Implementation of a Thermo- Hydrodynamic Model to Predict "Morton Effect"

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Abstract: In this paper the "Morton Effect" is analyzed. This phenomenon is a particular kind of rotor instability due to non-uniform journal bearing heating. A preliminary explanation about this effect has been done. After this short introduction the governing equation behind the phenomena are presented. Then, a particular approach suggested by the literature has been chosen and deeply analyzed by arguing this choice. Finally a thermal model, a rotor dynamic model and a stability criterion has been implemented with COMSOL Multiphysics®. The result is a "Comsol App" that allows to easily predict the stability of the rotor given a set of parameters.

Keywords: Morton effect, rotor instability, journal bearing, Comsol Application Builder.

1. Introduction

Morton effect [1][2][3][4] is a particular synchronous rotor instability caused by a non-uniform journal heating. One portion of the journal always stays at a minimum film thickness, while the opposite section stays at the maximum film thickness. Lower film thickness generates higher shear stresses and then higher temperatures: this is the hot spot of the journal (Figure 1). The opposite section stays at the maximum oil film and at the minimum temperature: this is the cold spot. If the hot spot stays 180° out of phase from the overhung mass then a potential instability can occur and a positive feedback can be generated: overhung unbalanced mass reduces the minimum film thickness; this condition increases the viscous shear rate and the temperature having a consequence of a reduction in the thickness.

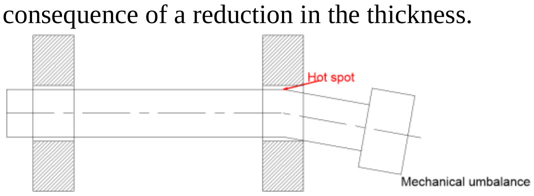

Figure 1. Mechanical unbalance and thermal bending acts in the same direction generating a positive feedback and a potential instability condition.

Figure 2 can help to better understand the phenomena: the center of journal moves along the elliptical trajectory depicted with blue color. While the rotor depicts this trajectory a limited part of the shaft is continuously heated (red point in the figure) while the opposite rotor section stays at a lower temperature due to higher film thickness.

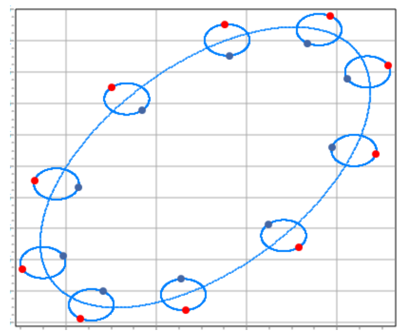

Figure 2. Elliptical journal trajectory with hot (red) and cold (blue) spot.

In this paper a specific approach able to solve the problem and suggested in literature has been considered and implemented with COMSOL.

2. Governing equations

Many "physics" are involved in the Morton problem:
- CFD physics: a fluid film is interposed between journal and bearing. Inside this film a pressure is generated to sustain the rotor weight. The basic equation that describes this interaction is know as Reynolds equation;
- thermal physics: when the rotor runs, fluid film rises it's temperature due to the shear stress inside the lubricant;
- mechanical physics and multibody dynamics: during its motion the center of the journal moves around a certain path due to the external forces and also due to the reaction caused by the fluid film bearing.

It is easy to understand that these main physics are related: when the fluid rise its temperature the fluid viscosity decreases and the pressure

field changes. This change produces a modification in the shaft trajectory.
In the next sections more details about equations governing the problem are presented.

2.1 Continuity equation and momentum equations : Reynolds equations

Pressure distribution across fluid film can be computed using classical Reynolds equation:

$$\nabla_T \cdot \left(\frac{-\rho h^3}{12\mu} \nabla_T p + \frac{\rho h}{2} (v_a + v_b) \right) = \frac{\partial(\rho h\ \ )}{\partial t}$$

where ρ is the density, h is the lubricant thickness, μ is the dynamic viscosity, p is the pressure inside the fluid, v_a v_b are the speed of the journal and bearing, and ∇_T is the tangential projection of the gradient operator.
The first terms describes the flow generated by the pressure gradient (*Poiseuille term*); the second term is related to the mean speed surface and is called *wedge term*, while the last term is related to the squeeze of the fluid (*squeeze term*) [5].
In the previous equation the pressure could have negative values but, since a fluid cannot sustain negative pressure, cavitation occur and the clearance is filled with a liquid vapor mixture. This means that the previous equation is valid only in a part of the domain, while in the remaining domain the pressure stays at a constant zero value:

$$\begin{cases} \nabla_T \cdot \left(\frac{-\rho h^3}{12\mu} \nabla_T p + \frac{\rho h}{2} (v_a + v_b) \right) = \frac{\partial(\rho h\ \ )}{\partial t} & \\ & (x,y) \in \Omega_1 \\ p(x,y) = 0 & (x,y) \in \Omega_2 \end{cases}$$

Ω_1 and Ω_2 are unknown and has to be numerically computed [2].

2.2 Energy equation and temperature distribution

The pressure field computed using the previous Reynolds equation requires to know the lubricant viscosity μ, but its value is strongly related to the temperature distribution across the fluid film. This temperature distribution can be computed using the energy equation that states that the energy generation rate is equal to the sum of the outflux energy rate plus accumulation energy rate. The equation that summarize this statement is:

$$\rho c \left(\frac{\partial T}{\partial t} + v \cdot \nabla T \right) = k \nabla^2 T + \mu \Phi_v$$

c is the specific heat at constant pressure, v is the (vector) velocity, k is thermal conductivity and $\mu\Phi_v$ is the rate of irreversible conversion of mechanical energy into internal energy per unit volume by viscous dissipation.

2.3 Motion equation

The governing equation of motion for a rigid rotor-bearing system is:

$$\boldsymbol{M}\ddot{\boldsymbol{X}} + \boldsymbol{C}\dot{\boldsymbol{X}} + \boldsymbol{K}\boldsymbol{X} = \boldsymbol{M} u \omega^2 \begin{pmatrix} \sin \omega t \\ \cos \omega t \end{pmatrix}$$

where $\boldsymbol{M}$ is the mass matrix, $\boldsymbol{C}$ is the damping matrix and $\boldsymbol{K}$ is the stiffness matrix. Previous equation can be better understood considering the figure 3.

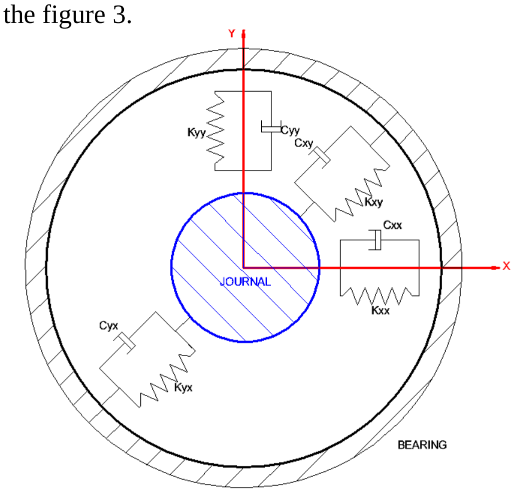

Figure 3. Schematic representation of the fluid film bearing coefficients.

Journal is sustained by the fluid and the fluid acts like a spring and dumper. Journal moves inside the bearing in relation to the fluid film properties and also to the forces acting on the rotor (unbalance included).

3. Methods

Previous equations describe entirely the physics involved in the study of the journal bearing system. It is also necessary to add a stability analysis (in time or frequency domain) to investigate the system stability.

The method adopted to study the stability of the overhung shaft starts from that equations. COMSOL can couple such different physics and can solve the problem, but in this work a different approach based on some simplification of the previous equations has been adopted. The work, as already mentioned, comes from a previous study presented in [6][7] and implements the algorithm inside COMSOL Application Builder.
In the next step the simplified equations and the whole methodology has been analyzed.

3.1 Static equilibrium position

The static equilibrium position is reached when the pressure developed inside the fluid film bearing equates the load applied to the shaft. For a short bearing (ratio between length and diameter less than 0.5) the solution of the Reynolds equation states that the minimum film thickness can be evaluated using this non-linear equation [8]:

$$F_0 = \mu\omega RL\left(\frac{L}{C}\right)^2 \frac{\varepsilon}{4}\frac{\sqrt{16\varepsilon^2+\pi^2(1-\varepsilon^2)}}{(1-\varepsilon^2)^2}$$

while the attitude angle ϕ_0 can be computed as:

$$\tan(\phi_0) = \frac{\pi\sqrt{1-\varepsilon^2}}{4\varepsilon}$$

F_0 is the applied load, ω is the speed, R the bearing radius, L the bearing length, C is the radial clearance and ε is the journal eccentricity ratio (e/C).

3.2 Orbit and motion

The orbit described by the journal center is the result of the dynamic motion. Stiffness and damping matrix can be evaluated using again the short bearing approximation. Literature [9][10] suggests that:

$$k_{xx} = K_{xx}\frac{C}{F_0} = \frac{f_{r0}}{\varepsilon(1-\varepsilon^2)}\left(f_{r0}^2+1+2\varepsilon^2\right)$$
$$k_{yy} = K_{yy}\frac{C}{F_0} = \frac{f_{r0}}{\varepsilon(1-\varepsilon^2)}\left(f_{t0}^2+1-\varepsilon^2\right)$$
$$k_{yx} = K_{yx}\frac{C}{F_0} = \frac{f_{t0}}{\varepsilon(1-\varepsilon^2)}\left(f_{r0}^2-1+\varepsilon^2\right)$$
$$k_{xy} = K_{xy}\frac{C}{F_0} = \frac{f_{t0}}{\varepsilon(1-\varepsilon^2)}\left(f_{r0}^2+1+2\varepsilon^2\right)$$

$$c_{xx} = C_{xx}\frac{C\omega}{F_0}\frac{2f_{t0}}{\varepsilon(1-\varepsilon^2)}\left((2+\varepsilon^2)f_{r0}^2+1-\varepsilon^2\right)$$
$$c_{yy} = C_{yy}\frac{C\omega}{F_0} = \frac{2f_{t0}}{\varepsilon(1-\varepsilon^2)}\left((2+\varepsilon^2)f_{t0}^2-1+\varepsilon^2\right)$$
$$c_{xy} = c_{yx} = C_{xy}\frac{C\omega}{F_0} = \frac{2f_{r0}}{\varepsilon(1-\varepsilon^2)}\left((2+\varepsilon^2)f_{t0}^2-1+\varepsilon^2\right)$$

where f_{r0} and f_{t0} are the dimensionless force coefficient in radial-tangential coordinates

$$f_{r0} = \frac{4\sigma\varepsilon^2}{(1-\varepsilon^2)^2}$$
$$f_{t0} = \frac{\pi\sigma\varepsilon}{(1-\varepsilon^2)^{3/2}}$$

and σ is the Modified Sommerfeld Number

$$\sigma = \frac{(1-\varepsilon^2)^2}{\varepsilon\sqrt{16\varepsilon^2+\pi^2(1-\varepsilon^2)}}$$

3.3 Temperature distribution

Previous energy equation can be simplified under the assumption of a negligible axial heat flow and steady state condition. Final result is:

$$\frac{dT}{d\xi} + \frac{2H}{\rho c\omega h} - \left(\frac{2HT_{amb}}{\rho c\omega h} + \frac{2\mu\omega R_j^2}{\rho c h^2}\right) = 0$$

where ξ is the rotor angle, H is the heat transfer coefficient an T_{amb} is the average ambient bearing temperature and R_j is the journal radius.

3.4 Thermal unbalance

After the solution of the previous ODE the thermal distribution is available and thermal unbalance can be estimated as:

$$U_t = m_o y_o = m_o\frac{\alpha\Delta T}{R_j}LL_o$$

where m_0 is the overhung mass, α is the thermal conductivity of the shaft, ΔT is the mean temperature difference between hot and cold spot and L_o is the length of the overhung part of the shaft.

3.5 Total unbalance and stability study

Thermal unbalance and mechanical unbalance can be summed up to generate a total unbalance:

$$U = \sqrt{U_t^2 + U_m^2 - 2U_m U_t \cos(\omega t - \vartheta_{ch})}$$

ϑ_{ch} is the angle of the line that connect hot with cold spot and U_m is mechanical unbalance. The method solve the stability problem comparing this total unbalance with a threshold unbalance. This comparison allows to understand if the system is stable.

4. Use of COMSOL Multiphysics® Software

The code has been organized in 4 main classes: TRotor, TLubricant, TJournalBearing and TJournalBearingSystem. First three classes has main data related to Rotor, Lubricant and Journal Bearing. The last is the object that describes the entire system made by a rotor, two bearing, and lubricant. This object has a method that build the entire analysis. The implemented steps are mainly these:

1) Select the speed where the analysis has to be computed
2) Compute the effective viscosity using exponential Reynolds equation
3) Compute the static equilibrium position
4) Estimate amplitude and phase of the orbit
5) Solve ODE to compute thermal distribution for a different rotor angle and estimate a mean hot-cold difference in a revolution
    a. This computation is made running the model implemented in the model builder; temperatures obtained solving the ODE is used inside the cycle

    The model is described by means of a global ODE
6) Compute thermal unbalance using mean temperature difference
7) Compute total thermal unbalance
8) Compare Total unbalance with threshold unbalance

Figure 4 shows one form of the implemented app with main results: thermal distribution along the journal circumference while the shaft moves along its elliptical trajectory. It is also visible thermal unbalance, total unbalance and threshold unbalance.

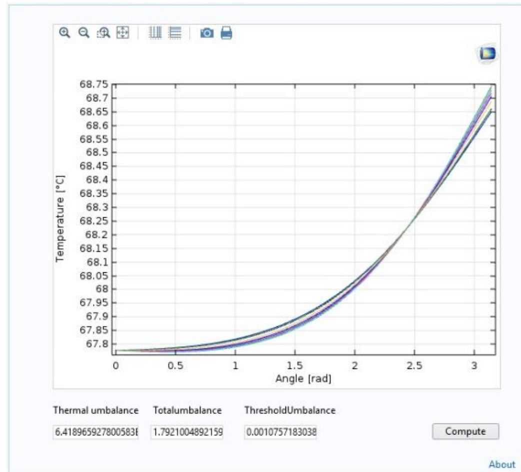

Figure 4. A form of the developed app with main results. Each curve represented with different colors is the circumferential temperature at different shaft position (time).

5. Discussion

Results obtained using this model has been compared with the results presented in [5] and [6]. Numerical values of the total thermal unbalance, at the same working condition are the same suggesting that the procedure has been correctly ported inside COMSOL environment.

6. Conclusions

The method suggested in [5] and [6] has been implemented in COMSOL. This implementation allows to understand the flexibility of the COMSOL Application Builder: the hard numerical computation required to solve differential equation is delegated to COMSOL engine, while the elaboration of the results is demanded to a dedicated algorithm specifically designed and implemented using app builder.

This could be considered as a first step in the development of a complete and accurate model. Future works could eliminate the approximations introduced to estimate temperature distribution and static equilibrium position, introducing COMSOL physics (previously presented) inside the current model.

7. References

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