Numerical Investigation of The Convective Heat Transfer Enhancement in Coiled Tubes

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Abstract: The present work is focused on the numerical analysis of forced convection in curved tubes. The study was performed by integrating the continuity, momentum and energy equations within Comsol Multiphysics 4.2a environment. The analysis was carried out under the assumptions of incompressible Newtonian, constant properties fluid and of periodically fully developed laminar flow. For the thermal problem, a uniform wall heat flux boundary condition was assumed. The main purpose was to study the influence of the curvature profile on the convective heat transfer mechanism: in order to evaluate the overall performances of the geometry the friction factor and the Nusselt Number were calculated. Moreover the flow pattern and the temperature distribution on the tube section were derived for different values of the Reynolds number. The aim was to deeply investigate the correlation between the heat transfer and friction factor enhancement and the effects of the wall curvature. In the present study, a toroidal geometry characterised by a tube diameter of 14 mm and a curvature ratio of 0.06 was considered. In addition, the numerical results were compared to the experimental values obtained on a coiled tube with the same tube diameter and the same curvature ratio. The study was performed by considering the Reynolds number range 2-100 and the Prandtl number range 25-50.

Keywords: Convection, Heat Transfer, curved tubes

Nomenclature

a	Toroid diameter	m
c_p	Specific heat	J/kgK
D	Tube diameter	m
De	Dean number. De=Re$\sqrt{\delta}$	
f	Friction factor	
h	Convective heat transfer coefficient	W/m^2K
m	Mass flowrate	Kg/s
Nu	Nusselt number	
Pr	Prandlt number: $Pr = c_p \cdot \mu/\lambda$	
q	Heat flux exchanged per unit surface	W/m^2
Re	Reynolds number	
T	Temperature	K
v_{av}	Average fluid axial velocity	m/s
δ	Curvature ratio: $\delta = D/a$	
λ	Fluid thermal conductivity	W/m·K
μ	Dynamic viscosity	Pa·s
ν	Kinematic viscosity	m^2/s

Subscripts

b	Bulk

1. Introduction

The convective heat transfer enhancement techniques represent an important research task in the heat transfer field. They can be classified into two main categories: active and passive techniques.

The first ones need a mechanical help or electrostatic fields; otherwise the passive techniques don't need an external power: they are based on modifications of the fluid flow induced by a specific conformation of the heat transfer surface.

Among the most widely used convection heat transfer enhancement passive techniques, wall curvature is found.

The effectiveness of this technique is due to the fact that the curvature profile of the tube induce the origin of a centrifugal force that cause a distortion of the velocity profile inducing local maxima in the velocity distribution. To this phenomenon an increase of the temperature gradients at the wall is associated and consequently a maximization of the heat transfer is obtained.

In addition it generates a secondary flow that promotes the mixing of the fluid in the boundary layer.

This technique represents a very interesting solution for mechanical applications where highly viscous fluid are involved like food, cosmetics and pharmaceutics industries where a laminar flow regime is often found.

The interest on this argument is witnessed by the great number of articles, regarding curved geometries, available in the scientific literature.

A numerical study focused on the effect of secondary flow on laminar flow heat transfer in helically coiled tubes both in the fully developed region and in the thermal entrance region was presented by Dravid et al. [1]. They also suggest the following correlation for the asymptotic Nusselt number:

$$Nu = \left(0.65\sqrt{De} + 0.76\right)Pr^{0.175} \qquad (1)$$

for $50 < De < 200$ and $5 < Pr < 175$

Janssen and Hoogendoorn [2] studied experimentally and numerically the laminar convective heat transfer in helical coiled tubes.

In their analysis the *Pr* number values varies in the range 10÷500 and the boundary condition of uniform wall heat flux was considered. In order to evaluated the heat transfer enhancement they report the local Nusselt number at the outside and inside of the helix and the peripherally averaged Nusselt number as a function of the Graetz number.

As a conclusion of their work they propose the following correlations for the asymptotic Nusselt number:

$$Nu = 1.7(De^2 Pr)^{1/6} \qquad (2)$$

for $De < 20$ and $(De^2 Pr)^{1/2} < 100$

$$Nu = 0.9(Re^2 Pr)^{1/6} \qquad (3)$$

for $20 < De < 100$, $20 < Pr < 450$ and $0.01 < \delta < 0.083$

$$Nu = 0.7 Re^{0.43} Pr^{1/6} \delta^{0.07} \qquad (4)$$

for $100 < De < 830$ and $20 < Pr < 450$ and $0.01 < \delta < 0.083$

A numerical study on the fully developed laminar convective heat transfer in a helicoidal pipe with a finite pitch has been performed by G.Yang et al. [3]. In this work it was found that the convective heat transfer was influenced in particular by three parameters: the Dean number, the torsion and the Prandtl number. The results showed that, as the Dean number and the Prandtl number increase, the Average Nusselt number rises; otherwise with the increasing of the torsion the average Nusselt number decreases softly for Pr lower than 1 and reduces widely for higher Pr number.

Garimella et al. [4] analysed the heat transfer coefficients for laminar and transition flows for forced convective heat transfer in coiled annular tubes. The results showed that the convective heat transfer coefficients in the case of coiled ducts, in particular in the laminar region, are higher than the ones obtained with straight tubes.

Xin and Ebadian [5] performed an analysis with the purpose of investigating the influence of the geometric parameters and the Prandtl number on the local and average convective heat transfer characteristics in helical ducts. In this work a significant change in the Nusselt number for the laminar flow region was observed with the progression of the Prandtl and the Dean numbers. Some correlations were suggested for the fully developed region:

$$Nu = (2.153 + 0.318 De^{0.643}) Pr^{0.177} \qquad (5)$$

for $20 < De < 2000$ and $0.7 < Pr < 175$ and $0.0267 < d/D < 0.0884$

$$Nu = 0.0619 Re^{0.92} Pr^{0.4}\left(1 + 3.455\frac{d}{D}\right) \qquad (6)$$

for $5 \times 10^3 < Re < 10^5$ and $0.7 < Pr < 5$ and $0.0267 < d/D < 0.0884$

Recently Rainieri et al. [6] experimentally studied the combined effect of wall curvature and wall corrugation in two Reynolds number ranges (in particular in the ranges 5–13 and 150-1500). In this work it was found that the combined effect permits to obtain interesting results in the higher Reynolds number range but for lower values of Re the best solution is represented by the wall curvature only.

2. Geometry and Model implementation

In the present study, a toroidal geometry characterized by a tube diameter of 14 mm and a curvature ratio of 0.06 was considered (Figure1). Since the geometry shows a symmetric construction only one half of the tube has been simulated. Moreover, by considering a periodically fully developed flow it is possible to analyze only a portion of the toroid: in this case a portion of 10 degrees was considered. The domain of integration and the computational grid are reported in figure 2.

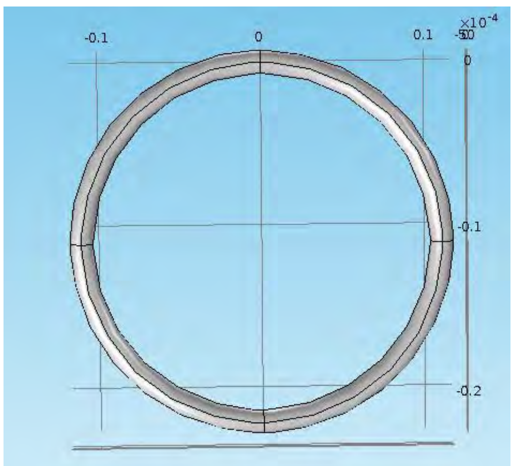

Figure 1. The toroidal geometry.

The study was performed by integrating the continuity, momentum and energy equations within Comsol Multiphysics 4.2a environment, under the assumption of constant properties and incompressible Newtonian fluid and of periodically fully developed laminar flow for the hydrodynamic and the thermal problem.
By considering negligible viscous dissipation the formulation of these equations is the following:

$$\boldsymbol{\rho(u\cdot\nabla)u=\nabla\cdot[-\rho I+\mu(\nabla u+(\nabla u)^T)]}+\mathbf{F}$$

$$\boldsymbol{\rho\nabla\cdot u=0}$$

$$\boldsymbol{\rho C_p\frac{\partial T}{\partial t}+\rho C_p u\cdot\nabla T=\nabla\cdot(k\nabla T)} \quad (7)$$

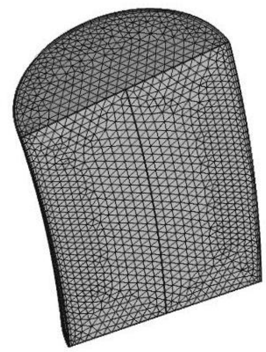

Figure 2. The geometry under test and the computational mesh.

For what concerns the pressure the suitable condition is defined with the following constraint:

$$p_{entrance}-p_{exit}-\Delta p=0 \quad (8)$$

where Δp is the pressure drop between the entrance and the exit sections; its progression implies the increase of the mass flow rate and consequently of the Re number.
For the hydrodynamic problem it's also added the no slip condition at the wall and a *pointwise* constraint for the pressure in one point of the entrance section.
The thermal problem is approached by assuming a uniform wall heat flux boundary condition. As a consequence the temperature distribution has a profile which repeats periodically as follows:

$$T(x,y,z,t)_{entrance}-T(x,y,z,t)_{exit}+\Delta T=0 \quad (9)$$

This condition in Comsol Multiphysics 4.2a environment is obtained by applying a linear extrusion of the temperature from the entrance section to the final one and then using a temperature condition on the final section:

$$linext(T_{entrance})-T_{exit}+\Delta T=0 \quad (10)$$

For the thermal problem, the boundary conditions are completed by applying a *pointwise* constraint for temperature in one point of the entrance section. Considering the energy balance, ΔT is related to the wall heat flux density as follows:

$$\Delta T = \frac{q \cdot S}{m \cdot c_p} \tag{11}$$

where S is the heat transfer surface, q is the heat transfer per unit area, m is the mass flow rate.
Firstly the continuity and the steady state Navier-stokes equations are solved in the domain with the boundary condition before described. Then for a given ΔT the heat transfer per unit area is calculated from equation (11) and then the energy equation is solved. The thermal problem has been approached by using an unsteady solver in order to achieve the convergence to the solution.

3. Data processing

In order to evaluate the overall performances of the geometry, the friction factor and the Nusselt Number were calculated. They are obtained respectively, as follows:

$$Nu = \frac{\bar{h} \cdot D}{\lambda} \tag{12}$$

$$f = \frac{\Delta p}{\rho} \cdot \frac{D}{l} \cdot \frac{2}{{v_{av}}^2} \tag{13}$$

where v_{av}, the average axial velocity is calculated in this way:

$$v_{av} = \frac{1}{A_c} \int_{A_c} v dA \tag{14}$$

where A_c is the cross section area.
The average convective heat transfer coefficient is obtained as follows:

$$\bar{h} = \frac{q}{(T_{w,av} - T_{b,m})} \tag{15}$$

where $T_{w,av}$, the surface averaged wall temperature and $T_{b,m}$, the arithmetic mean of the inlet and of the outlet bulk temperature are defined as follows:

$$T_{w,av} = \frac{1}{A} \int_A TdA \tag{16}$$

$$T_{b,m} = \frac{1}{2}\left(\left(\frac{\int_{A_c} T\boldsymbol{u} \cdot \boldsymbol{n} dA}{\int_{A_c} \boldsymbol{u} \cdot \boldsymbol{n}\, dA}\right)_{inlet} + \left(\frac{\int_{A_c} T\boldsymbol{u} \cdot \boldsymbol{n} dA}{\int_{A_c} \boldsymbol{u} \cdot \boldsymbol{n}\, dA}\right)_{outlet}\right) \tag{17}$$

The different analysis were realized considering an increasing value of pressure drop corresponding to an increasing Reynolds number value:

$$Re = \frac{\rho \cdot v_{av} \cdot D}{\mu} \tag{18}$$

4. Results and discussion

The flow pattern and the temperature distribution on the cross section were derived for different values of the Reynolds number and are reported in Figures 3-10.

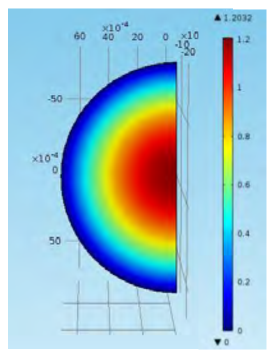

Figure 3. Axial velocity: Re=2.9

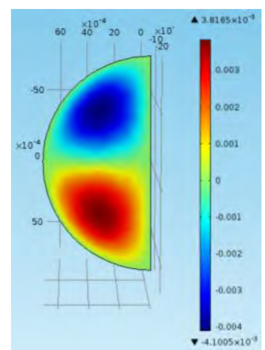

Figure 4. y- velocity: Re=2.9

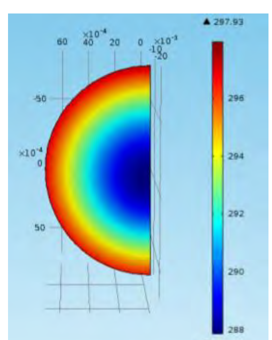

Figure 5. Re=2.9 Temperature distribution: Pr=25

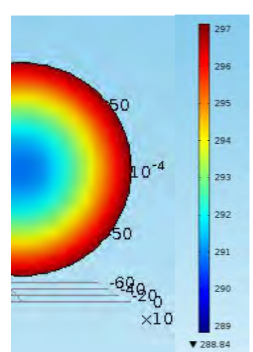

Figure 6. Re=2.8 Temperature distribution: Pr=50

From picture 3 it is possible to notice that for low Reynolds numbers the distortion in the axial velocity distribution, due to the centrifugal force induced by the wall curvature, is still negligible: the velocity profile presents the characteristic parabolic shape. The negligibility of the distortion can be also seen in the temperature distributions (figure 5,6). From figure 4, that shows the y-velocity distribution, it can be noticed the presence of a secondary flow, but its size is still not significant to influence the heat transfer phenomenon.

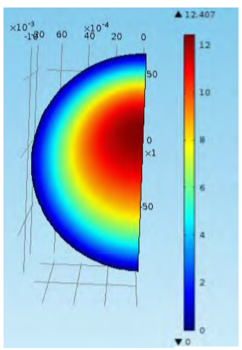

Figure 7. Axial velocity: Re=97.33

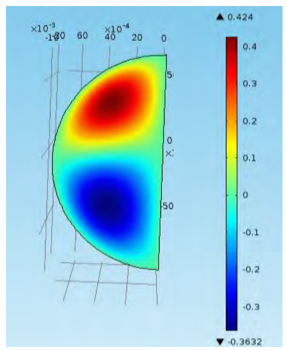

Figure 8. y-velocity: Re=97.33

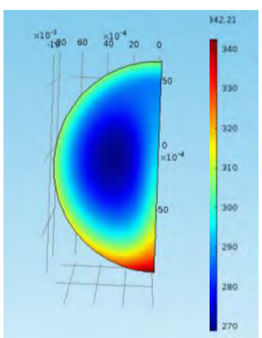

Figure 9. Pr=25 Temperature distribution: Re=97.33

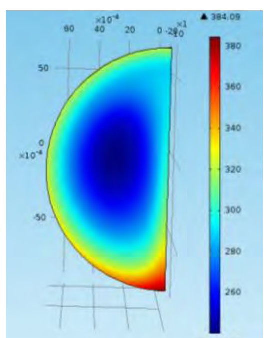

Figure 10. Pr=50 Temperature distribution: Re=97.33

With the increasing of the Reynolds number the secondary flow became more significant and the distortion of the velocity profile is not still negligible. As a consequence, also the temperature profile is distorted showing an increase of the gradients at the wall.

In figure 11 the average Nusselt number for two different Prandtl numbers is reported. The results are compared with the analytical solution holding for the straight smooth wall tube under uniform heat flux boundary condition [7], with the prediction of Janssen and Hogendoorn and with the values obtained experimentally for a parallel study performed using a coiled tube with the same tube diameter and the same curvature ratio and Ethylene Glycol (Pr number varies in the range 170-230) as working fluid.

The values obtained within Comsol Multiphysics 4.2a environment are supported by the experimental results: the local Nusselt number reaches values higher than the ones expected for straight pipes. The augmentation effect was due to the wall curvature.

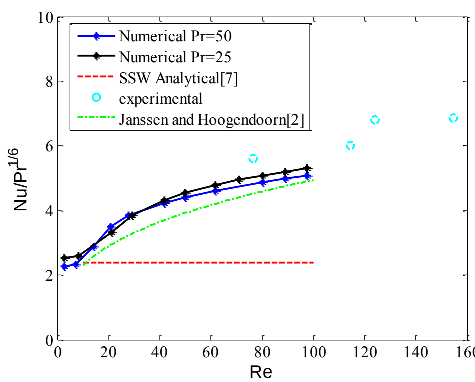

Figure 11. Average numerical Nusselt number versus *Re* for two different Prandtl numbers and comparison with the analytical solution for the straight smooth wall tube under uniform heat flux boundary condition [7] (SSW:straigth smooth wall), with the prediction of Janssen and Hogendoorn [2] and with experimental results.

Moreover, the numerical results are in good agreement with the correlations proposed by Janssen and Hoogendoorn [2]. Finally, from this graph it's possible to see that the exponent 1/6 represents a good correlation for the Pr number influence.

Furthermore the friction factor is reported in figure 12 versus Reynolds number. The results support the values obtained experimentally: they show that the increasing over SSW analytical solution of the friction factor due to the wall curvature profile is negligible for this flow regime.

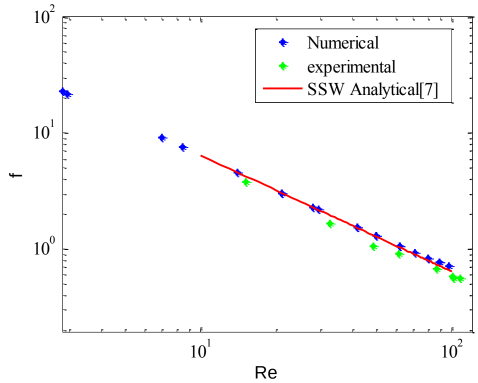

Figure 12. Friction factor versus Reynolds number for the numerical analysis compared to the one obtained experimentally and to the analytical solution holding for the straight smooth wall tube [7].

5. Conclusions

In the present study, a toroidal geometry characterized by a tube diameter of 14 mm and a curvature ratio of 0.06 was considered**.** In addition, the numerical results were compared to the experimental values obtained on a coiled tube with the same tube diameter and the same curvature ratio.

The numerical data obtained in this work, also supported by the experimental results, show that the curvature of the wall tube represents an efficient solution for enhancing the heat transfer in case of laminar flow regime.

The effectiveness is ascribable to the fact that the curvature profile of the tube induce the origin of a centrifugal force that cause a distortion of the velocity profile, inducing local maxima in the velocity distribution. To this phenomenon an increase of the temperature gradients at the wall is associated and consequently a maximization of the heat transfer is obtained.

This fact is of interest especially in processes in which highly viscous fluids are involved, such the ones used in the food, chemical and pharmaceutics industries.

Moreover, the results are in good agreement with the correlation proposed by Janssen and Hoogendoorn for helical coiled tubes.

Further analysis, by varying the geometry and the fluid properties, that could allow generalizing the results are in progress.

6. References

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