2D Modeling of Elastic Wave Propagation in Solids Containing Closed Cracks with Friction

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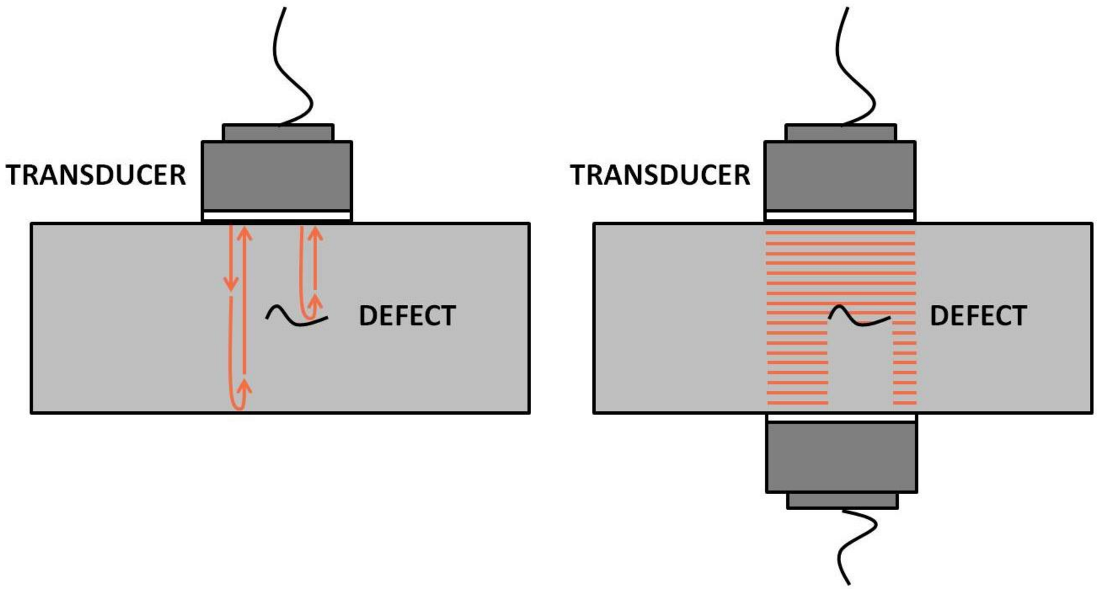
Ultrasonic Non-Destructive Testing (NDT)

A family of techniques based on the **propagation of ultrasonic elastic waves** in order to test materials or objects for the **presence of defects**, without destroying the material under study.



Conventional techniques mostly rely on reflection, transmission or scattering of ultrasonic waves at defects.

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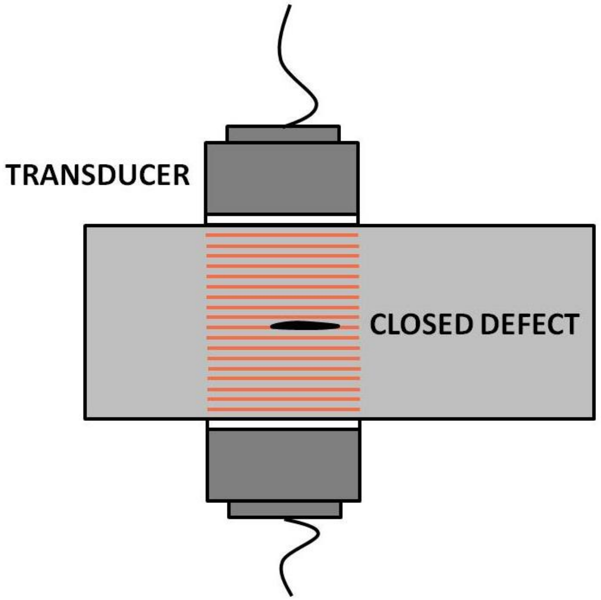
Contact-type defects

- Solid-to-solid interfaces with **imperfect adhesion**
- Defects can be either **opened or closed**



Conventional techniques are less sensitive to contact-like defects

Nonlinear ultrasonic techniques investigating nonlinear features induced by clapping and frictional behavior of contact-like defects:

- Harmonic generation
- Frequency modulation
- Distorted scaling with amplitude

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Modeling of contact-type defects

Crucial to push nonlinear ultrasonic NDT the next step forward!!!

Realistic and effective numerical modeling allows:

1. **Comparison between numerical and experimental results** for full defect characterization and localization, lifetime-predictions, etc.
2. **Further investigation of microscale contacts** to predict and explain the presence of macroscopic nonlinear events

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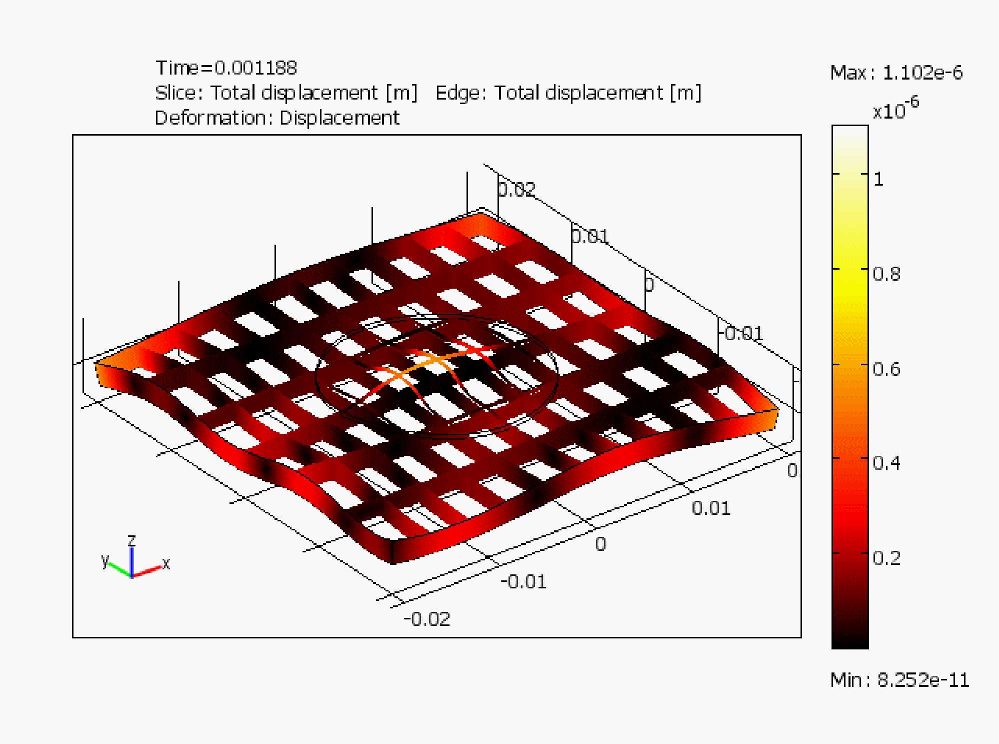
Previous modeling in COMSOL®

S. Delrue & K. Van Den Abeele, **Three-dimensional finite element simulation of closed delaminations in composite materials**, Ultrasonics 52 (2012) 315-324.



Clapping model (using virtual spring and damper forces) for defects with smooth surfaces

More realistic model, including:

- Roughness of internal crack faces
- Friction between crack faces
- Memory effects & Hysteresis

Outline

Theoretical & Numerical Model

Illustrative Example

Conclusion & Future Work

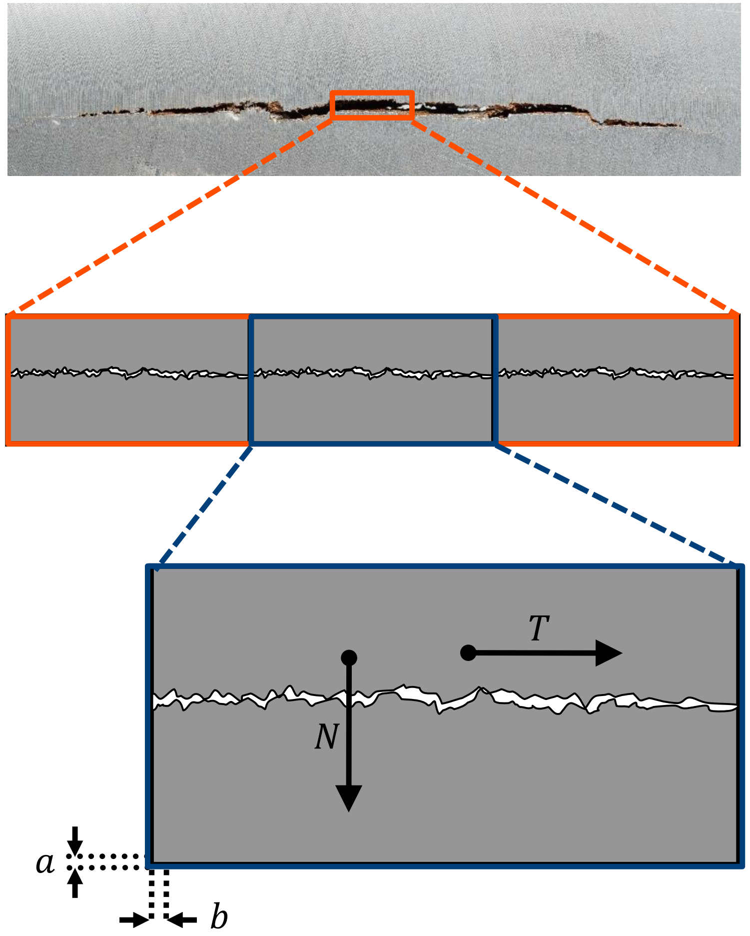
Modeling of contacts with friction

Friction modelling in COMSOL®		
Method	**Pro's**	**Con's**
Contact pairs with friction	• Already included in COMSOL® • Strictly valid only for stationary problems	• Convergence problems for transient models
Tangential virtual spring and damper forces	• Similar implementation as the earlier developed clapping model	• Realistic frictional behavior (e.g. Coulomb friction) can only be approximated
Method of Memory Diagrams (MMD)	• Extended model taking into account roughness of crack faces and associated effects of memory and hysteresis	• Too complex to be implemented directly into Comsol, but possible using Matlab interface

V. Aleshin, O. Bou Matar & K. Van Den Abeele, **Method of memory diagrams for mechanical frictional contacts subject to arbitrary 2D loading**, Int. J. Solids. Struct. 60-61 (2015) 84-95

The concept

Theory



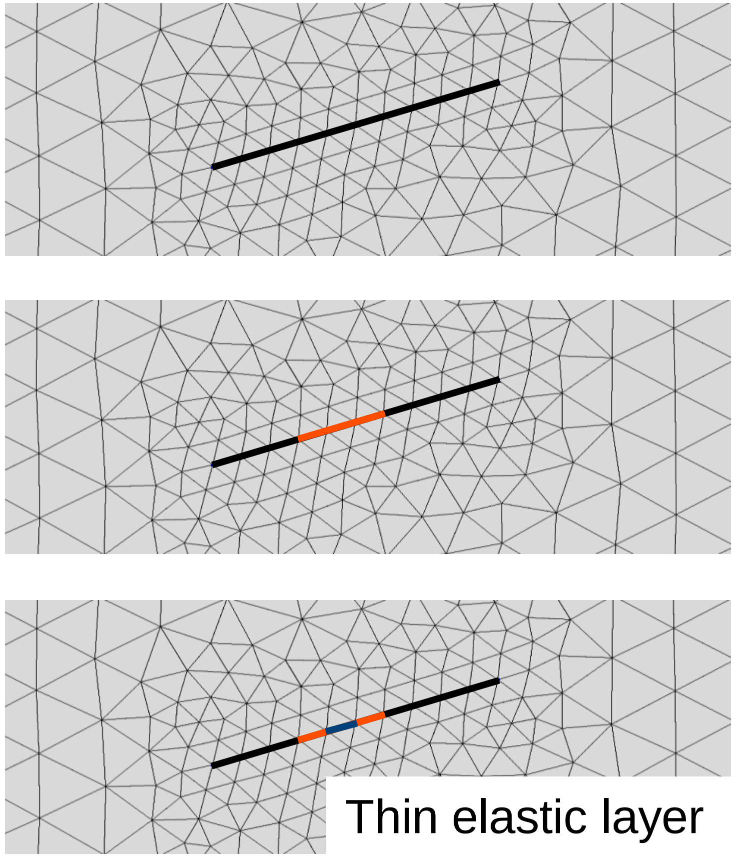
Real crack in a sample

Approximation by a number of mesoscopic cells

In each cell, MMD provides link between:

- Forces per unit area (normal N and tangential T)
- Displacements (normal a and tangential b)

Implementation



Normal and tangential reaction curves

Normal reaction curve

Quadratic force-displacement relation:

$$N(a) = C^2 a^2, \qquad a \geq 0$$

with $C = 6 \times 10^{10}\ Pa^{-1/2}\ m^{-1}$

(Yuan et al., Nondestructive Testing and Evaluation 30, 2015)

Tangential reaction curve: Partial slip

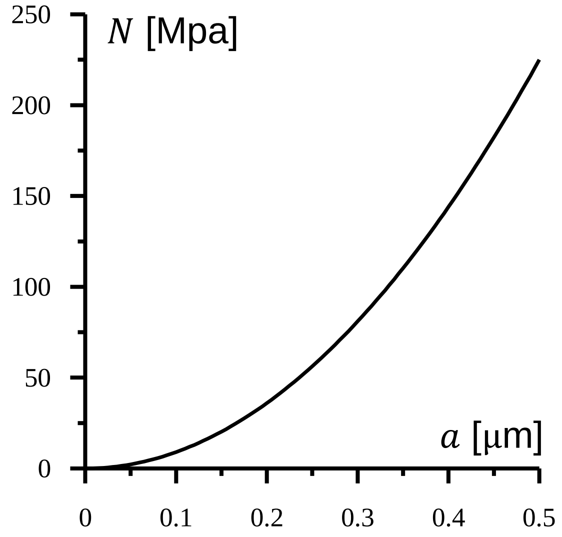
MMD for arbitrary 2D loading:

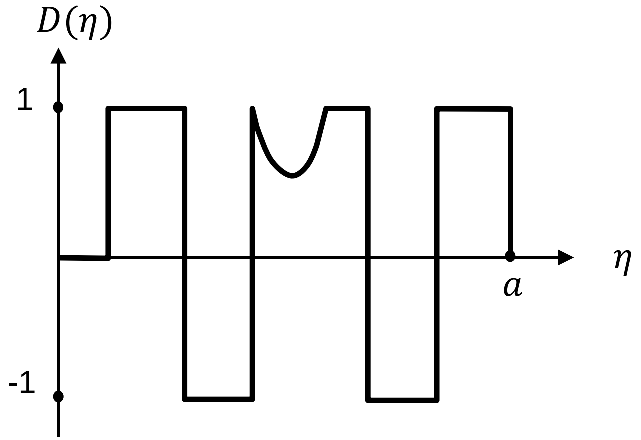
$$b = \theta\mu \int_0^a D(\eta) d\eta$$

$$T = \mu \int_0^a D(\eta) \left.\frac{dN}{da}\right|_{a=\eta} d\eta$$

with μ the friction coefficient, θ a material constant depending on Poisson's ratio ν, and $D(\eta)$ the memory diagram.

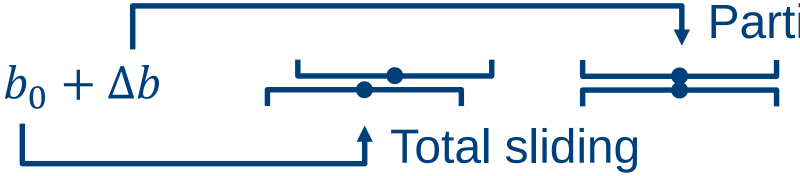
(Aleshin et al., Int. J. Solids & Structures 60-61, 2015)



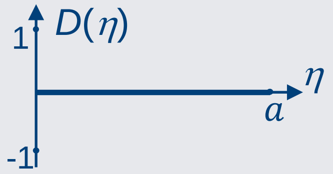
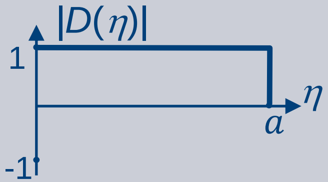
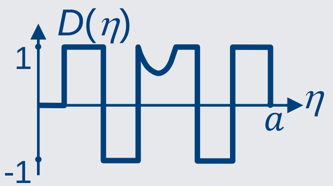


The full constitutive crack model

Note: $b = b_0 + \Delta b$



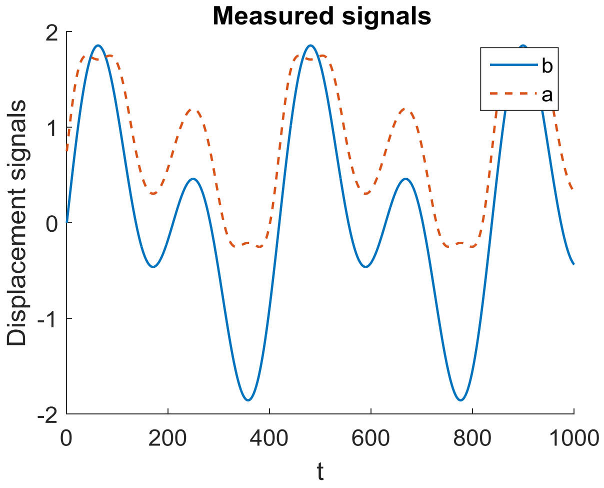
Algorithm

Defect state	If	Then	Memory diagram
Contact loss	$a < 0$	$\Delta b := 0$ & $b_0 := b$ $T = N = 0$	
Total sliding	$a \geq 0$ $\lvert\Delta b\rvert \geq \theta\mu a$	$\Delta b := \pm\theta\mu a$ & $b_0 := b - \Delta b$ $T = \pm\mu N$	
Partial slip	$a \geq 0$ $\lvert\Delta b\rvert < \theta\mu a$	$b_0 := b_0$ & $\Delta b := b - b_0$ $T := MMD(\Delta b)$	

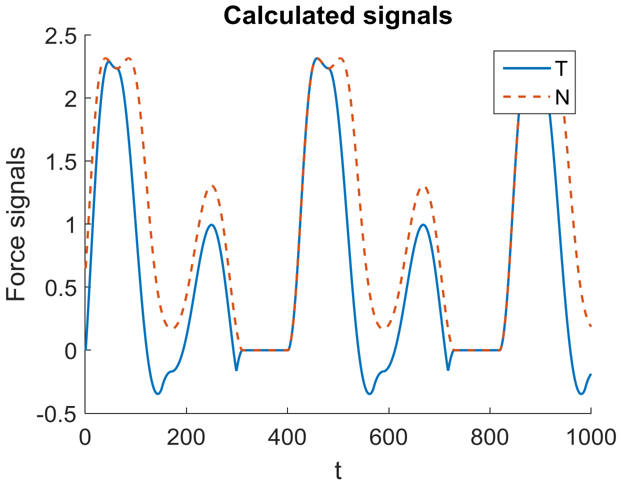
Based on Coulomb's condition written for displacements

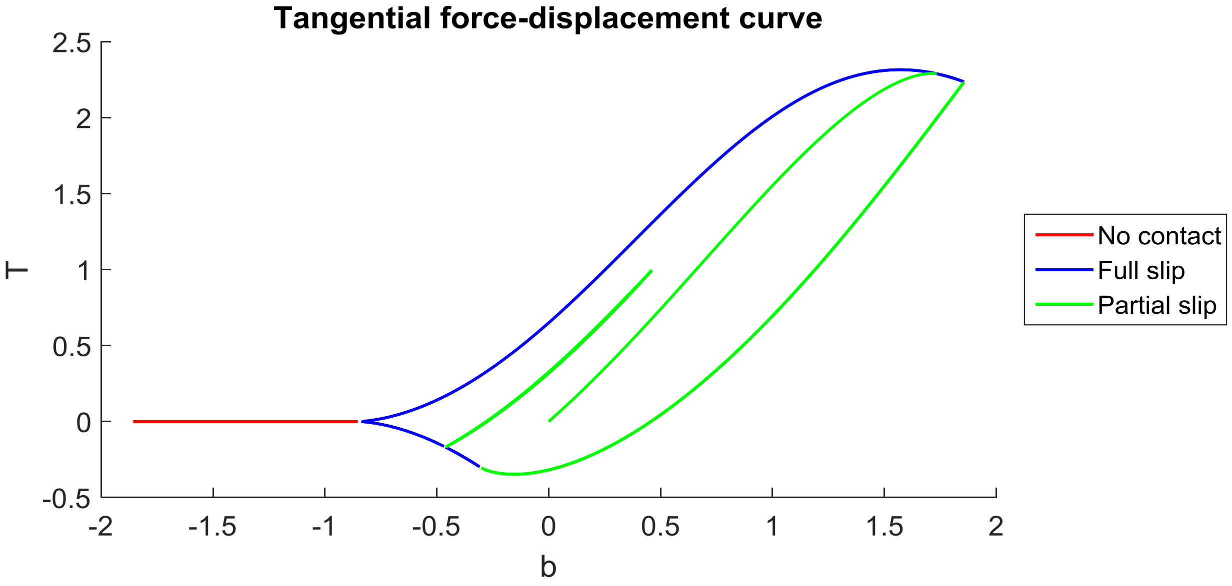
Theoretical example

Input:



Output:





Comsol modeling approach

Structural mechanics module	
Uses stresses and forces to calculate strains and displacements in a solid sample with crack inserted using thin elastic layer.	
Output	Δu_n and Δu_t
Input	σ_n and σ_t

Output → ($=$) → Input

Input ← ($\approx$) ← Output

Thin elastic layer accepts external MATLAB® functions

Crack model	
Uses contact mechanical models for normal and MMD for tangential interactions, system is driven by displacements	
Input	a and b
Output	N and T

Crack states: contact loss, partial slip or total sliding

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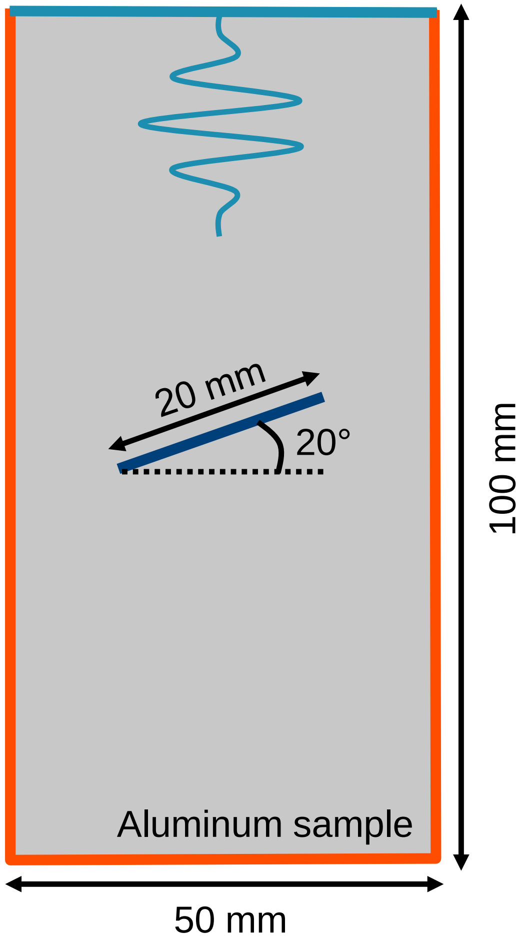
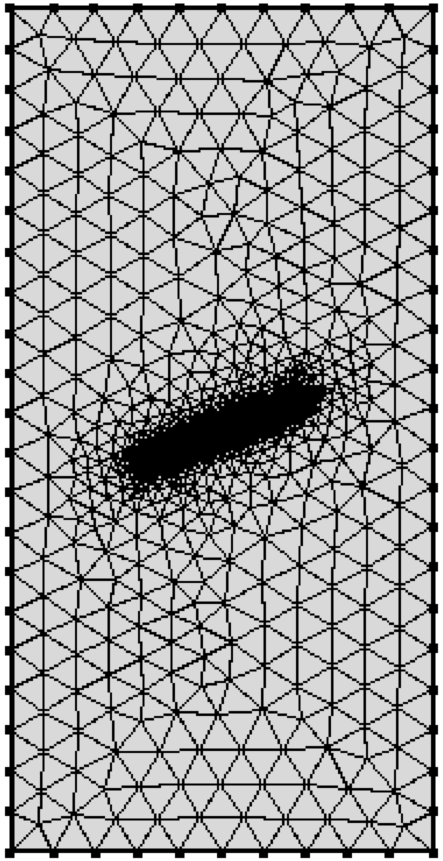
Model geometry

Shear wave excitation:
5 cycle sine wave burst at 100 kHz

Crack of finite extent
(modelled as thin elastic layer)

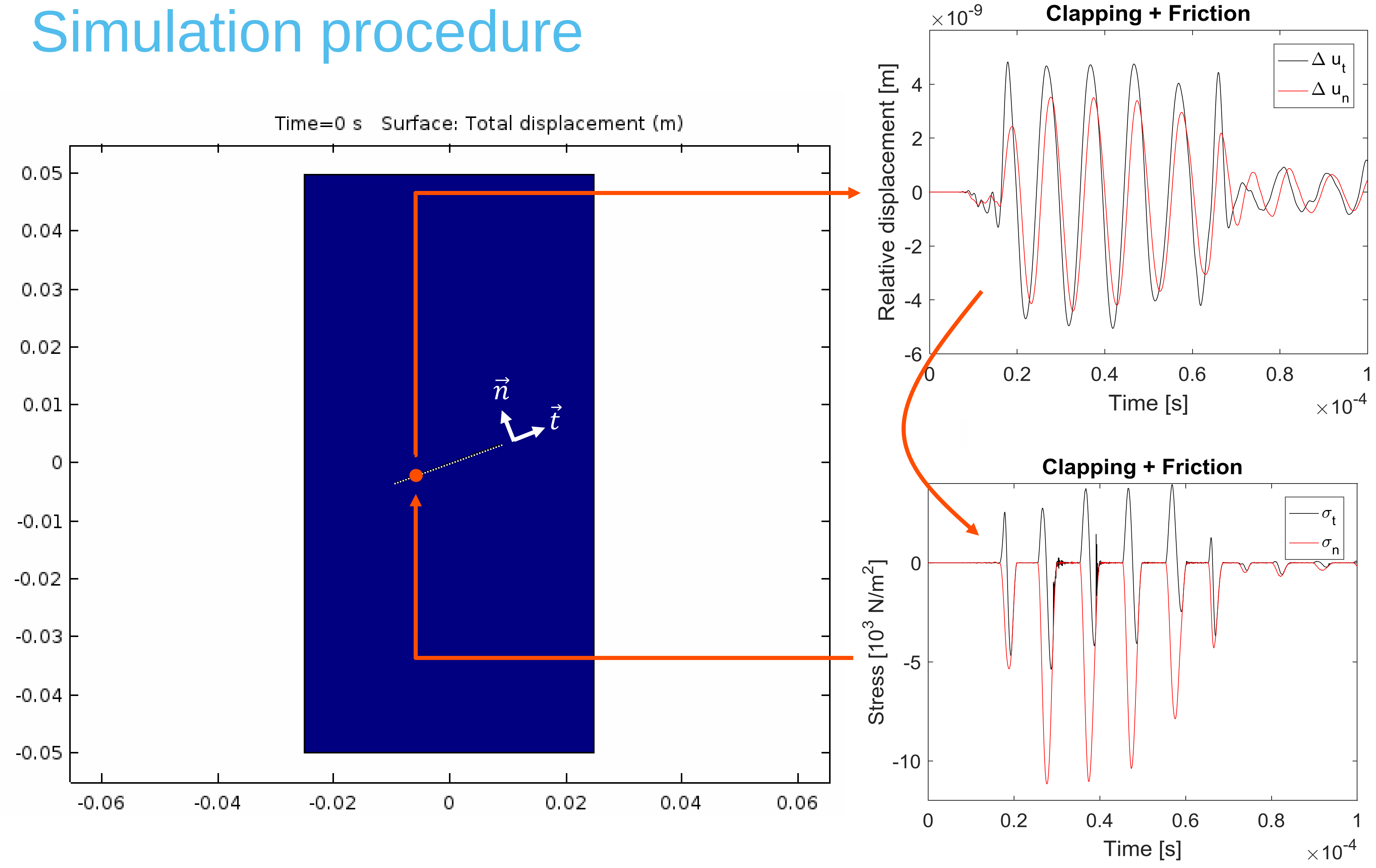
Low reflecting boundaries



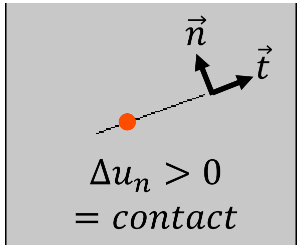


Finer mesh at crack interface is required to obtain a stable and accurate solution

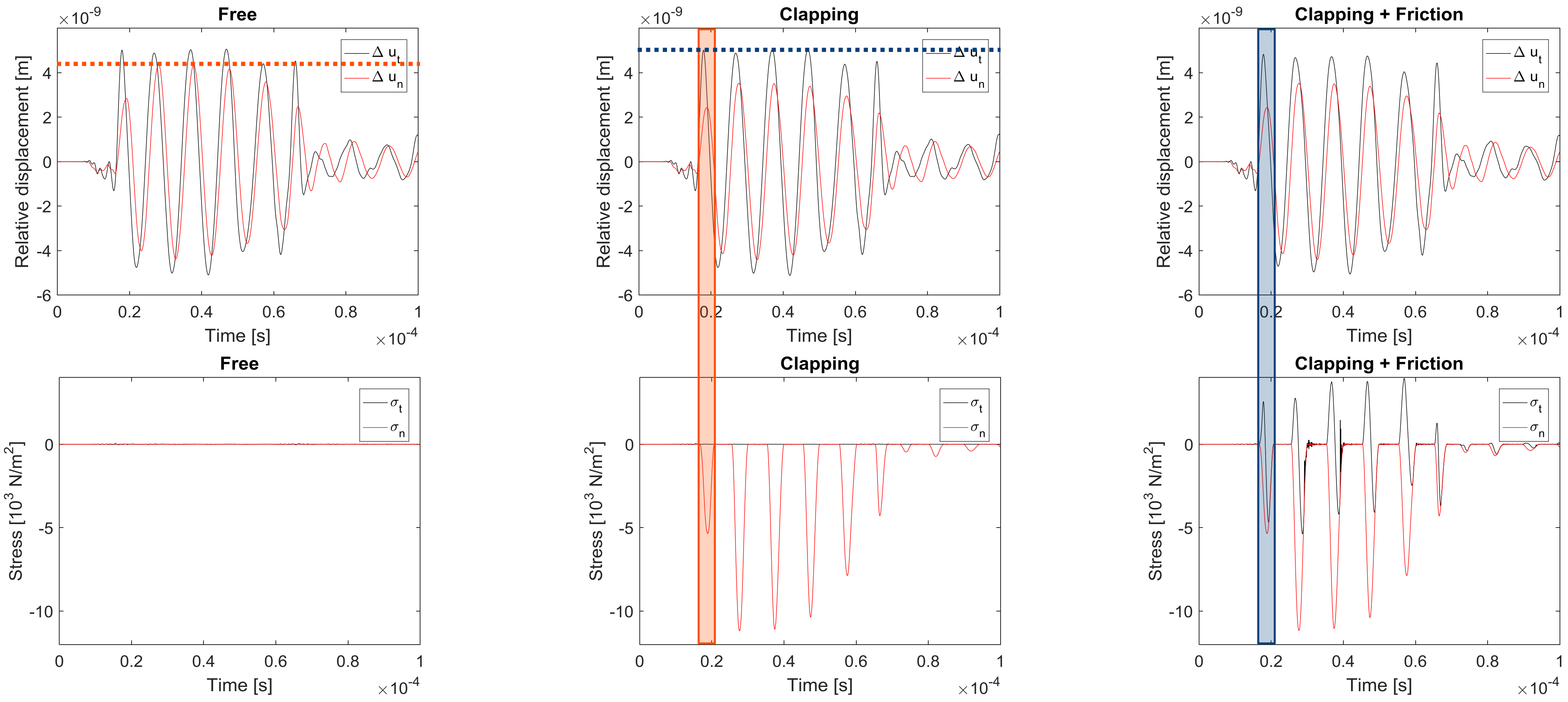
Simulation procedure



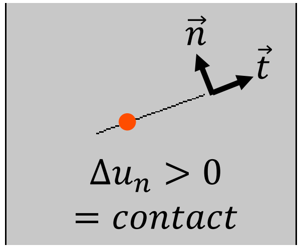
Low amplitude excitation ($A_1 = 10$ nm)



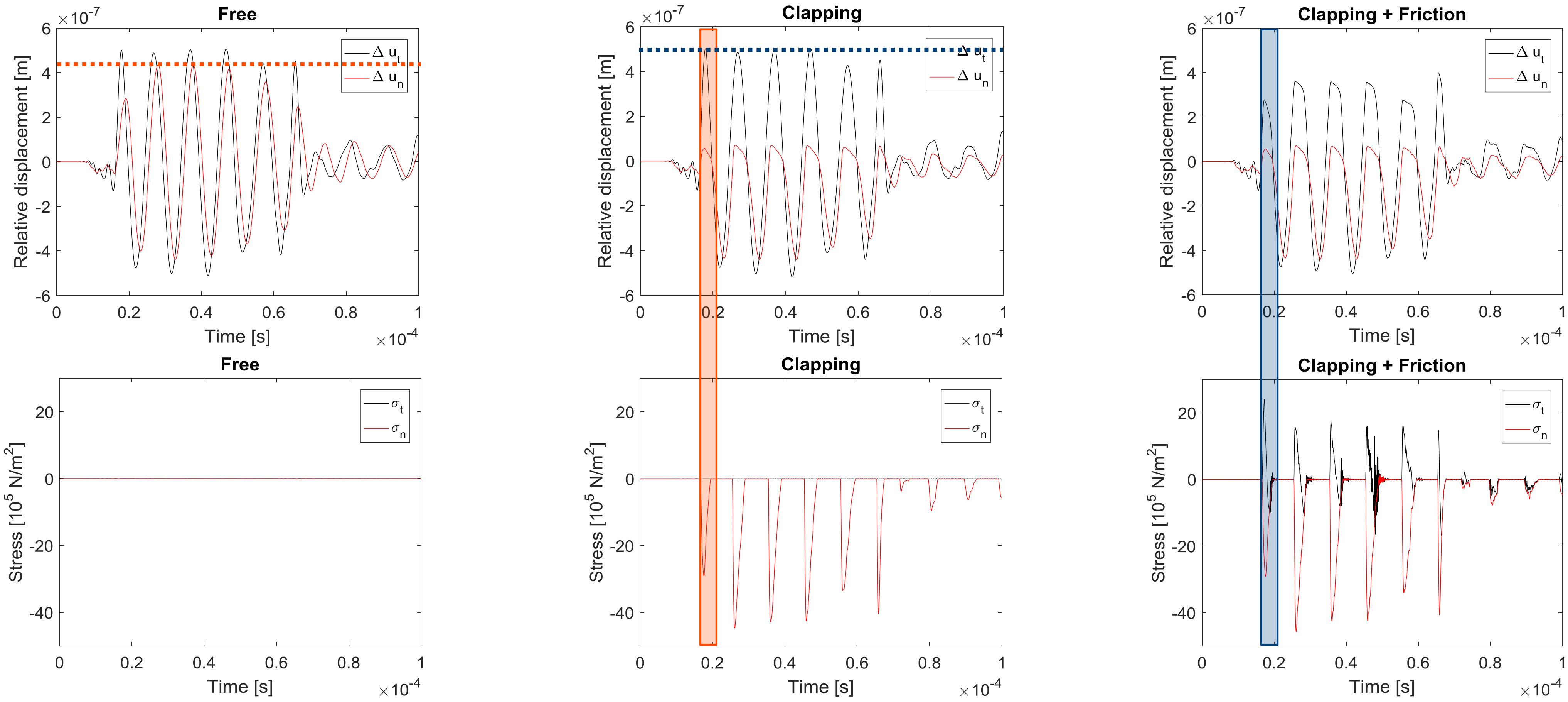
Contact ⇒ Normal interaction + Tangential interaction



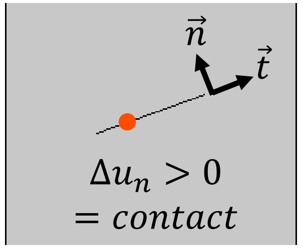
High amplitude excitation ($A_2 = 1\ \mu m$)

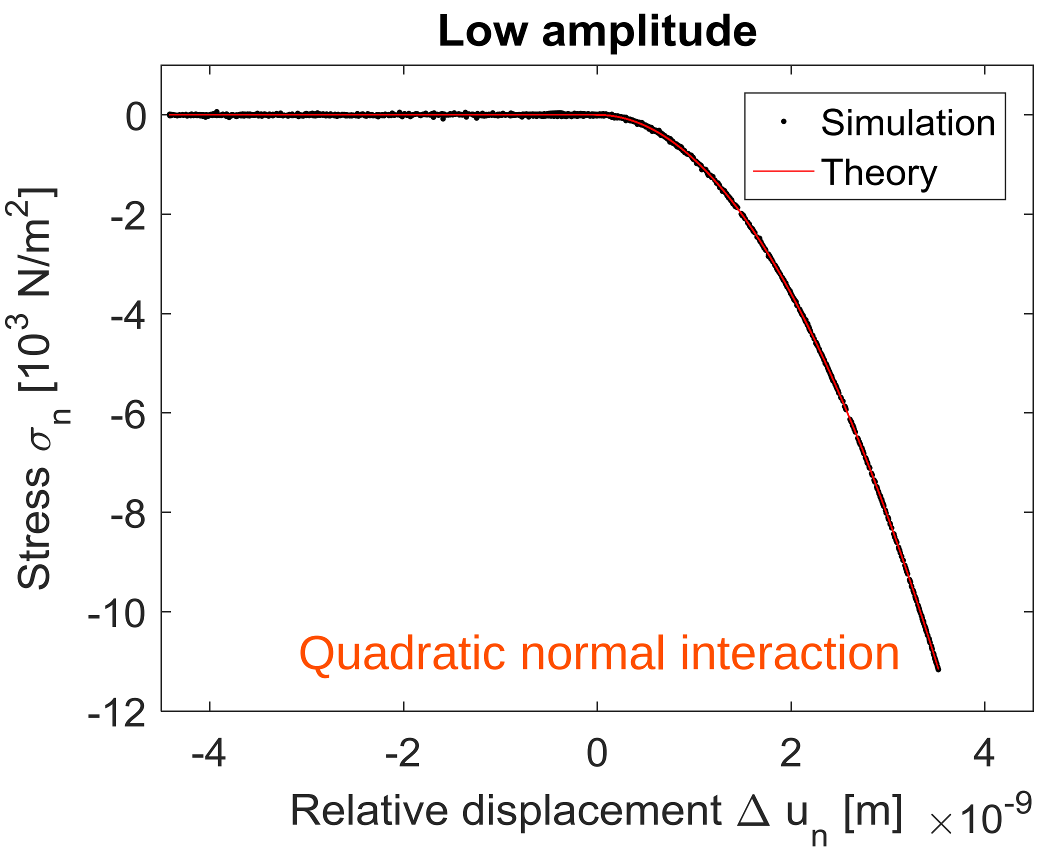


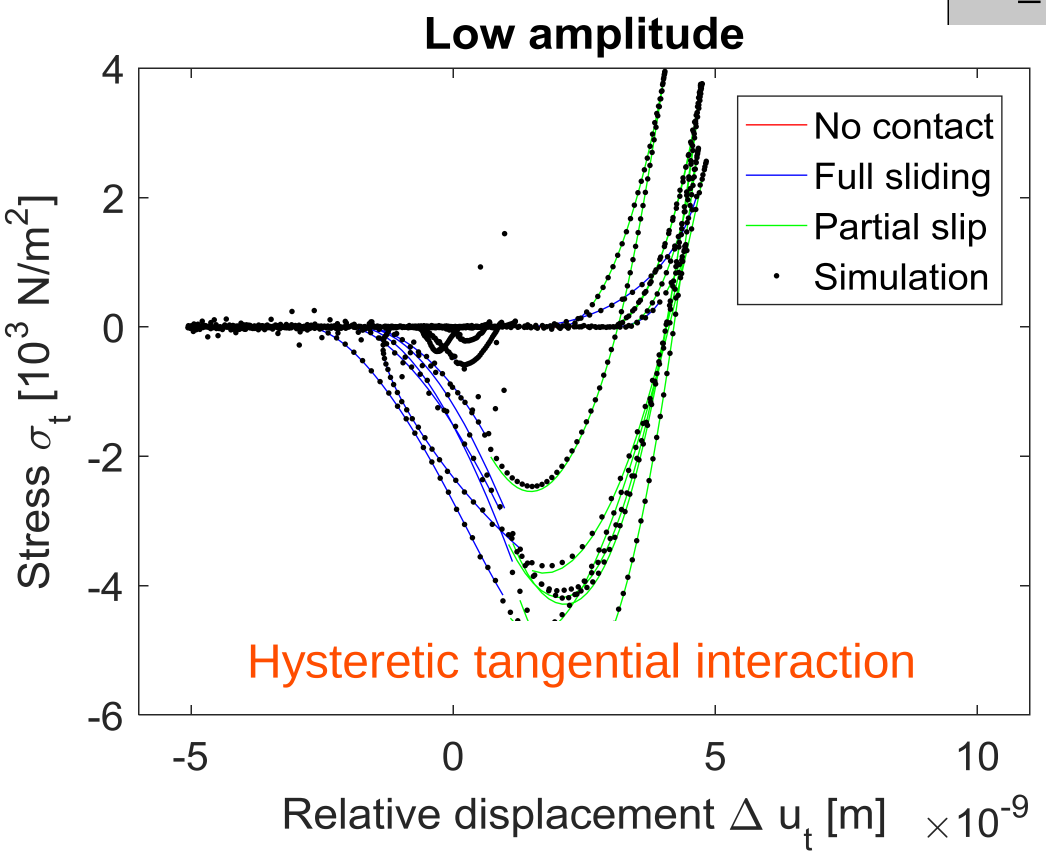
Contact ⇒ Normal interaction + Tangential interaction



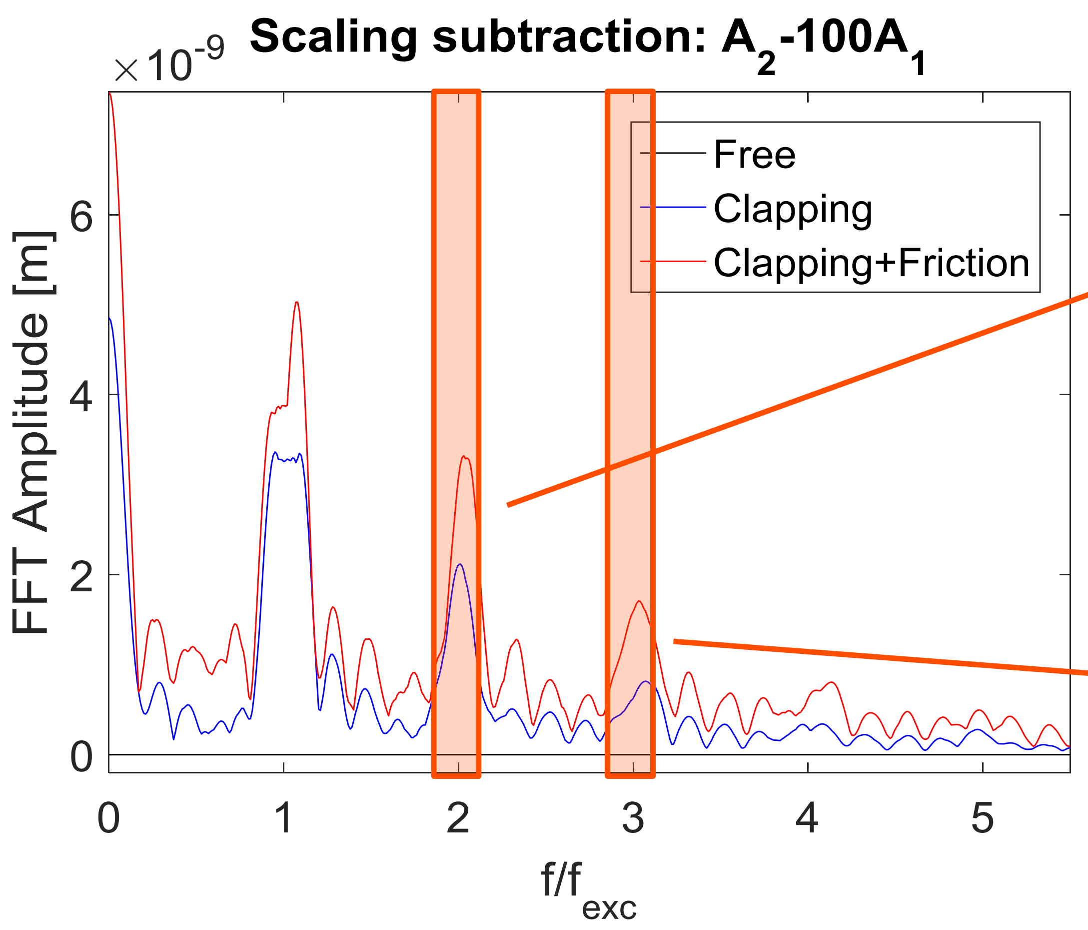
Normal & tangential stress-displacement relations

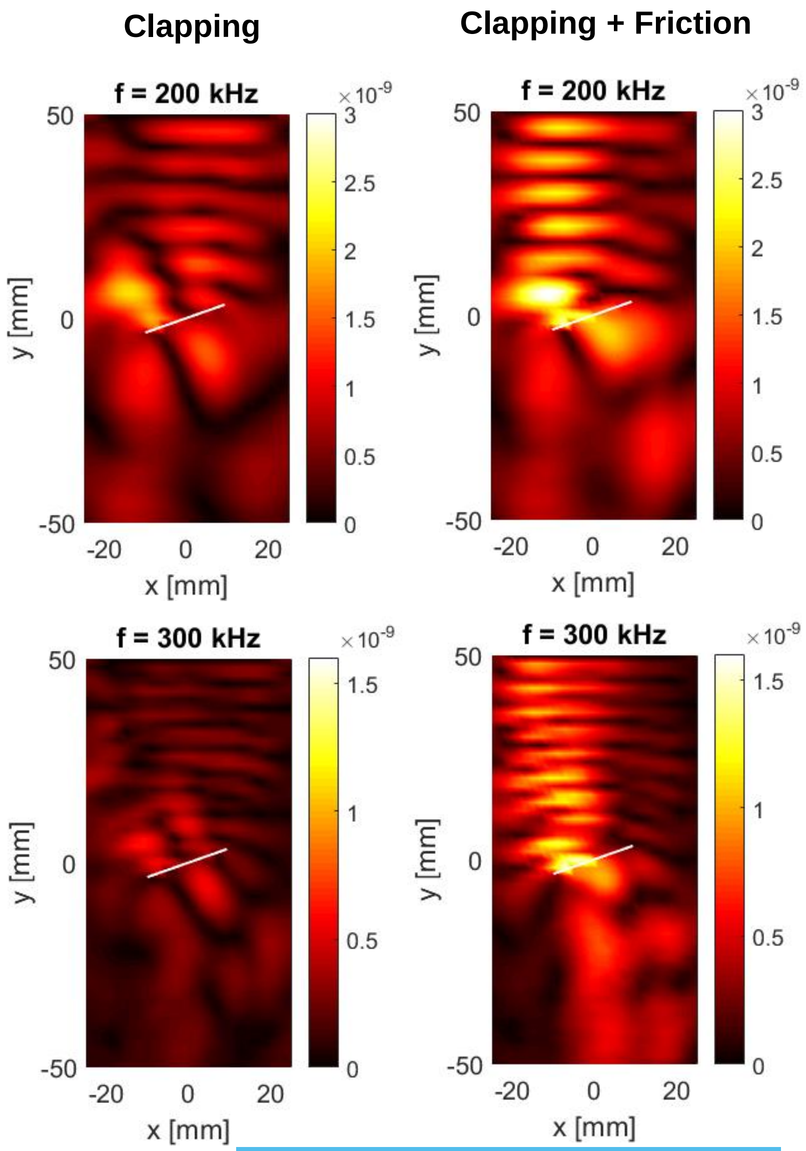






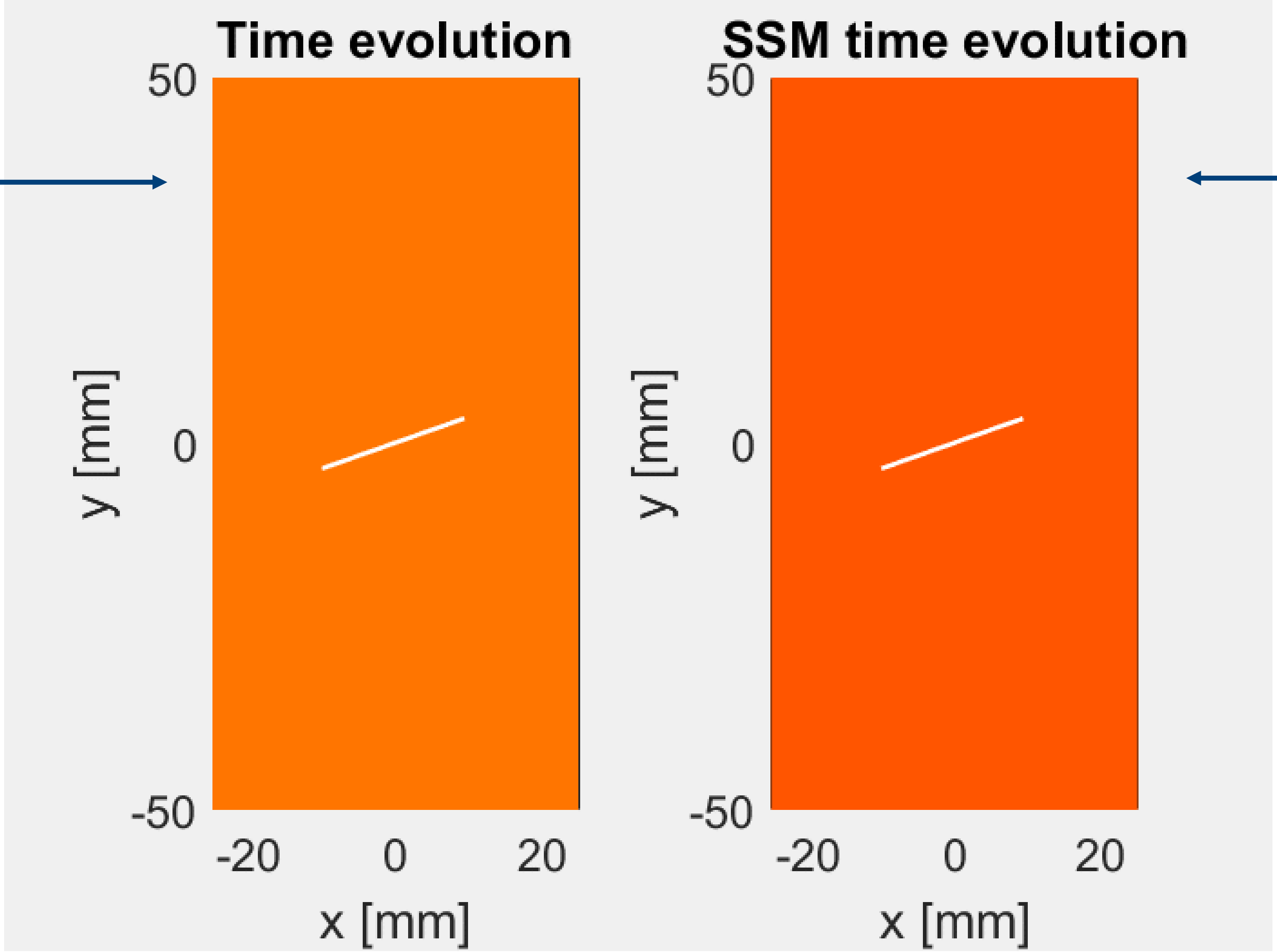
Enhanced harmonics using scaled subtraction





Crack as a nonlinear source

Shear wave propagation in solid sample with crack



Nonlinear wave propagation

Crack starts to behave as a **nonlinear source** once excited by the shear wave propagation

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Conclusion

A model for **elastic wave propagation in solids with cracks** was developed in COMSOL®:

- The **wave model** was implemented in the **Structural Mechanics Module**
- The **crack model** was implemented in **MATLAB®**
- Both models were connected using the **LiveLink™ for MATLAB®**

The crack model is **very realistic**, since:

- It takes into account **clapping, friction, roughness and hysteresis**
- It allows the modeling of three defect states: **contact loss, partial slip and full sliding**

We have a numerical laboratory for elastic wave problems in materials with cracks!!!

Future Work

Numerical modeling in which various **nonlinear mechanisms** (e.g. clapping, friction) are **switched on/off**
→ **Full physical understanding** of **nonlinear phenomena** associated with wave-defect interaction

Accurate modeling of novel and promising **nonlinear ultrasonic techniques**
→ Support the **further development and optimization** of nonlinear NDT applications

Extend the constitutive crack model **to the 3D case**

Thank you for your attention!

WPSP
Wave Propagation and Signal Processing

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