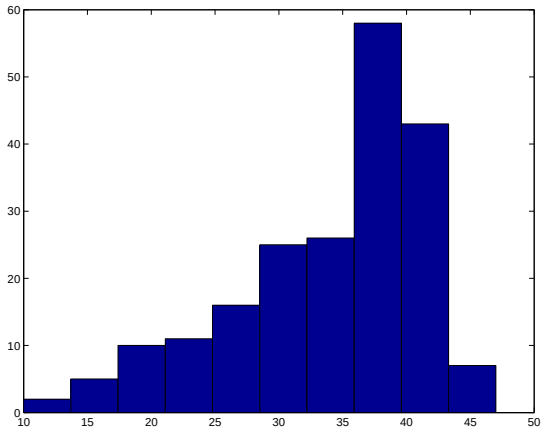

Fig. 5. Posterior distribution for tumour in center size 1cm

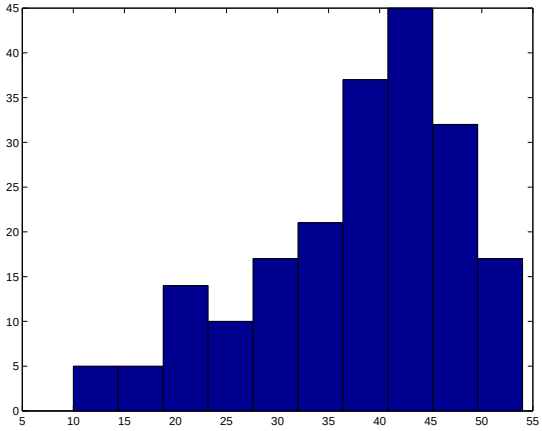

Fig. 6. Posterior distribution for tumour in center size 2cm

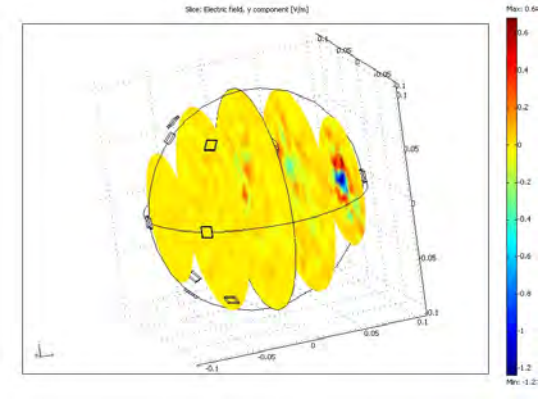

Fig. 7. Electric field

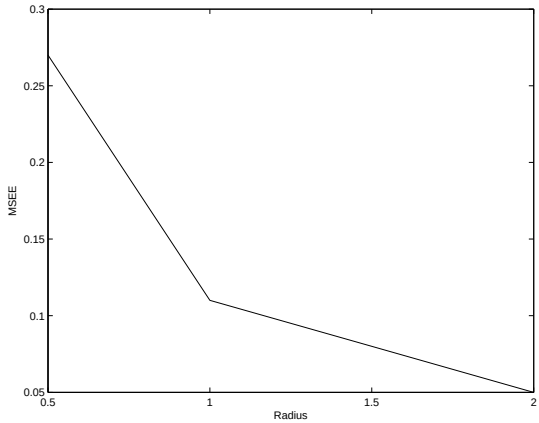

Fig. 8. MSE of radius estimate as a function of tumour size

(MSE) for tumour size estimate as a function of tumour size. However, note that in reality the larger tumours have irregular (star-like patterns) and the effect of that was not included in this work. Consequently the actual estimation error may not decrease as rapidly as in our simulation results.

In Figures 9 and 10 we illustrate mean-square-error (MSE) estimates of tumour size and location as a function of signal-to-noise ratio (SNR).

VI. CONCLUSIONS AND FUTURE WORKS

In this paper we proposed frequency domain parametric three-dimensional model of microwave propagation in breasts. The results presented in this paper suggest that it may be possible to obtain accurate estimates of tumour position and size using parametric model. Of course certain issues (e.g. skin reflection) have been disregarded and will be addressed in future work. Furthermore these results can be useful when designing antennas since these models can be used to determine the necessary power of the transmitted signal that is necessary in order to achieve certain estimation accuracy.

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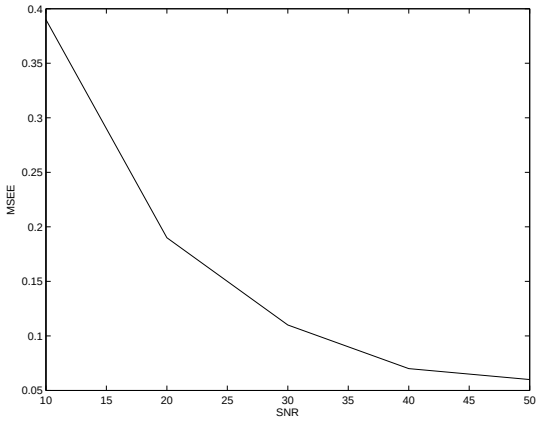

Fig. 9. MSE of radius estimate

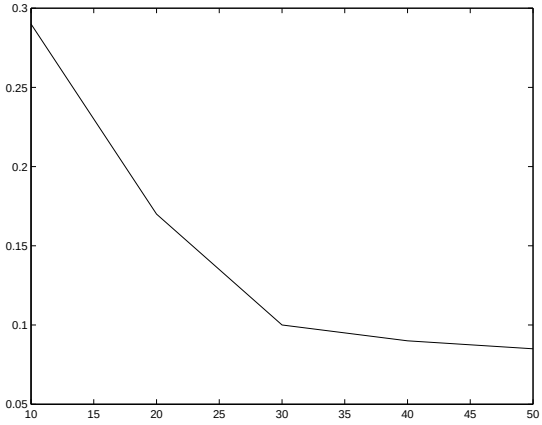

Fig. 10. MSE of location estimate