Simulation of Piezoelectric Transformers with COMSOL

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Abstract:

In this work COMSOL is utilized to obtain the Mason lumped parameter model for a piezoelectric transformer (PT) design. The Mason lumped parameters are relevant in the design process of power converters. The magnitude of the impedance is simulated for a specific: interleaved multilayer thickness mode PT. Interleaved indicates that the primary section of the PT has been interleaved into the secondary section. Furthermore the primary section is build with interdigitated electrodes (IDE). The PT design has been prototyped and the measurements results are compared with simulations.

Two methods for simplifying the PT model are given in order to decrease the simulation time.

This paper aims to aid electrical engineers with less knowledge within the field of mechanics, to be able to simulate a PT design with COMSOL and extract the key electrical parameters.

Keywords: Model, Simplification, IDE, Mason, COMSOL Multiphysics 4.2a.

1. Introduction

Within power electronic electromagnetic transformers have been the dominating component for converting and transforming of electrical power. The trend of power converters goes in the direction of higher efficiency and smaller volume. Research has shown that piezoelectric transformers (PT) can compete with traditional electromagnetic transformers on both efficiency and power density [1-4]. PTs are therefore an interesting field of research.

A PT utilizes two interconnected piezoelectric elements. One is set into motion, by the inverse piezoelectric effect, and the other is harvesting the energy from the motion by the direct piezoelectric effect. By proper design the PT is capable of transferring energy. The conversion ratio is given by geometry, polarization and placement of electrodes. The piezoelectric constitutive equations [5] in stress-charge form is given by equations (1) and (2). These equations describe the relation between the mechanical and the electrical. The symbols are explained in Table 1.

$$T = c^{E} \cdot S - e \cdot E \qquad (1)$$

$$D = e \cdot S + \varepsilon^{S} \cdot E \qquad (2)$$

Table 1 piezoelectric constitutive equation symbols

T	Stress	[N/m^2]
S	Strain	-
D	Electric displacement	[C/m^2]
E	Electric field	[V/m]
c^E	Elastic stiffness at const. electric field	[N/m^2]
e	Piezoelectric constant	[C/m^2]
ε^S	Permittivity at const. strain	[F/m]

From the constitutive equations analytic equations can be derived to solve piezoelectric problems e.g. PTs. In this work COMSOL 4.2a and the module "*Piezoelectric Devices*" (pzd) is utilized to solve the piezoelectric problem based on the constitutive equations.

This paper aims to aid electrical engineers with less knowledge within the field of mechanics, to be able to simulate a PT design with COMSOL and extract the electrical key parameters.

2. Piezoelectric transformers

In this chapter the basic of PTs are revealed with focus on electrical parameters.

PTs are based on a piezoelectric material. This material has an electromechanical coupling and through this coupling a charge displacement is generated, which is proportional to the deformation of the material. A PT is basically two piezoelectric elements which is joined together to form a transformer. The primary side element is then exited by an electrical AC voltage, which induces a deformation of the two joined elements. This deformation generates an output voltage on the secondary side element. With a proper design of the PT a desired voltage conversion can be obtained from the primary to the secondary side.

In order to convert power at a high efficiency, the PT is operated in one of its resonance modes [4, 6-8]. The PT resonates each time it is possible to generate a standing wave in the element. But the design is usually optimised for one specific resonance mode, in order to obtain the highest efficiency [6, 8].

The PT resembles a distributed network, but for simplicity and mathematical representation, only the resonance mode of interest is modelled. One of the most used PT models is the lumped parameter model, which was derived by Mason in 1942 [9] and is illustrated in Figure 1.

For a PT based converter-design's point of view the values of interest is the lumped parameters of figure 1. The Mason lumped parameters can be calculated in the frequency domain from the primary's and secondary's magnitude of the input impedance [7, 10]. An example of a magnitude plot for the Mason equivalent with the secondary or the primary side shorted is illustrated in figure 2. Three points for each section (six in total) are necessary to calculate the lumped parameters: a low frequency (f_0) magnitude (Z_0), far below the frequency of the first resonance mode, magnitude (R) at the resonance frequency (f_z) and the anti-resonance frequency (f_p). From these values the lumped parameters are calculated using equation (3) to (10).

$$R = \left| Z_{pri}\left(f_{z_{pri}} \right) \right| \tag{3}$$

$$C_{T_{pri}} = \frac{1}{2\pi f_{0_{pri}} Z_{0_{pri}}} \tag{4}$$

$$C_{T_{\sec}} = \frac{1}{2\pi f_{0_{\sec}} Z_{0_{\sec}}} \tag{5}$$

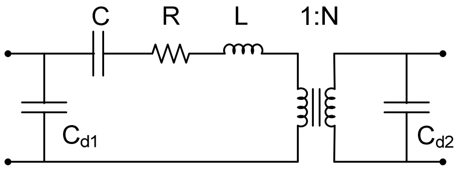

Figure 1 Lumped parameter model, which describes the behavior of the PT in a narrow band around the operating resonance mode.

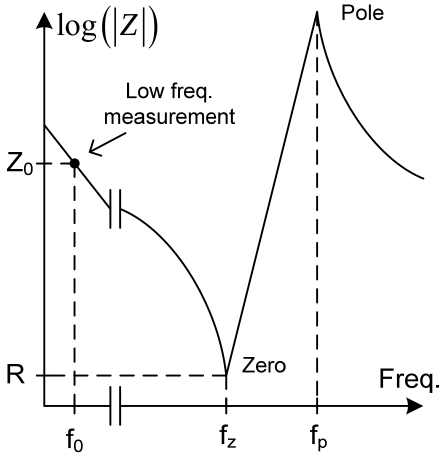

Figure 2 Magnitude plot of a PT around a resonance mode with either secondary or primary section shorted.

$$C_{d1} = C_{T_{pri}} \cdot \left(\frac{f_{z_{pri}}}{f_{p_{pri}}} \right)^2 \tag{6}$$

$$C_{d2} = C_{T_{\sec}} \cdot \left(\frac{f_{z_{\sec}}}{f_{p_{\sec}}} \right)^2 \tag{7}$$

$$C = C_{T_{pri}} - C_{d1} \tag{8}$$

$$L = \frac{1}{\left(2\pi f_{z_{pri}} \right)^2 C} \tag{9}$$

$$N = \sqrt{\frac{1}{\left(2\pi f_{Z_{\sec}} \right)^2 \left(C_{T_{\sec}} - C_{d2} \right) \cdot L}} \tag{10}$$

From the lumped parameters other PT related properties can be calculated : resonance frequency (11), matched load (12), efficiency in matched load (13), soft switching factor or ZVS factor (14). Derivation of the equations can be obtained from the following references [6, 11, 12].

$$f_r \approx \frac{1}{2\pi\sqrt{L \cdot C}} \tag{11}$$

$$R_m = \frac{1}{2\pi f_r C_{d2}} \tag{12}$$

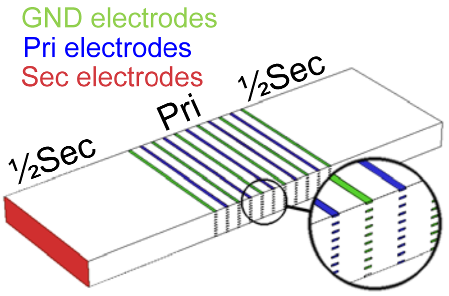

Figure 3 Sketch of the PT design with simplified primary section build-up.

$$\eta_m \approx 1 - \frac{2RC_{d2}N^2}{\sqrt{LC}} \tag{13}$$

$$V_P' \approx \left(0.304 \cdot \frac{N^2 C_{d2}}{C_{d1}} + 0.538\right) \cdot \left(0.585 \cdot \eta + 0.414\right) \tag{14}$$

3. PT geometry

In this chapter the geometry of the PT design is shown. The thoughts and calculations behind the geometry are out of the scope of this work and will not be discussed.

The specific PT design is intended for high voltage output applications. The design is an interleaved multilayer thickness mode transformer, indicates that the primary section of the PT has been interleaved into the secondary section. Furthermore the primary section is build with interdigitated electrodes (IDE) to allow tape casting manufacturing process. The transformer is build up by 60 piezoelectric ceramic sheets of 33 µm in thickness after sintering, leading to a total height of about 2 mm. Each sheet is printed with conducting IDE, the width of each electrode is 50 µm, the distance between each electrode is 200 µm and the edge margin is 0.33 mm. The IDE divides the primary section into 40 layers with a primary section width of 10.1 mm. The total width of the PT is 30.0 mm and the depth is 10.0 mm. A simplified sketch of the PT is shown at figure 3 with electrode margin not included.

The electrical equivalent of the PT design is basically the same as the Mason model. However the split secondary is modelled with a 2 winding center tap transformer on the secondary side. From the electrode arrangement the center tap is then grounded. Figure 4 illustrates the electrical equivalent of the interleaved PT design.

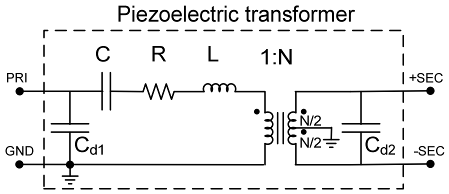

Figure 4 Electrical equivalent of the interleaved PT design.

Table 2 PT geometry specifications

W	30.0 [mm]	Total width
H	2.0 [mm]	Total height
D	10.0 [mm]	Total depth
Wp	10.1 [mm]	Primary section width
Lp	40	Primary layers
We	50 [µm]	Electrode width
He	8 [µm]	Electrode height
Wd	200 [µm]	Electrode distance
Lt	60	Tape layers
Ht	33 [µm]	Sintered tape thickness
Em	0.33 [mm]	Edge margin

4. COMSOL

In this chapter the impedance plots, necessary for obtaining the Mason lumped parameters, are simulated in COMSOL for the specified PT geometry.

For simulating the PT design in COMSOL the "*Piezoelectric Devises (pzd)*" physics under structural mechanic is used. The study type required for a frequency sweep is the "*Frequency Domain*".

4.1 Material parameters

The piezoelectric material parameters are derived from Ferroperm's PZ26 [13]. Material parameters for the polarized material are shown in (15) (16) (17) and (18).

Losses for the material are included by adding "*Damping and Loss*" under the "*Piezoelectric Material Model*". Good experience is observed with Rayleigh damping with the value from (19).

4.2 Polarization

Before a material has piezoelectric behaviour the material needs to be polarized. The poling process usually implies a high electrical field to the material that will align the electric dipoles in

$$\rho = 7700\left[{kg}/{m^2}\right] \tag{15}$$

$$c^E = \begin{bmatrix} 16.8 & 11.0 & 7.85 & 0 & 0 & 0 \\ & 16.8 & 7.85 & 0 & 0 & 0 \\ & & 12.2 & 0 & 0 & 0 \\ & & & 3.01 & 0 & 0 \\ & & & & 3.01 & 0 \\ & & & & & 2.88 \end{bmatrix} \cdot 10^{10} \tag{16}$$

$$e = \begin{bmatrix} 0 & 0 & 0 & 0 & 9.86 & 0 \\ 0 & 0 & 0 & 9.86 & 0 & 0 \\ -2.8 & -2.8 & 17.7 & 0 & 0 & 0 \end{bmatrix} \tag{17}$$

$$\varepsilon^S_{relative} = \begin{bmatrix} 828 & & \\ & 828 & \\ & & 1282 \end{bmatrix} \tag{18}$$

$$\begin{aligned} \alpha_{dM} &= 0 \\ \beta_{dK} &= 1.11e-9 \end{aligned} \tag{19}$$

the same direction and hence the electromechanical coupling arises [14].

The default polarization direction of piezoelectric material in COMSOL is along the z-axis. To change the poling direction it is necessary to define a new coordinate system under "*Definitions*". Figure 5 illustrates effect of different poling directions by applying a DC voltage across the material.

The poling directions of the PT are shown for a simplified 4 layer PT model in figure 8 together with electrodes. The blue and green volumes are space taken by piezoelectric ceramic and electrode material. However the piezoelectric ceramic in the blue and green-colored volumes are not able to be polarized and is referred to as inactive material. For inactive (not polarized) material the parameters of the piezoelectric material changes, however it is assumed that inactive material can be modeled as polarized material except that the piezoelectric coupling matrix (17) is zero.

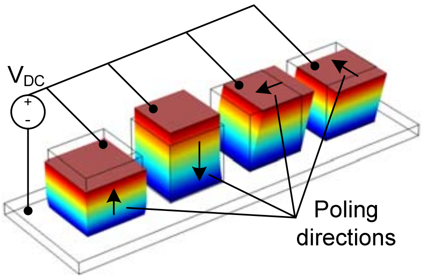

Figure 5 Simulation of four piezoelectric cubes on a conducting plate subjected to a voltage much lower than poling voltage. The electric potential is given by the colors. Deformation depends on the poling direction of the material.

4.3 Electrode boundary conditions

For simulating the impedance of the primary side the electrode of the secondary (Sec) must be shorted. One way of connecting two or more electrodes together is by use of the "*Floating potential*" condition. The selected boundaries are then electrical connect without specifying an actual voltage. The ground (GND) electrodes are set by the "*Ground*" condition and the primary (Pri) electrodes are set by the "*Electrical potential*" condition.

For simulating the impedance of the secondary side the primary (Pri) and ground (GND) electrodes must be shorted together with the "*Floting potential*" condition. One of the secondary electrodes is connected to the "*Ground*" condition while the other is connected to the "*Electrical potential*" condition.

4.4 Model simplification and transformation

Simplification of a model in COMSOL can reduce the simulation time by decades with only little effect on the results. Simplification is therefore important. There are two major ways to simplify a PT design: Dimension reduction and layer reduction. Both reductions are done by transforming the geometry. The resulting impedance plots are then inverse-transformed to reflect the results of the non-transformed geometry.

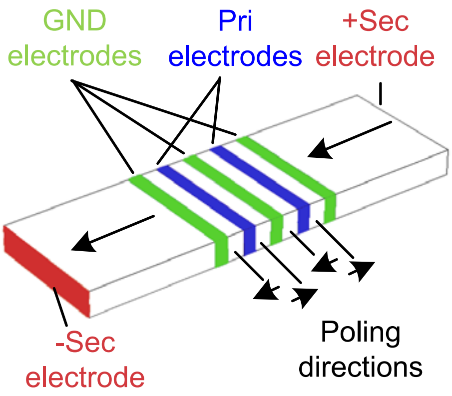

Figure 8 Simplified PT model with 4 primary layers, illustrating poling directions (arrows) and electrodes (gray area).

Dimension reduction is a 2-dimension model of 3-dimension geometry. In a 2-dimension model the third dimension of the PT is equal to one meter. To obtain the correct impedance plot (Z) the 2-D impedance plot (Z_{2D}) is divided by the actual depth (d_z) of the PT in the third dimension (20).

$$Z(f) = \frac{Z_{2D}(f)}{d_z} \tag{20}$$

Layer reduction is another way of simplifying the geometry in order to speed up the simulation time. Layer reduction can be applied in both 3-D and 2-D models. Figure 8 illustrates an example of layer reduction applied for the primary section. The original design has 40 primary layers but is reduced to 4 primary layers. The impedance of a layer reduced section (Z_{LR}) is transformed to a full layer section impedance (Z) by the square of the ratio between the amount of layers in the reduced section (L_{LR}) and the amount layers without reduction (L) (21).

$$Z(f) = Z_{LR}(f) \cdot \left(\frac{L_{LR}}{L}\right)^2 \tag{21}$$

5. Results

In this chapter the results from measurements and simulations are compared for the PT design. The results of interest are the lumped parameters of the Mason model (see figure 1) for the first mode of resonance. Impedance plots are measured and simulated for both primary and the secondary side in order to derive the lumped parameters.

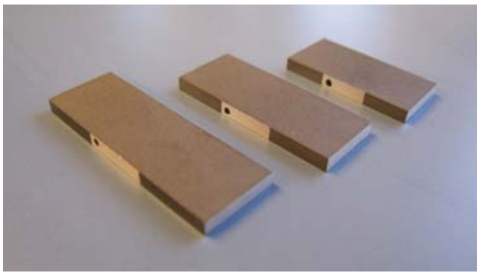

Figure 6 Prototypes of the piezoelectric interleaved multilayer thickness mode transformer of different sizes: 30 mm, 25mm and 20 mm.

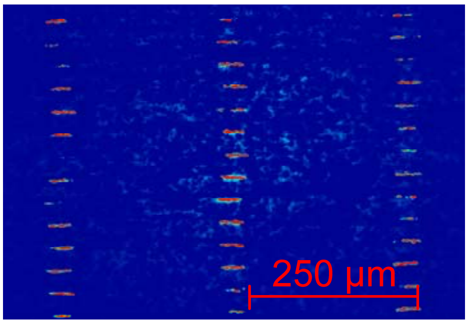

Figure 7 PT 30mm design: Cross-section view of primary electrodes under microscope. Image colors are post processed. Red colors are IDE electrodes.

Figure 6 show a picture of piezoelectric interleaved multilayer thickness mode transformer in three sizes. The PT to the right in the picture is designed after the specifications in Table 2. However the total width of the PT is 30.24 mm. This width is used for the later COMSOL simulations.

Figure 7 is a microscope image of the primary cross section, the red lines are the IDE electrodes. The image verifies the electrode width (We), electrode height (He) and electrode distance (Wd).

Figure 9 shows a simulation of the PT displacement at its first resonance mode. The displacement is exaggerated by a factor of 10^8 for the purpose of illustration. The dark blue area in the center of the PT indicates the node point where movement is almost zero.

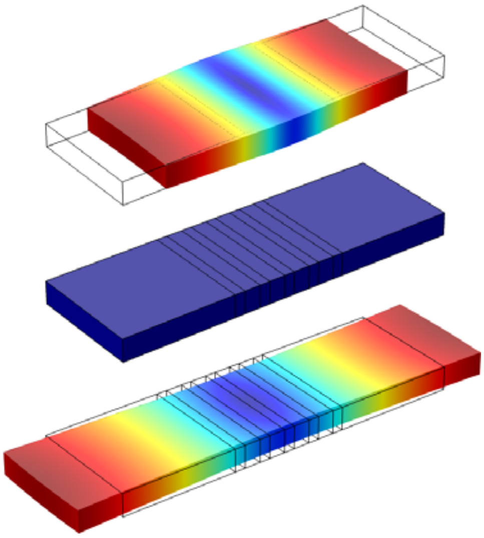

Figure 9 Simulation of displacement at first resonance mode 53.0kHz at phase 0° (top), 90° (middle) and 180° (bottom). Displacement is scaled by a factor of 10^8.

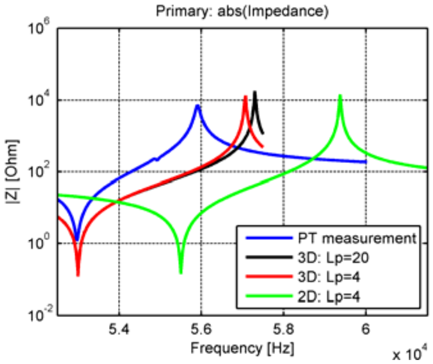

Figure 10 Magnitude plots of primary impedance.

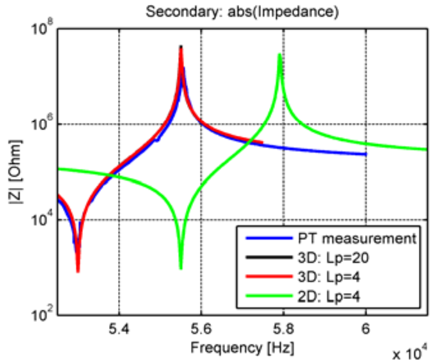

Figure 11 Magnitude plots of secondary impedance.

resonance frequency is measured for the PT prototype with a network analyzer.

In COMSOL the impedance magnitude plots are obtained by simulating the magnitude of the ratio between voltage and current for the electrodes of interest over a range of frequencies. The electrode voltage (V) is defined by its boundary condition and for the same electrodes the current is calculated by a surface integral of the inward current ("*pzd.nJ*"). Equation (22) gives the magnitude of the impedance.

$$|Z| = abs\left(\frac{V}{\iint_{surface} pzd.nJ}\right) \qquad (22)$$

Magnitude of the primary impedance is plotted in Figure 10 and magnitude of the secondary is plotted in Figure 11, the low frequency measurement is not shown at the plots. The corresponding lumped parameters are calculated in table 3**.** In COMSOL the PT is simulated with different model simplifications, ideally given the same result. The variations between the simulations are also given in Table 3.

No. 1. PT measurement
No. 2. 3D, 20 primary layers
No. 3. 3D, 4 primary layers
No. 4. 2D, 4 primary layers
No. 5. Variations between simulations

Table 3 Resulting lumped parameters

	No. 1	No. 2	No. 3	No. 4	No. 5
R [Ω]	1.19	0.12	0.12	0.14	17 %
C [nF]	2.71	8.98	9.10	8.01	14 %
L [mH]	3.33	1.00	0.99	1.03	4 %
C_{d1} [nF]	23.8	53.2	57.0	55.4	7 %
C_{d2} [pF]	15.1	14.4	14.4	14.0	3 %
N	42.5	80.3	80.8	80.4	1 %
f_r [kHz]	54.3	54.3	54.3	56.7	4 %
R_m [kΩ]	195	203	203	200	15 %
η_m [%]	97.9	99.2	99.2	99.1	0.1%
V'_p [%]	99.0	153	145	143	7 %

6. Discussions

By comparing the results some general trends can be observed. Evaluating the results from the magnitude of the impedance plots; Starting with the secondary side: a good correlation between measurement and the 3D simulations, however the 2D simulation is shifted a bit in frequency.

Comparing the magnitude of the primary impedance shows that the measurement and the 3D simulations share the same resonance frequency but the anti-resonance frequency is higher for the 3D simulations which is equivalent to a higher effective coupling factor [5]. Again, the 2D simulation is shifted in frequency compared to the 3D simulations. The magnitude at resonance frequency is too optimistic for all the simulations, a higher Rayleigh damping values (19) is required then. Also the number of primary layers has an effect on the anti-resonance frequency.

The results from the different simulation models are expected to be close to equal, however the variations of the individual lumped parameters are below 17 % in worse case.

The higher deviation observed between measurement and simulation from secondary to primary is a combination of model complexity and material parameters. The primary section has a complex electric field distribution around the IDE electrodes which again leads to a non-constant and multi direction polarization of the material. The material parameters are affected by the “degree” of polarization therefore a much more complicated model of the PT and the material parameters are need to obtain better correlation of results.

7. Conclusions

A PT design is simulated using COMSOL with focus on extracting electric key parameters in order to construct the Mason equivalent model of the PT. Methods to simplify PT models in order to speed up simulation time is given. In this work the simplification errors is less than 17% in worse case. Experimental measurements indicate that care has to be taken about polarization and material parameters when simulating PTs with complex electrical field distributions.

8. References

[1] T. Bove, W. Wolny, E. Ringgaard, and K. Breboel, "New type of piezoelectric transformer with very high power density," in *Applications of Ferroelectrics*, 2000, pp. 321-324 vol. 1.

[2] A. M. Flynn and S. R. Sanders, "Fundamental limits on energy transfer and circuit considerations for piezoelectric transformers," *IEEE Transactions on Power Electronics,* vol. 17, pp. 8-14, 2002.

[3] W. Shao, L. Chen, C. Pan, Y. Liu, and Z. Feng, "Power density of piezoelectric transformers improved using a contact heat transfer structure," *IEEE Transactions on Ultrasonics, Ferroelectrics and Frequency Control,* vol. 59, pp. 73-81, 2012.

[4] E. Horsley, M. Foster, and D. Stone, "State-of-the-art piezoelectric transformer technology," in *EPE*, 2007, pp. 1-10.

[5] A. Meitzler, "IEEE standard on piezoelectricity," *Society,* 1988.

[6] C. Lin, "Design and analysis of piezoelectric transformer converters," PhD Dissertation, 1997.

[7] C. Lin and F. Lee, "Design of a piezoelectric transformer converter and its matching networks," 1994, pp. 607-612 vol. 1.

[8] R. L. Lin, "Piezoelectric transformer characterization and application of electronic ballast," PhD Dissertation, Virginia Polytechnic Institute and State University, 2001.

[9] W. P. Mason, *Electromechanical transducers and wave filters*: Van Nostrand Reinhold, 1946.

[10] G. Zerong, J. Lingling, L. Huabo, and W. Ting, "Measurement of PT equivalent circuit model parameters based on admittance circle," 2011, pp. 20-23.

[11] K. S. Meyer, M. A. E. Andersen, and F. Jensen, "Parameterized analysis of Zero Voltage Switching in resonant converters for optimal electrode layout of Piezoelectric Transformers," 2008, pp. 2543-2548.

[12] M. S. Rødgaard, T. Andersen, and M. A. E. Andersen, "Empiric analysis of zero voltage switching in piezoelectric transformer based resonant converters," in *PEMD*, England, 2012.

[13] Ferroperm. Available: http://www.ferroperm-piezo.com/

[14] D. Jones, S. Prasad, and J. Wallace, "Piezoelectric materials and their applications," *Key Engineering Materials,* vol. 122, pp. 71-144, 1996.