Control of Real Distributed Parameter Systems Modeled by COMSOL Multiphysics®

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Introduction. Dynamics of distributed parameter systems (DPS) depends on both position and time. The time-space coupled nature of the DPS can be mathematically described by partial differential equations (PDE) with boundary and initial conditions. Resulting models obtained through finite element method (FEM) solution of PDE in the software environment COMSOL Multiphysics® give possibilities not only for analyzing the dynamics, but also for optimization and control of these systems as DPS. Base conception of FEM modeling and control is presented for two various real DPS: casting mould and extruder body.

LDS representation of DPS

DPS very frequently are found in various technical and non-technical branches in the form of lumped-input/distributed-output system (LDS).

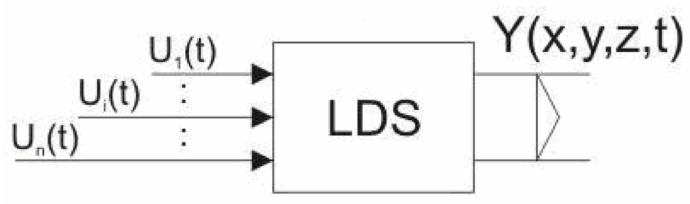

Fig. LDS structure of DPS

$$Y(\bar{x},t)=\sum_{i=1}^{n}Y_i(\bar{x},t)=\sum_{i=1}^{n}\mathcal{G}_i(\bar{x},t)\otimes U_i(t)$$

$$Y(\bar{x},s)=\sum_{i=1}^{n}Y_i(\bar{x},s)=\sum_{i=1}^{n}S_i(\bar{x},s)U_i(s)$$

For points located in surroundings of lumped input variables:

$$\left\{Y_i(\bar{x}_i,k)=\mathcal{G}H_i(\bar{x}_i,k)\oplus U_i(k)\right\}_{i=1,n}\qquad \left\{Y_i(\bar{x}_i,z)=SH_i(\bar{x}_i,z)U_i(z)\right\}_{i=1,n}$$

Space dependency in steady-state: $\left\{\mathcal{H}HR_i(\bar{x},\infty)=\dfrac{\mathcal{H}H_i(\bar{x},\infty)}{\mathcal{H}H_i(\bar{x}_i,\infty)}\right\}_{i=1,n}$

Feedback control loop based on LDS

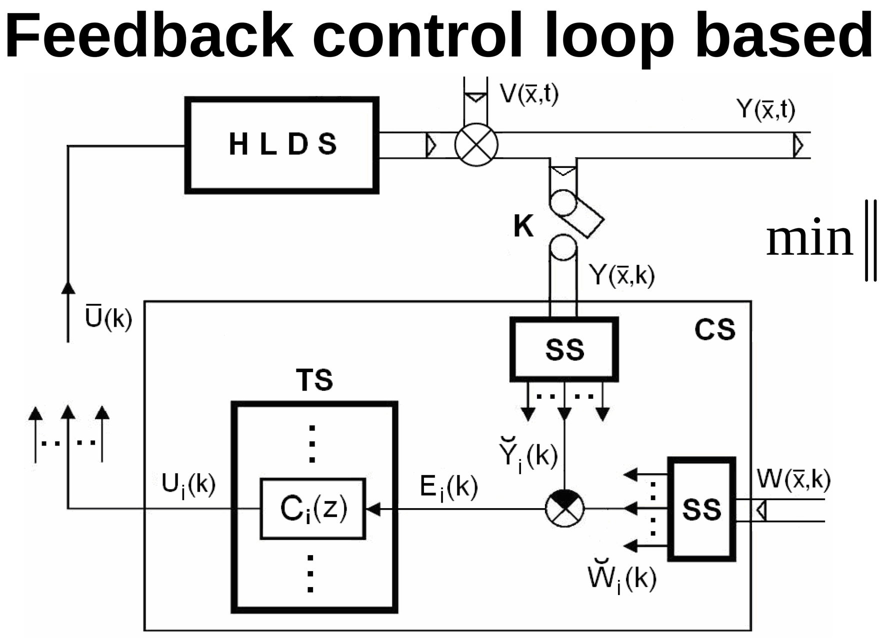

Fig. DPS feedback control loop

The goal of control:

$$\min\left\|\bar{E}(x,\infty)\right\|=\min\left\|W(\bar{x},\infty)-Y(\bar{x},\infty)\right\|$$

Control synthesis:

* Time Synthesis (TS)
* Space Synthesis (SS)

$$\min_{\tilde{Y}_i}\left\|Y(\bar{x},k)-\sum_{i=1}^{n}Y_i(\bar{x}_i,k)\mathcal{H}HR_i(\bar{x},\infty)\right\|=\left\|Y(\bar{x},k)-\sum_{i=1}^{n}\tilde{Y}_i(k)\mathcal{H}HR_i(\bar{x},\infty)\right\|$$

$$\min_{\tilde{W}_i}\left\|W(\bar{x},\infty)-\sum_{i=1}^{n}W_i(\bar{x}_i,\infty)\mathcal{H}HR_i(\bar{x},\infty)\right\|=\left\|W(\bar{x},\infty)-\sum_{i=1}^{n}\tilde{W}_i(k)\mathcal{H}HR_i(\bar{x},\infty)\right\|$$

$$\bar{E}(k)=\left\{E_i(k)\right\}_i=\left\{\tilde{W}_i(k)-\tilde{Y}_i(k)\right\}_i$$

Modeling and control of temperature fields in casting mould

The quality of the castings is affected strongly by the distribution of temperature in the mould, which has both time and space dependence.

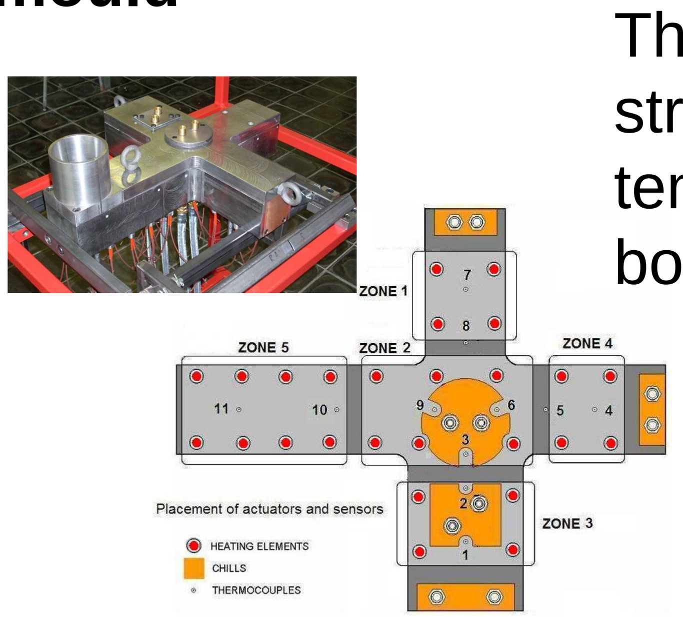

Fig. Benchmark casting plant and bottom side of the steel casting mould.

PDE with BC and IC:

$$\frac{\partial T(\bar{x},t)}{\partial t}-a\nabla^2 T(\bar{x},t)=\sum_{i=1}^{5}U_i(\bar{x},t)$$

$$-n(-\lambda\nabla T)=h(T_{ext}-T)$$

$$T(\bar{x},0)=T_{init}$$

$\left\{U_i(\bar{x},t)\right\}_{i=1,5}$: heat sources

FEM modeling of temperature fields of casting mould in **COMSOL Multiphysics®** with *Heat Transfer Module.*

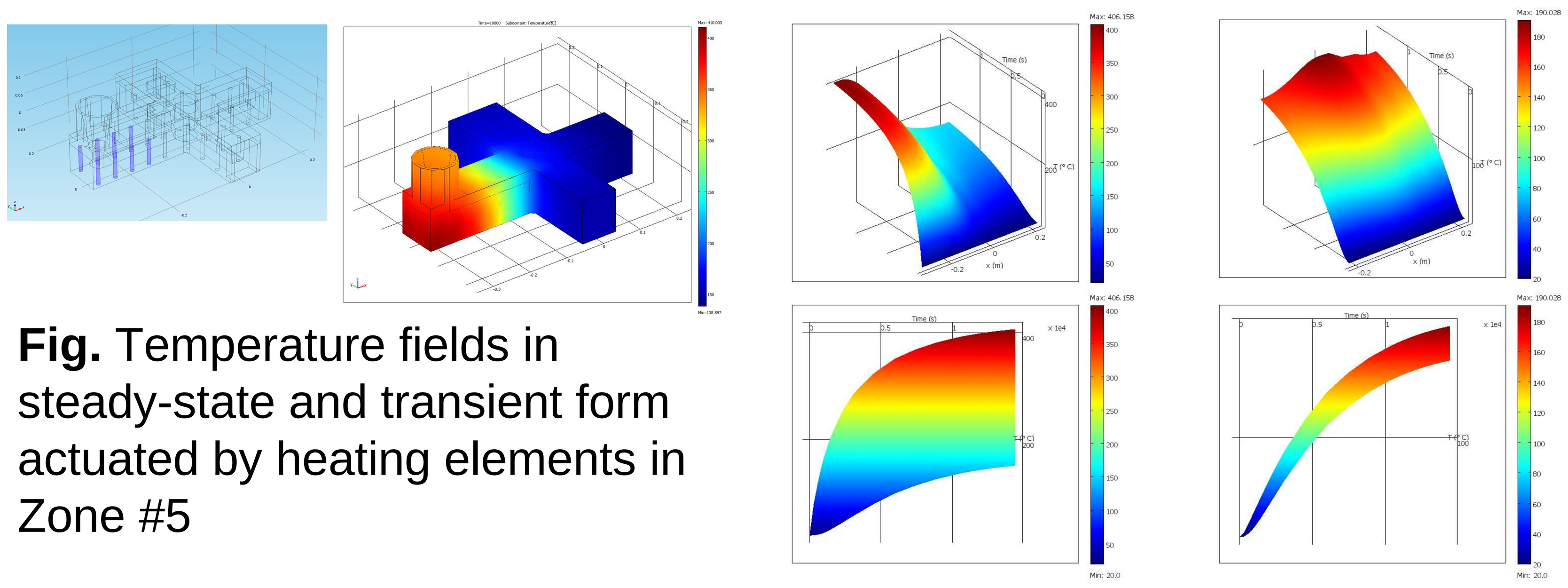

Fig. Temperature fields in steady-state and transient form actuated by heating elements in Zone #5

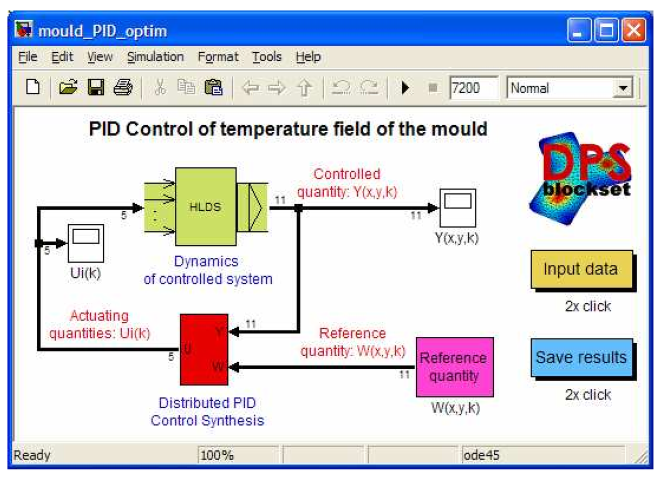

Fig. DPS feedback control loop with PID controllers in TS

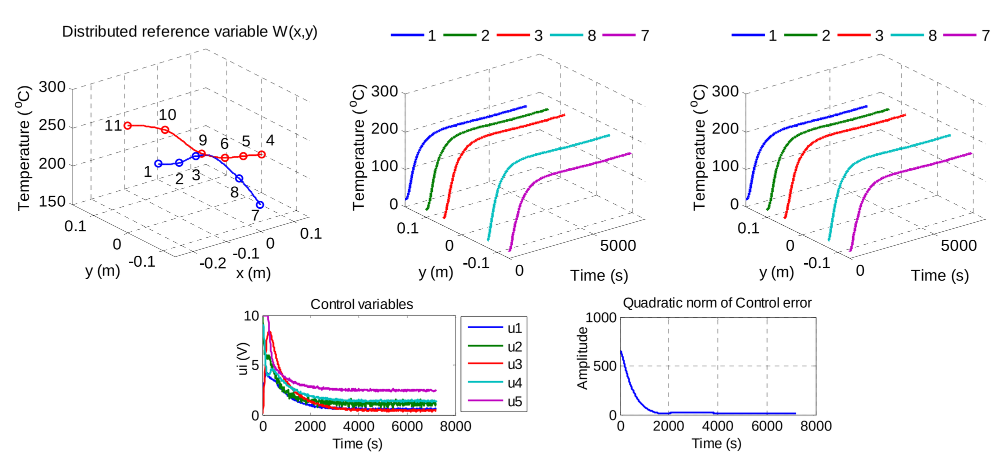

Fig. PID control of temperature field of the mould

Modeling and control of temperature fields in extruder

Extrusion is the most common plastics and rubber processing technology. The extruder barrel temperature field has a major influence on the product quality as well as the process itself.

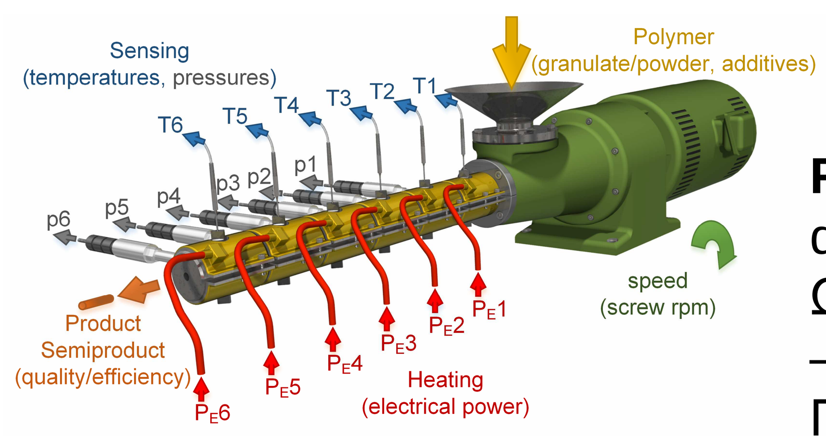

Fig. Extruder assembly

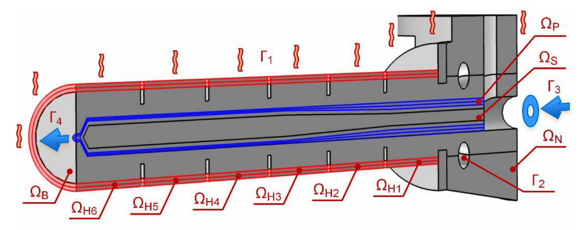

Fig. The computational model with the domains: Ω_B – barrel, Ω_P – plastic, Ω_S – screw, Ω_N – hopper, and $\Omega_{H1}\cdots_{H6}$ – heater bands; further boundaries: Γ_1 – outer surface, Γ_2 – cooling channel, Γ_3 – cold plastic inlet, and Γ_4 – hot plastic outlet.

Model of the extrusion process focused on the relation between the heater inputs and barrel temperature field: PDE with boundary conditions:

$$\rho c\left(\frac{\partial T}{\partial t}+\nabla T\ v\right)-\nabla(\lambda\nabla T)=\dot{q}\qquad \begin{aligned}\dot{q}(x,t)&=h.(T(x,t)-T_{ext}), & x\in\Gamma_1\\ \dot{q}(x,t)&=h.(T(x,t)-T_{CW}), & x\in\Gamma_2\end{aligned}$$

COMSOL Multiphysics® & LiveLink™ for MATLAB® for co-simulation and control

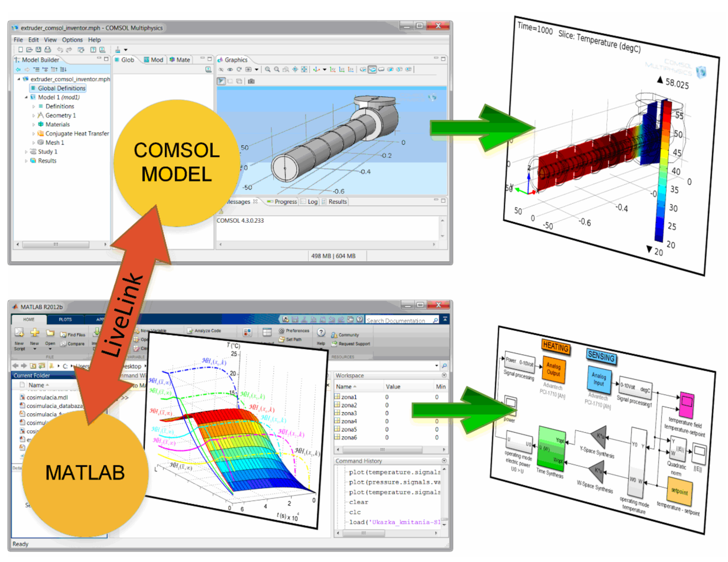

Fig. Co-simulation joining COMSOL Multiphysics® and MATLAB® & Simulink®

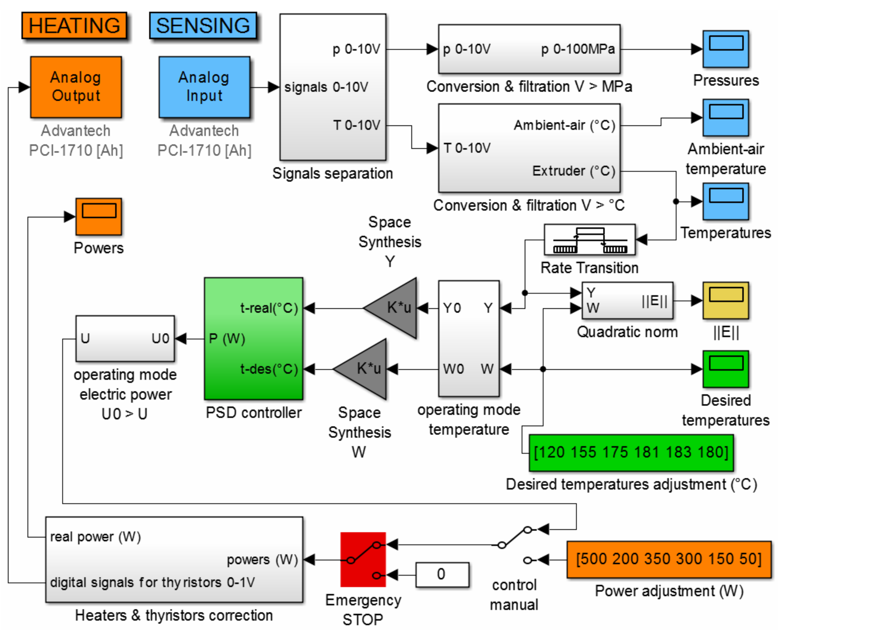

Fig. DPS feedback control scheme

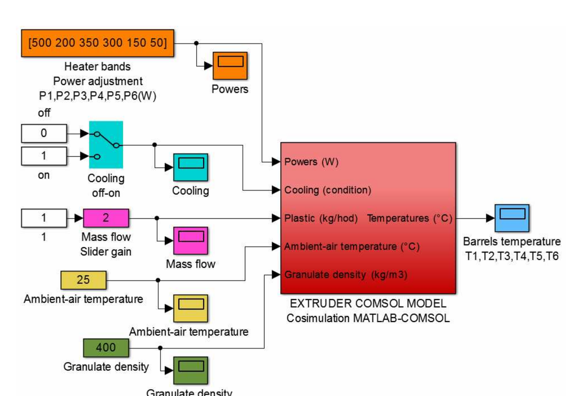

Fig. Co-simulation scheme

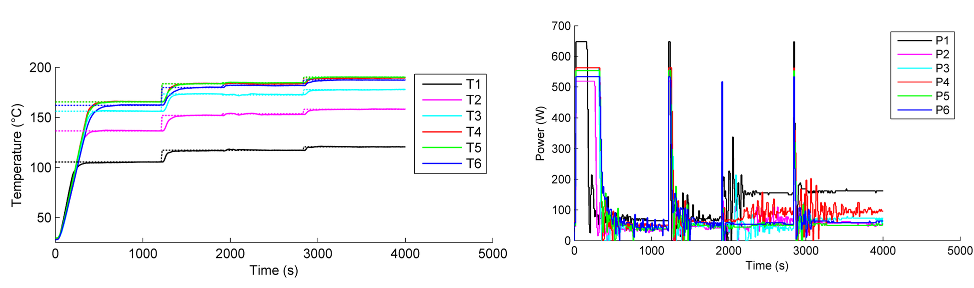

Fig. Real-time control of temperatures in extruder body

Conclusions. FEM modeling in COMSOL Multiphysics® and design of control of DPS based on LDS approach is suitable for various real technological processes.

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