Key-Holes Magnetron Design and Multiphysics Simulation

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Abstract: This paper describes the design and the characterization of an 8 slots resonant cavities Magnetron, which undergoes the thermal-structural effects due to the cathode heating. The proposed study involves Thermal Stress (TS), Eigen-frequency (EF) and Particle Tracing (PT) analysis based on a COMSOL Multiphysics (MP) simulation.

Magnetrons are well known and more utilized High Power (HP) Radiofrequency (RF) Vacuum Tube (VT) oscillators. In order to generate high power signals, they employ thermoelectric cathodes which can reach very high temperatures, necessary to produce the enough surface charge density [1].

Since device efficiency depends critically to the operating temperature, a Multiphysics (MP) approach has been adopted.

A PT and an EF analysis with a computation of the power density distribution of the electric field resonant modes and particle trajectories and velocities have been performed, considering the thermal-structural modifications induced by the cathode heating to the entire structure.

Keywords: Magnetron, Eigen-frequency, Thermal Stress, Moving Mesh, Charged Particle Tracing.

1. Introduction

MAGNETRONS are well known and more utilized Vacuum Tube (VT) oscillators which generates High Power (HP) radiofrequency (RF) signals by transferring energy from an electron stream to a RF field. Typical Magnetrons are constituted by a cylindrical cathode inserted coaxially in a cylindrical anode, in which a set of resonant cavities is excavated. Electrons are emitted by the cathode, from which are extracted by anode-cathode voltage. Electron trajectory is then deviated by the interaction with a static magnetic field in the axial direction, in order to let electrons oscillating between the anode resonant cavities, describing epicycloids. During this process, the electron velocity decrease, and with it, also its energy decreases since is transferred to a RF oscillating field.

In this study we describe an eight key holes X-Band Magnetron operating in π mode, with copper anode and tungsten cathode. In order to simplify the model, cathode surface is modeled as a copper layer. The comprised material between anode and cathode is a high vacuum medium that is air at pressure of $P = 10^{-10}$ bar.

The resonant mode of the coupled cavities structure is called Normal Mode (NM), it identifies the field distributions of coupled oscillators. Several NM's are allowed in the anode-cathode space region. NM's are called with their resonant electric field phase shift between two contiguous cavities. Typical NM's for magnetrons are the π and 2π mode.

By selecting opportune NM frequency, axial static magnetic induction *B* and anode cathode voltage *V*, electrons can oscillate and are not captured by the anode. This condition occurs when the static field equals a particular value called critical field related to the chosen NM.

In order to obtain the critical field, the anode-cathode voltage must equal the critical voltage *Vc* and the axial magnetostatic induction must equal the critical induction *Bc*.

Critical voltage of designed device can be described by (1):

$$V_C = \frac{1}{2}\pi B r_m d f \tag{1}$$

With $r_m = (r_a^2 - r_k^2)/(2r_a)$, where, r_m is the effective medium radius of anode and cathode in which r_k and r_a are respectively the cathode and anode radii, *B* is the Magnetic induction field applied along the axial direction, *d* the anode cathode distance and *f* the desired NM operative frequency which will correspond to the frequency of the generated microwave power.

The critical magnetic induction is given by (2):

$$B_C = \sqrt{\frac{2mV}{ed^2}} \tag{2}$$

Where *V* is voltage applied between anode and cathode. Critical values are much higher than the typical operative values, which are chosen in order to maximize the Magnetron efficiency [1].

For this device, the design π mode frequency is f = 9 GHz, then Vc = 130 [KV] with Bc = 1359 [G]. The chosen operative values are V = 60 [KV] and B = 1330 [G].

The power dissipation of the cathode produces a considerable temperature increase and induces a thermal expansion of both the cathode and the anode, which is heated by the heat transfer operated by the non ideal vacuum between anode and cathode regions. If the cavity temperature exceeds a certain threshold, oscillation failure or device damage may occur.

The thermal expansion of the materials may also induce significant stresses and strains with consequent displacement of the resonant structure, which alter the desired Electromagnetic (EM) behavior of the device.

In order to perform the characterization of the coupled cavities resonant system of Magnetrons, an Eigen-frequency (EF) study is needed, in order to find the frequencies relative to the allowed resonant normal modes. By performing an Eigenvalues (EV) analysis we can estimate these resonant modes, describing their electric field power density distribution in the transversal cross section of the Magnetron. This study is to be performed, obviously in the steady state condition, as the resonances are defined, by using a stationary study.

Moreover, a Thermal Stress (TS) stationary analysis allows to determine the temperature and the deformation when the heat generated by the cathode power dissipation has been diffused on all the reachable Magnetron components and the system has become thermally stable, since the external temperature is by then steady over all the Magnetron outer boundaries, which are exposed to the external environment.

For the Magnetron operation, an electron cloud is generated by the thermo-electrical effect on the cylindrical cathode lateral surfaces. These electrons are accelerated, in the radial direction, by the static anode-cathode electric field and deflected by the axial magnetic field, in order to be extracted from their initial position, avoiding that they fall on the anode.

Since the ES fields are altered by the particles presence, we have a two-way coupling between the particles and field: the field exerts a force on the particles and the particles exert a space charge on the field. Moreover, the variables of the problem are also dependent on the Coulomb forces interactions between the particles. For these reasons, is necessary to solve the problem for the particles and fields simultaneously by couple a Particle Tracing (PT) and an Electrostatic (ES) analysis [5]. These coupling effects inside the acceleration-deflection space require a time dependent (TD) analysis, in order to perform a PT characterization of the particle trajectories representation with a description of energy and velocity. It allows besides to estimate the cathode electron density and the charge distribution of the electron clouds in the transversal cross section of the Magnetron.

The Thermal Stress deformation of the geometric shape and consequent modification of the potentially distribution of the EM field, in addition to the alteration of the resonance, produces alternations of the particle spatial distribution and motion, causing variation of the output power. By performing a TS analysis, the shape deformation can be estimated and used to define a new geometry in which execute the EF, ES and PT simulation.

2. Use of COMSOL Multiphysics

A Finite Element Method (FEM) based Multiphysics simulation using COMSOL can couple TS, EM, ES and CPT analysis by Moving Mesh (MM) dedicated interface and storing temperature information.

In order to decrease computational time and resources maintaining accuracy, the device model is organized by using several strategies allowed by COMSOL. The architecture of the model is based on TS, MM, Electromagnetic Waves (EMW), ES and CPT COMSOL modules.

2.1 Thermal Stress

The TS module is employed to describe the Thermal-Structural formulation of the problem by using the following features [2]:

- Heat Sources: The cathode represents a constant volume heat sources made by tungsten. The heat power density is established by design requirements, in order to allow the cathode boundaries to reach the necessary temperature for thermo-electrical effect. The heat power density has been set to Q = 0.405 [GW/m^3], in order to obtain the operative temperature of cathode thermo-electrical emission, which is T=1050°C.

- Heat transfer in Fluids: The non ideal vacuum atmosphere inside the Magnetron volume between anode and cathode is modeled only to describe the heat transfer from cathode to anode.
- Fixed constraints: The external metallic surfaces of Magnetrons are locked to rigid structures in order to support the device. Thus they represent fixed constraints for the generation of the compressive forces induced by the thermal expansion. The bases of the cylinder which represents the cathode are also connected to an internal support, so that represent other fixed boundaries. For this reason all the anode external surface and the surface base of the cathode are modeled as fixed constraints.
- Temperature: The external lateral surfaces of the Magnetrons are typically cooled by high efficiency fluid cooling systems. The temperature of Magnetron lateral outer boundaries is constant and equals the cooling fluid temperature, in order to model the thermal steady state. This temperature has been fixed to 35°C, consistently with typical Magnetron cooling systems.

2.2 Moving Mesh

The MM module is employed to give the rules on how to move the mesh in function of the displacement computed by the TS analysis. In order to perform the RF analysis on the deformed geometry, the MM module uses the following features [3]:

- Prescribed deformation: The combining structure and the SSPA's represent the volumes subjected to deformation. The displacement vectors (u, v, w) computed by the TS module are employed to specify this volumetric deformation. Its prescribed mesh displacement is set to dx = u, dy = v, dz = w.
- Free deformation: The non ideal vacuum volume (which is not subjected to any structural elastic formulation by the TS analysis) is free to move. Initial deformation is set to $dx_0 = 0$, $dy_0 = 0$ and $dz_0 = 0$.
- Prescribed Mesh Displacement: This condition specifies that the boundary is to be deformed by the thermal stress computation, though is attached to the free deformation air boundary. The resonant structure boundaries, in the region delimited by cathode external surface and anode internal surface, are subjected to deformation. This superficial displacement has been specified by setting the prescribed mesh displacement to d_x = u, d_y = v, d_z = w.

2.3 Electromagnetic Waves

The EMW module is describes the EM modeling, considering the surfaces losses, by employing the following features [4]:

- Impedance boundary condition: The lateral cathode surfaces and the internal lateral anode surface are modeled in order to consider the losses due to the partial penetration of the electric field in the lossy material which constitutes the anode and cathode walls. This condition allows to exclude the anode and cathode domain to the EMW calculation, avoiding the meshing and saving computational cost. The specified thickness of the anode and cathode boundaries is fixed to 10 mm.
- Scattering boundary condition: The surface bases of the interaction region are made of non ideal vacuum. So that, the plane wave which crosses this boundary must be free to feed forward. This condition makes a boundary transparent for a scattered wave and potential resonance errors are avoided.

2.3 Electrostatics

The ES module is employed to describe the Electrostatic field formulation of the problem by using the main following features [6]:

- Electric Potential: The anode electric potential is set to zero and the cathode electric potential to V_k = -60 [MV].

- Dielectric shielding: This feature is used to separate the anode electric potential to the cathode electric potential and is applied on the external boundary which connects anode and cathode. The surface thickness has been set to 5.0 [mm].

2.4 Charged Particle Tracing

The CPT module is employed to calculate the particle trajectories considering ES computation, by employing the main following features:

- Inlet: In order to simplify the particle emission, by neglecting the statistical behavior, the electrons are released from the cathode boundary only on its normal direction with a null medium value initial velocity. Since the cathode current density is constant in time and on the cathode surface, the charge release is represented by a short pulse sequence with initial null value, so that the number of particle per release is given by (3):

$$N = \frac{I\Delta t}{e} \tag{3}$$

  Where I is the design cathode current, e the elementary charge of the electron and Δt is the time interval between two consecutive charge releases. For this design have been set I = 110 [A] and $\Delta t = 1.8 \cdot 10^{-11}$[s], thus we have $N=1.23\cdot 10^{10}$ particles per release. In order to decrease computational costs, this number has been set to $N^1=123$ and, as explained below, a charge multiplier factor has been added in order to respect the space charge effect. The particle release times have been set to start from $t=\Delta t$ and stop to $t=30\,\Delta t$, by steps of Δt.
- Particle field interaction: This node adds two-way coupling between the particles and field. In order to model at the same time the space charge effect (exerted by the electrons on the ES field) and the force (exerted by the electric field on the charges) this node has been added in the CPT COMSOL module. Since the number of charges per release has been set to $N^1=123$, a charge multiplier factor of $n = 10^8$ has been specified in this feature. The charge multiplication factor can be calculated as $n=N/N^1$.
- Electric Force: This feature is employed to define the electric part of the Lorentz force $F=e(-\nabla V)$. The particles are accelerated in the same orientation as the electric field. The force is specified via the electric potential computed time dependently by the ES module, by setting "V = mod1.V" [5].
- Magnetic Force: This feature is employed to define the magnetic part of the Lorentz force $F=ev\wedge B$ [5]. The particles are deflected by the operating magnetic field imposed as a domain condition. This force is specified via the operating magnetic induction which is B = 1330 [G].
- Particle-Particle Interaction: This feature is employed to include the Coulomb interaction force between charged particles to the total force. The particle position is step by step updated, and the process repeats until the specified end time for the simulation is reached [5].
- Wall: In this feature, the anode disposition to absorb charges has been included by setting the Disappear modality of the walls. This condition well allows to identify the electrons trajectory in the operative oscillating normal mode of the Magnetron.

2.5 Analysis

In order to consider the computed temperature resulted from the TS analysis, the information has been inserted as the default temperature in all the features in the EMW and ES module settings, where required. By the MM module the meshes of the model has been moved in function of the displacement computed by the TS analysis, in order to couple the TS with RF, ES and CPT simulation by performing the RF, ES and CPT analysis on the deformed geometry.

Due to the simplicity of the geometry, default settings have been adopted, by choosing Physic controlled mesh with Finer element size. Complete mesh consists of 45258 elements. Minimum element size is 4.08E-4 m, which corresponds to 0.012 λ, where λ is the wavelength at 9 GHz.

The RF EF analysis has been performed specifying the initial guess of 5 GHz, by inserting this value in "search for EF around" and the number of desired EF's has been set to 10, in order to have a large resolution to visualize the main normal modes, which are the π and 2π modes .

The solver is organized in performing two steps: First, a stationary analysis to compute the thermal TS and MM in fully coupled mode, then an EF step to perform a stationary analysis which calculates the resonant electric fields.

Because of the impedance boundary condition with a finite conductivity value, the model solves a nonlinear Eigen-value problem. It is necessary to provide a frequency at which to initially evaluate the frequency-dependent surface losses [4]. In the Eigen-value solver an initial guess of 5 GHz has been indicated as a linearization point.

For the ES and CPT computation, the time dependent analysis has been set to start at time t=0 and end at $t=120 \cdot \Delta t$, by steps of Δt. Note that, in order to see the initial current transient, the particle release has been set to start at $t=\Delta t$.

3. Results

3.1 Temperature

By imposing a Heat density of Q = 0.405 [GW/m^3], for the cathode heating, the TS stationary analysis has shown a maximum temperature of 1050°C on the cathode internal boundaries, perfectly according with the thermo-electrical threshold for cathode electron release. This result is shown in Figure 1.

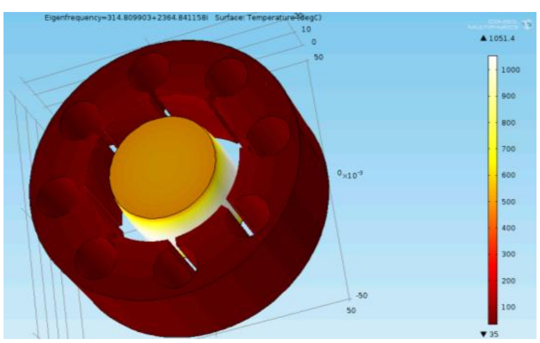

Figure 1. Temperature.

3.2 Stress and Displacement

By receiving the stored Temperature, the fully coupled stationary analysis has been shown the following results. In order to underline the deformation, stress and displacement are been plotted with a magnified scale, so that the deformation scale has been increased. In the following figures, black outlines represent the original conformation, and the stained volume represents the deformed structure.

The maximum stress is near the cathode base surfaces, since these ones are fixed constraints and cathode is the heat source. Maximum stress is about 6.0 [GN/m^2] as shown in Figure 2.

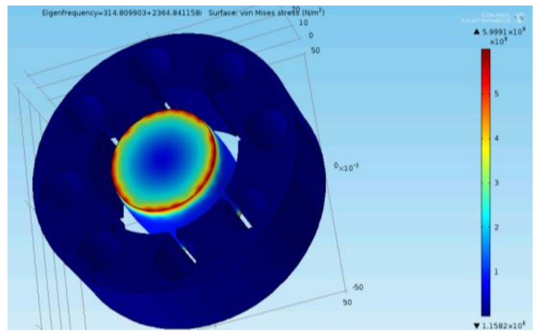

Figure 2. Stress – Magnified.

The maximum total displacement is located on lateral cathode surface, and is about 0.125 [mm] and is shown in Figure 3.

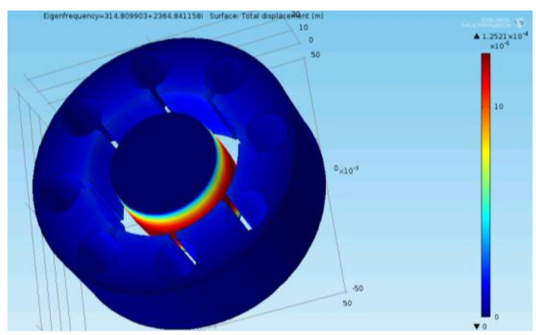

Figure 3. Displacement – Magnified.

These values are very small respect the wavelength, so will result in producing a little alteration of the structure oscillating properties, as shown by the following eigenfrequency analysis.

3.3 Electric fields

The simulation output shows the field power density distribution of the resonant modes in the transversal cross section of the Magnetron.

A cut plane has been set on the transversal section of the Magnetron and it has been used to visualize the Electric field streamlines, in order to evaluate the presence of the searched normal modes. In the plane, the modulus of the x and y component of the electric field have been plotted. A particular graphical representation has been adopted in order to perform this evaluation. The search of normal modes has been supported by superposing, on the same streamline graphic, a set of vectors composed by the x and y components of the electric field at the center of the cavities. In order to save space we report only the resonant field plot in thermal stress working condition, which qualitatively fits the plot in cold condition. Resonant field are plotted in Figure 4 and 5. Arrows represent the resonant electric field direction inside cavities.

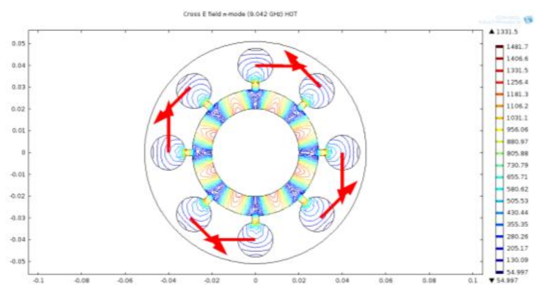

Figure 4. π mode - E field - cross section – working conditions.

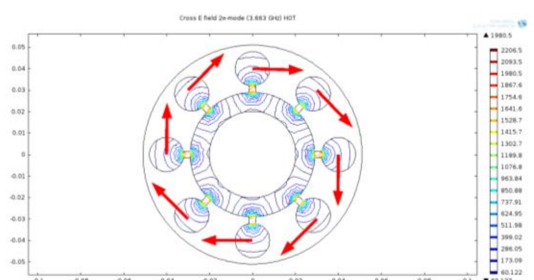

Figure 5. 2π mode - E field - cross section – working conditions.

3.4 Eigen-frequencies and Quality factors

The resulting Magnetron EF's and Quality factors in cold conditions are: $f_{2\pi}$ = 3.673 GHz, f_{π} = 9.061 GHz and $Q_{2\pi}$ = 1000, Q_{π} = 8300.
In thermo mechanical operative conditions, the displacement and the electrical properties alteration of the materials cause a variation of the Eigen-frequencies to: $f_{2\pi}$ = 3.663 GHz, f_{π} = 9.042 GHz and a variation of the quality factors to: $Q_{2\pi}$ = 950, Q_{π} = 8250.

3.5 Electrostatic Fields

Electric fields have been computed in order to consider the interaction between the electrostatic and magnetostatic prescribed fields with the space charge effect due to the presence of electron in the interaction area with a certain energy. The resulting maximum electric fields are: In cold conditions: E_{Max} = 7.94 [MV/m], and in thermo mechanical operative conditions: E_{Max} = 8.03 [MV/m]. This situation allows to work in X-band in pulsed wave with an appropriate duty cycle, in order to not exceed the Kilpatrik threshold, avoiding to trigger sparks.

3.6 Particle trajectories and velocities

Thermal excitation contributes to increase the maximum particle velocity magnitude from v_{cold} = 1.196·10^8 [m/s] in cold conditions, to v_{hot} = 1.227·10^8 [m/s] in thermo mechanical operative conditions. Particle trajectories and velocities are represented in Figure 6 and 7. A more visible trajectory is reported in Figure 8.

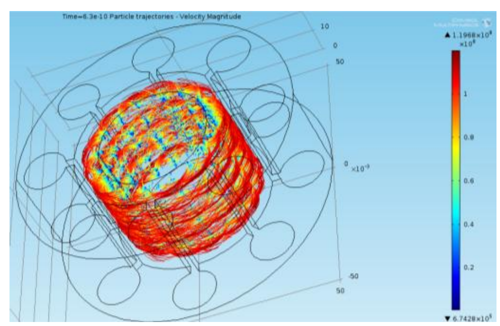

Figure 6. Particle Trajectories with velocity magnitude – without heating.

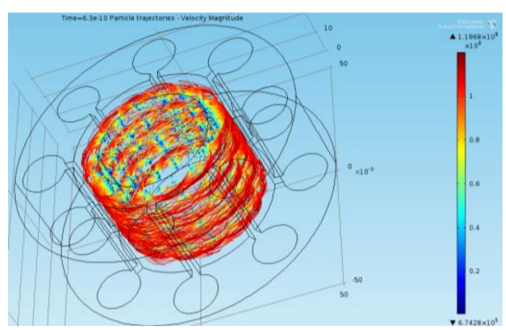

Figure 7. Particle Trajectories with velocity magnitude – working conditions.

3.7 Superposition of particle trajectories and resonant field

The superposition of the resonant field and the trajectories of the electrons have been plotted on a plane which bisects the Magnetron structure. This study allows to individuate the correct operating electrostatic and magnetostatic fields conditions. By setting the static conditions of π mode operation and plotting particle trajectories, a superposition with π mode oscillating field shows a correct synchronism condition. This result is reported in Figure 8.

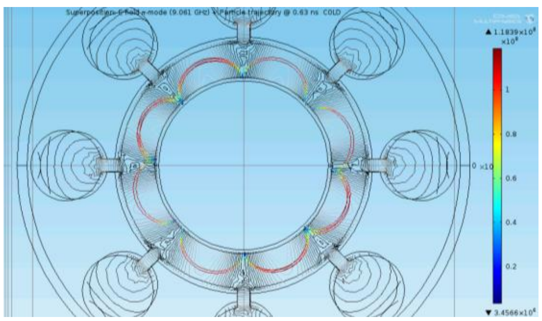

Figure 8. Cross section of a short particle tracing scrap over π mode resonant electric field in cold conditions.

3.8 Magnetron π mode Working Points

By performing this simulation, the Magnetron working points related to the π mode operation have been found. These points are the couples of electrostatic voltage V and magnetostatic induction B which ensures the superposition of particle trajectories with the resonant π mode electric field. Working points are reported below in Figure 9.

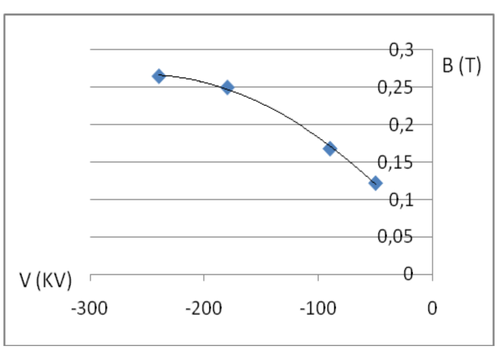

Figure 9. Magnetron π mode working points: Cathode voltage on the horizontal axis and axial magnetic induction on the vertical axis.

4. Conclusions

The Magnetron resonance and particle tracing have been studied using COMSOL, and many aspects has been investigated.

The frequencies of resonance relative to the main normal modes of the analyzed device have been found, in cold and in thermal-stress thermo mechanical operative conditions, by using an EF analysis and adopting particular graphical representation of the electric field, in order to facilitate the search.

The alteration of the normal mode frequency of resonance, due to the thermal-structural working condition, has been studied, obtaining a complete characterization of the proposed device.

The electric field power density has been computed and plotted in cold and thermal-stress operative conditions. Quality factors have been also plotted for all the found resonances.

Operative electrostatics and magnetostatics condition are been, applied and the interaction with space charge, have been computed together with the particle coordinates and velocities.

The Magnetron particle motion related to the π mode operation has been described.

In order to verify the correct excitation of the particles by the applied fields, the superposition of the resonant field and particle trajectories cross sections are been performed by post-processing the separate physics results.

The Magnetron π mode working points have been documented.

By applying the design condition: V=60KV, B=1330G in order to have I=110A; this device, with a typical efficiency of 40%, can produce a pulsed microwave peak power of 2.64 MW.

8. References

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