



# Modeling Acoustic Interface Wave Dispersion Using COMSOL

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COMSOL  
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# Outline



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  - Geo-acoustic Parameters
  - Scholte Waves
  - Experiment
- COMSOL Modeling
  - Geometry
  - PML considerations
- Post-processing
  - Spatial Transform
  - Comparison with data

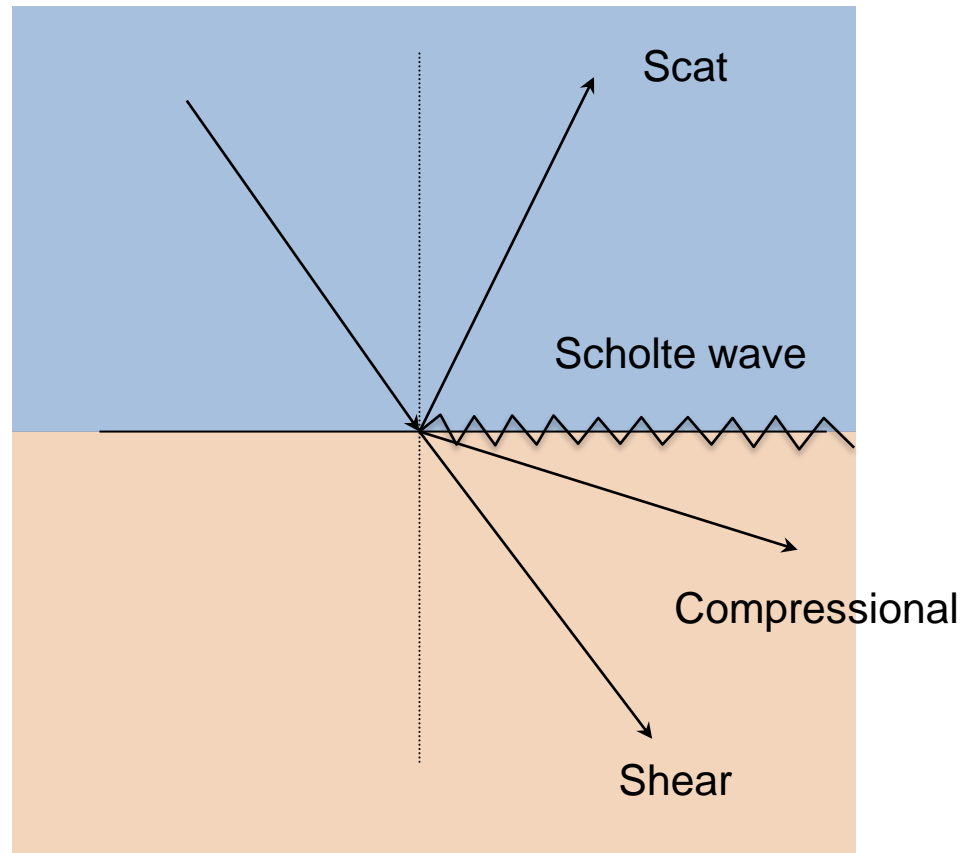


# Motivation



- Require accurate models of sediments in shallow water environments
- Compressional wave speed and density are easy to measure
- Need to incorporate shear waves in sediments for accurate numerical modeling
- Shear wave speed hard to measure due to high attenuation.
- Direct relationship between shear wave and Scholte wave
  - Scholte wave measurements → shear wave speeds

# Motivation



- Scholte wave phase speed is  $\sim 90\%$  of the shear wave speed
- Elliptical particle motion
- No low frequency cutoff



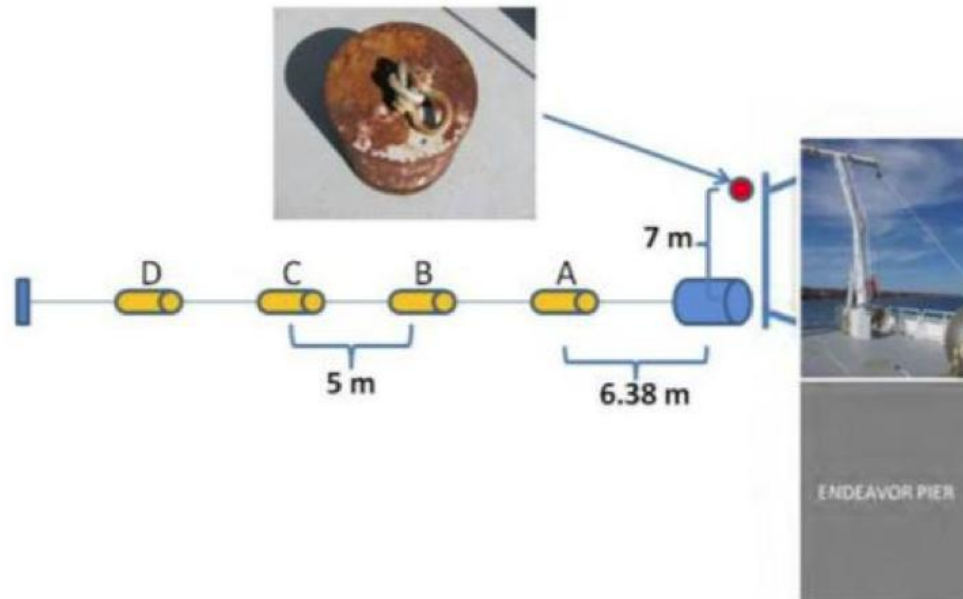
# Motivation



- Scholte wave phase speed is frequency-independent in an elastic half-space.
  - For a seafloor composed of layered sediments, Scholte wave phase speed is dispersive.
- Can use Scholte wave dispersion to invert for shear wave speeds in each sediment layer.
  - Dynamic stiffness matrix inversion model

# Experiment

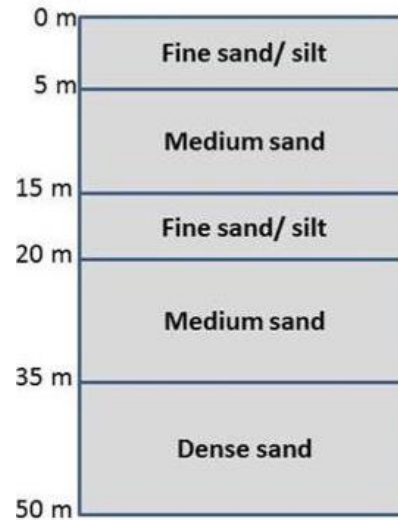
Plan view:



G. Potty and J. Miller. Measurement and modeling of Scholte wave dispersion in coastal waters. In Proc. of Third Int. Conf. on Ocean Acoustics, Beijing, China, 21-25 May 2012.

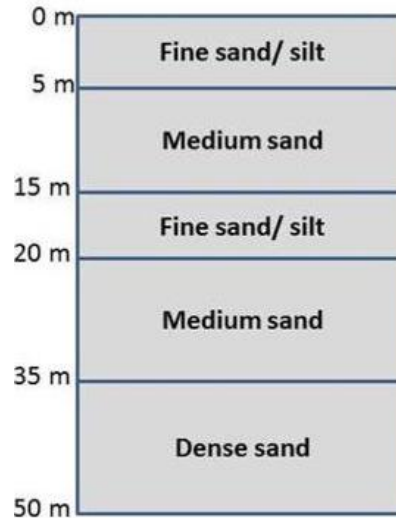


# Experiment



Medium	Layer thickness (m)	$c_p$ (m/s)	$c_s$ (m/s)	$\rho$ (kg/m <sup>3</sup> )
Water	6	1540	-	1000
Sediment				
Layer 1	2	1600	45	1650
Layer 2	4	1650	100	1840
Layer 3	7	1600	170	1710
Layer 4	9	1650	250	1940
Half-space	$\infty$	1836	380	2034

# Experiment



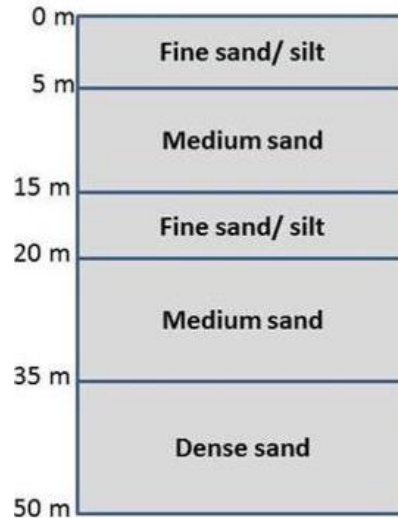
Core sample analysis

Medium	Layer thickness (m)	$c_p$ (m/s)	$c_s$ (m/s)	$\rho$ (kg/m <sup>3</sup> )
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# Experiment

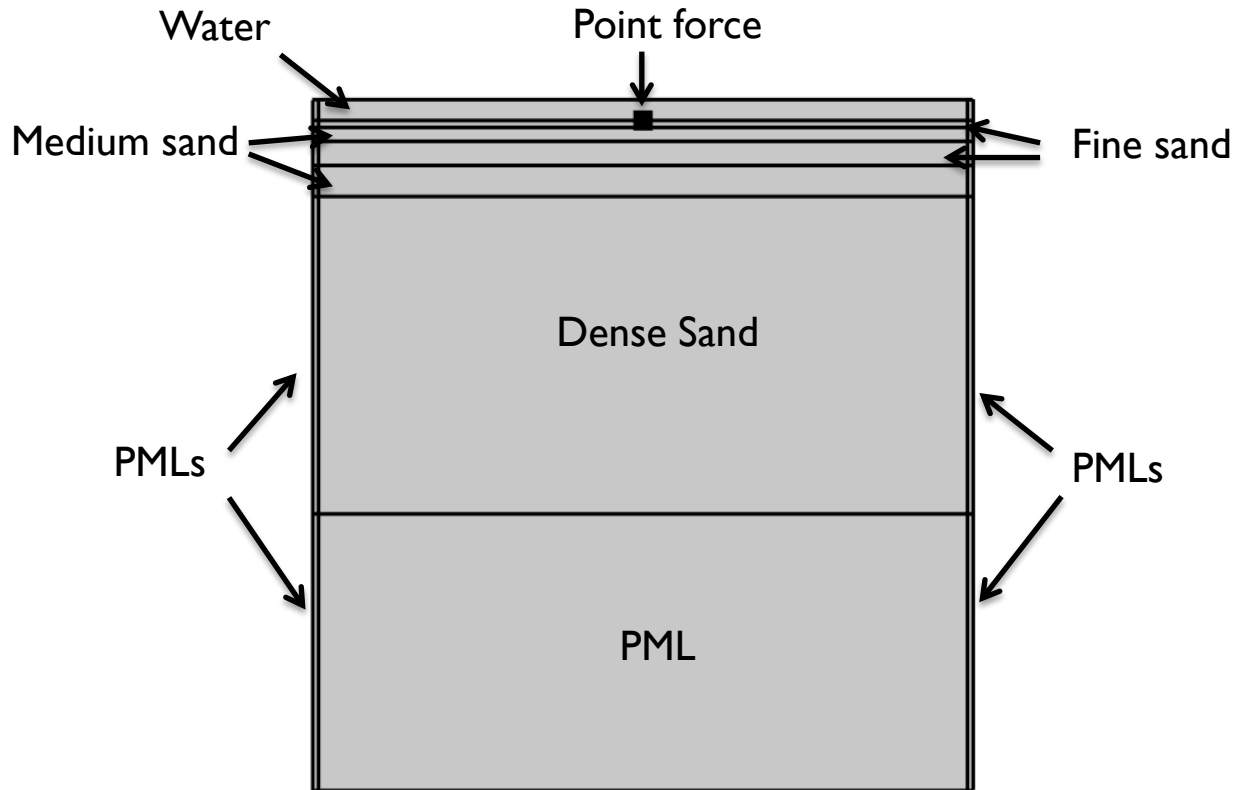


Dynamic stiffness matrix inversion

Core sample analysis

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# Modeling



- Pressure Acoustics, Frequency Domain: Water
- Solid Mechanics, Frequency Domain: Sand layers



# Modeling



- Interface conditions:

$$d = \nabla\Phi + \nabla \times \Psi \quad \text{Total particle displacement in elastic}$$

$$\frac{1}{\rho\omega^2} \frac{\partial p}{\partial n} = \nabla\Phi \cdot \hat{n} + (\nabla \times \Psi) \cdot \hat{n} \quad \text{Continuity of vertical displacement}$$

$$p = \sigma_n \quad \text{Continuity of normal stress}$$



# Modeling



- Interface conditions:

$$d = \nabla\Phi + \nabla \times \Psi$$

Total particle displacement in elastic

$$\frac{1}{\rho\omega^2} \frac{\partial p}{\partial n} = \nabla\Phi \cdot \hat{n} + (\nabla \times \Psi) \cdot \hat{n}$$

Continuity of vertical displacement

$$p = \sigma_n$$

Continuity of normal stress

Pressure Acoustics: Normal acceleration node

Solid Mechanics: Boundary load node



# Modeling



- Source
  - Solid Mechanics: point load node
  - Mass of weight  $\times$  gravitational acceleration
- Mesh criterion
  - Based on smallest wavelength (shear)
  - Maximum element size:  $\lambda/8$



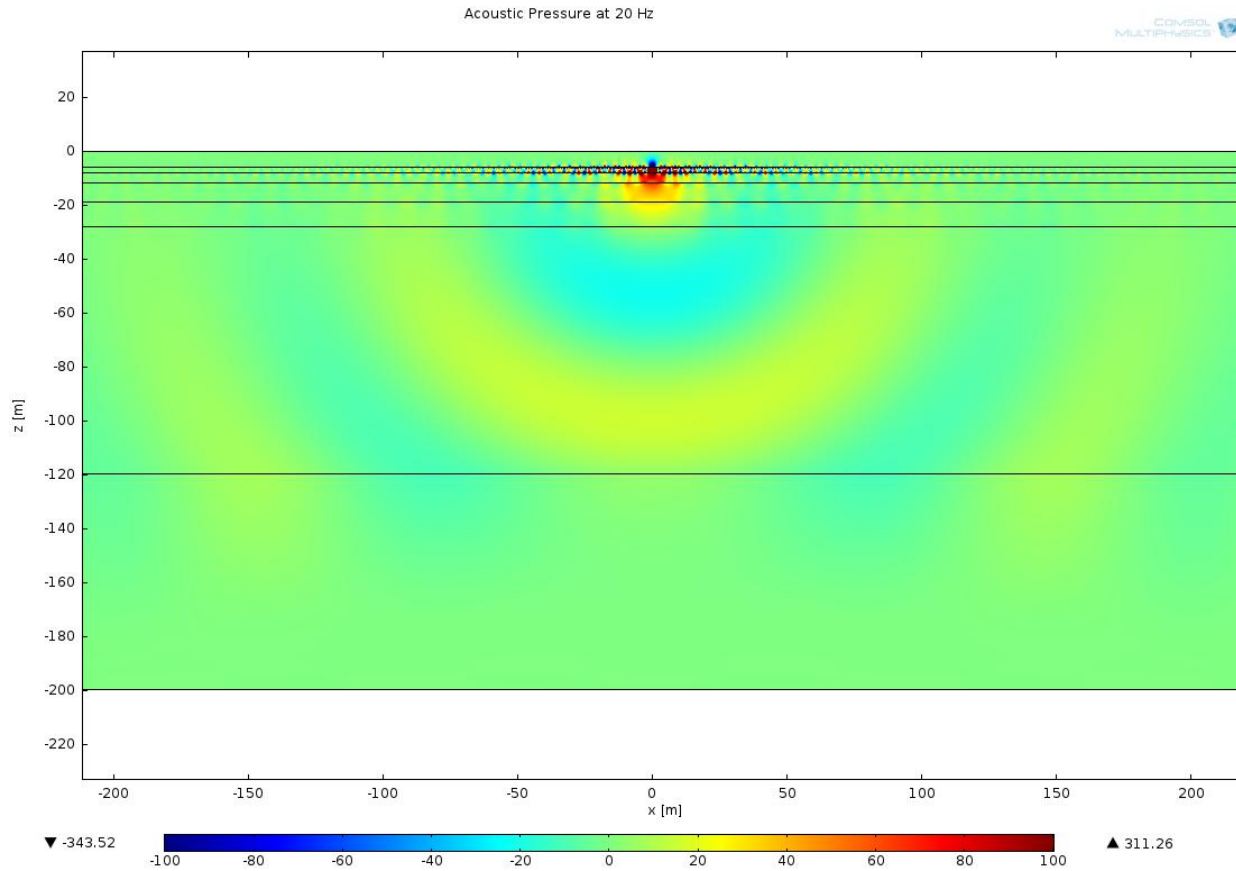
# Modeling



- PMLs do not effectively absorb Scholte waves and un-attenuated shear waves in elastic media.
- PML considerations:
  - Attenuate shear wave and use PMLs to only absorb compressional wave.
  - Make computational domain long enough so the Scholte wave is mostly dissipated at the boundary



# Modeling





# Post-processing



- Spatial Fourier Transform

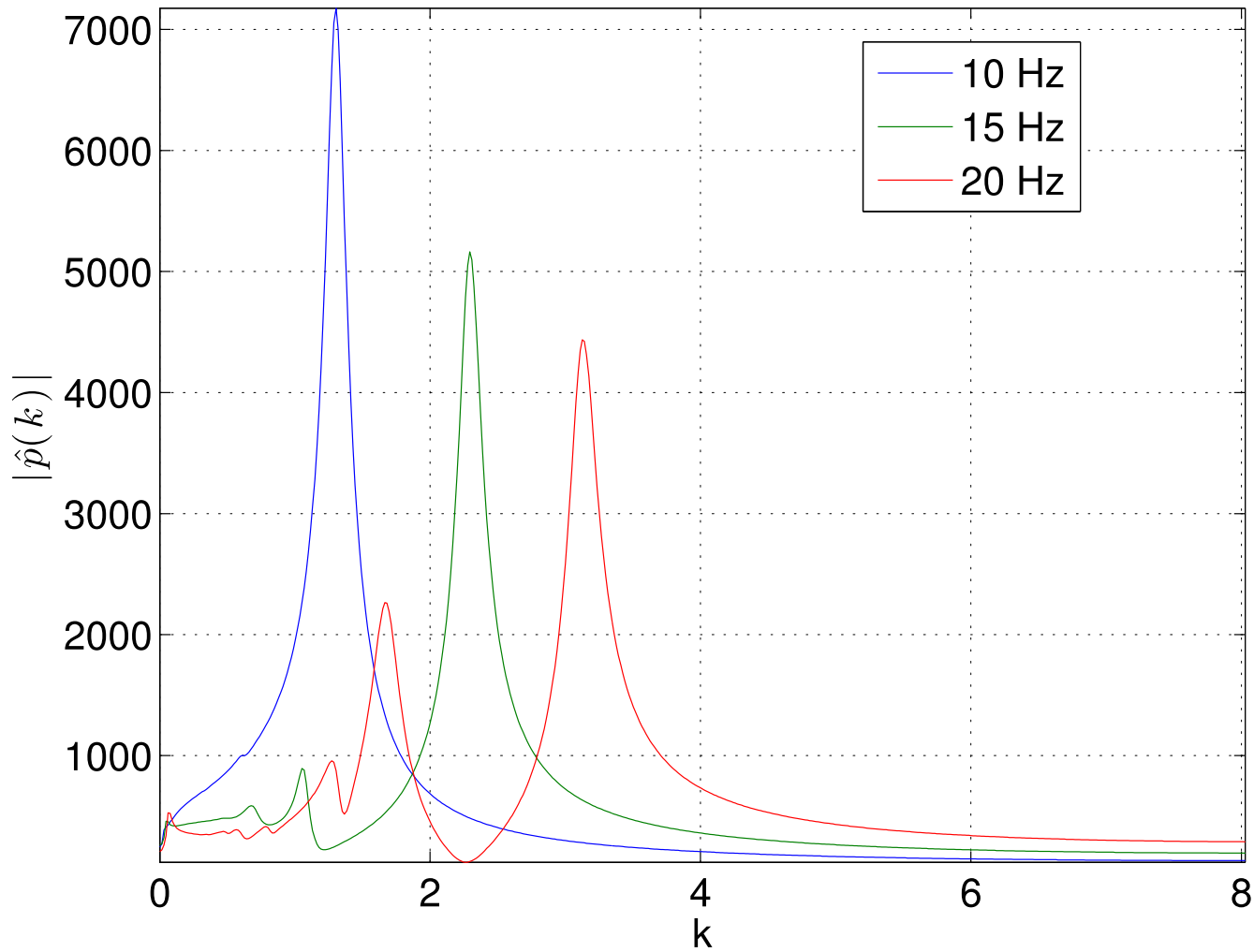
$$\hat{p}(k) = \int_{-\infty}^{\infty} p(x) e^{-i2\pi kx} dx$$

$$c_{ph} = \frac{2\pi f}{k}$$

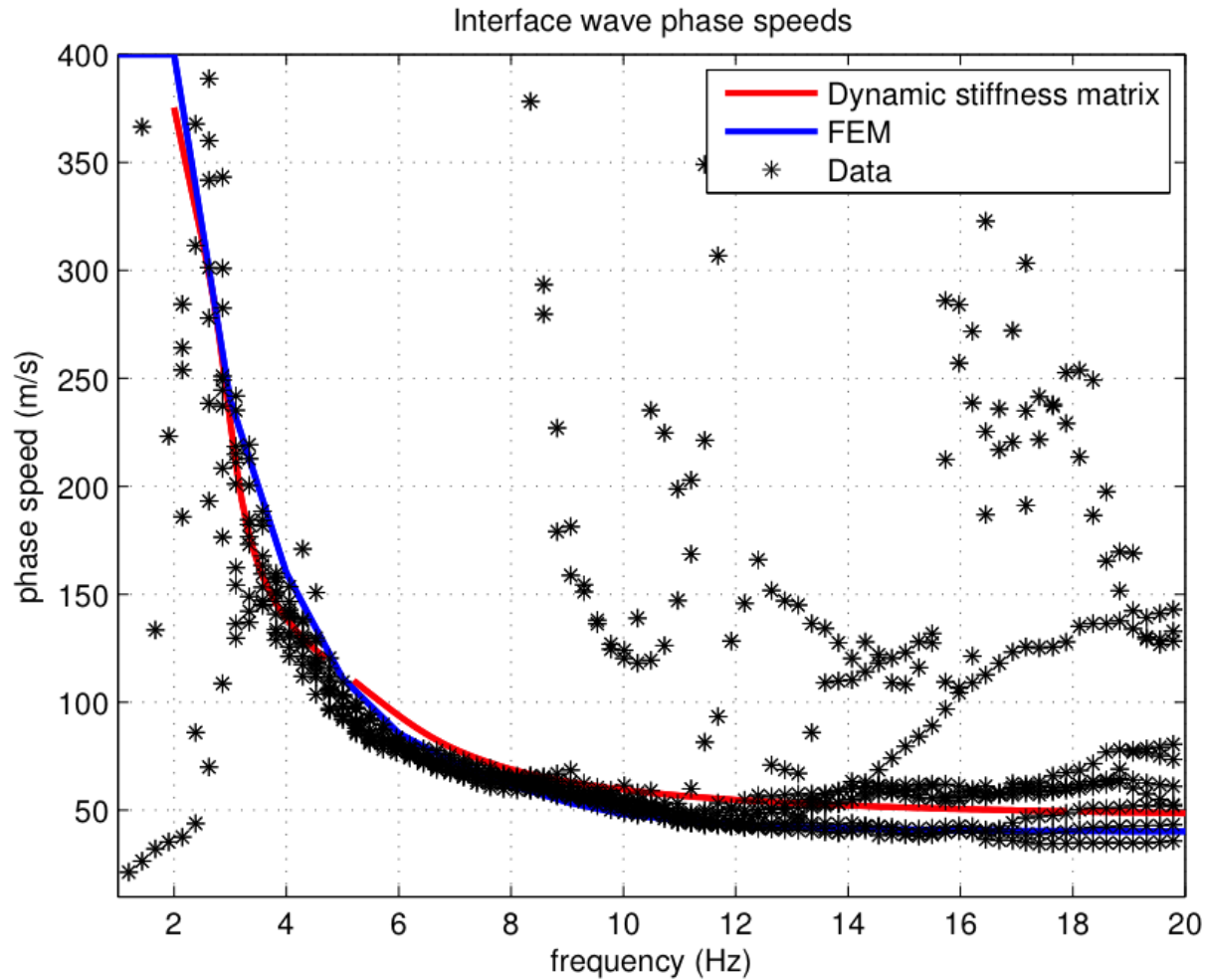


# Post-processing

Spatial fourier transform



# Post-processing





# Conclusion



- COMSOL used to model experiment at Narragansett Bay, RI.
- A spatial Fourier transform was used to calculate phase speeds
- Computed phase speeds agree well with measured data.