Finding Stationary Solutions with Constraints Using Dynamic Damped Systems

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Abstract

We consider the problem of solving the stationary solution of a nonlinear PDE with additional constraints either given by a time dependent problem or by extending to a (artificial) time dependent problem. However, COMSOL Multiphysics® built-in stationary solvers may fail to find a solution if the initial guess is inadequate and this motivates our study.

We have used the General Form PDE interface [3]. For the heat equation we used a time-dependent solver with Dirichlet Flux boundary conditions on a finite line with the initial conditions being unity.
The Nonlinear Schrödinger Equation (NLSE) was solved using the stationary solver with Zero Flux boundary conditions and the geometric domain was a torus. The global constraints were implemented as two global equations and the transverse potential was defined as a local variable using a cylindrical coordinate system. The initial data was at first imported from an interpolated 1-dimensional solution on the circle with a Gaussian profile. We used an auxiliary sweep in the angular momentum where the initial data were chosen from the previous solution in the sequence.

As the first example, we have investigated stationary solutions to an inhomogeneous non-linear heat equation. We compare the time dependent heat equation with DFPM, as well as the COMSOL built-in stationary solvers applied to the time-dependent heat equation. Some of these results are shown in Figure 1 and Figure 2.
We add relevant boundary conditions and the solutions are normalized and has a specific angular momentum, representing a rotating state along the torus [4], see Figures 3 and 4 for examples.

We have shown that a COMSOL implementation of DFPM successfully find a stationary solution to non-linear problems. In our future work we will use symplectic methods which may improve DFPM such that it will be the first choice in COMSOL simulations of this kind.

Reference

[1] S. Edvardsson, M. Gulliksson, and J. Persson, The dynamical functional particle method: An approach for boundary value problems, J. Appl. Mech., Vol. 79(2), p. 021012 (2012)
[2] P. Sandin et al., Numerical solution of the stationary multicomponent nonlinear Schrödinger equation with a constraint on the angular momentum, Phys. Rev. E 93, 033301 (2016)
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[4] P. Sandin et al., Effective NLSEs for the torus, to be submitted

Figures used in the abstract

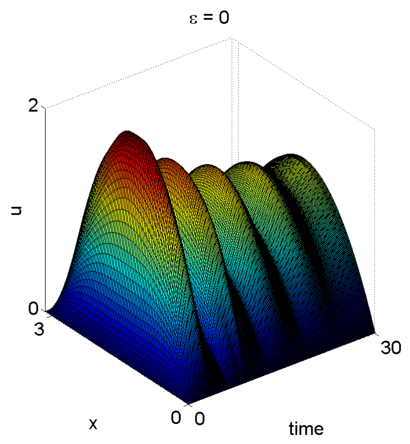

Figure 1: Oscillatory convergence of DFPM.

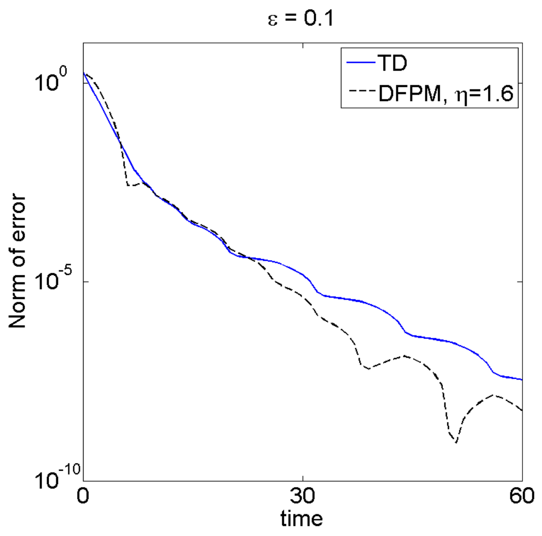

Figure 2: Comparison with the TD heat equation.

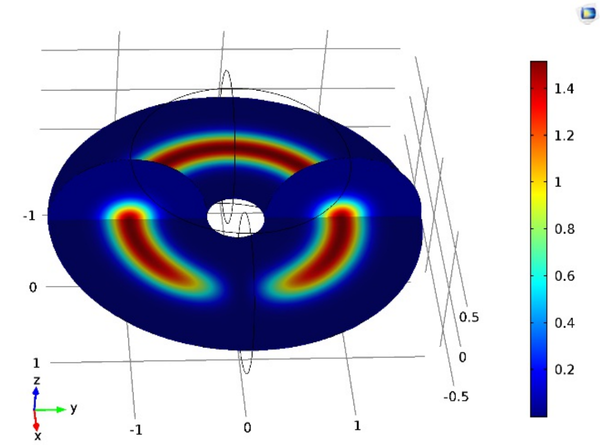

Figure 3: A dark soliton in the torus.

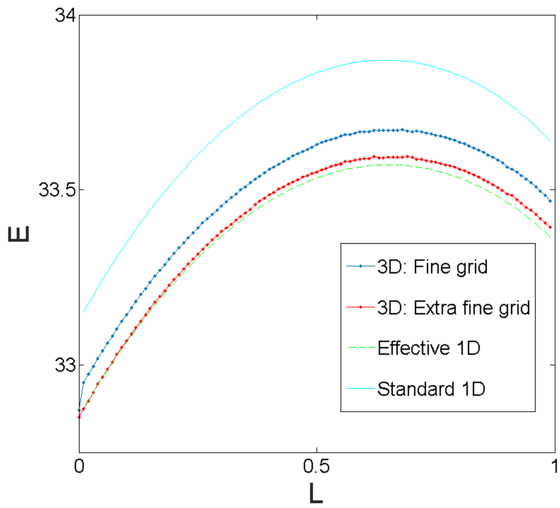

Figure 4: An energy vs angular momentum plot.