Finite Element Models of Elasto-Plastic Deformation in Volcanic Areas

Danila Scandura(1),*, Gilda Currenti(1), Ciro Del Negro(1)

(1) Istituto Nazionale di Geofisica e Vulcanologia, Sezione di Catania – Italy
* Corresponding author: Istituto Nazionale di Geofisica e Vulcanologia, Sezione di Catania,
Piazza Roma, 2, I-95123 Catania, scandura@ct.ingv.it

Abstract: In volcanic areas, the presence of heterogeneous materials and high temperatures affects the rheological behaviour of the Earth's crust that calls for considering the anelastic properties of the medium surrounding the magmatic sources. The elastic approximation is generally appropriate for small deformations of crustal materials with temperatures cooler than the brittle-ductile transition, between 300 and 500 °C depending mainly on composition and strain rate. Materials surrounding a long-lived magmatic source are heated significantly above the brittle-ductile transition and rocks no longer behave in a purely elastic manner, but permanently deform because of the plastic deformation. Therefore, the thermal state of the volcanoes can greatly influence the surface deformation field, making the elastic approximation inappropriate to model the observed ground deformation.

A thermo-mechanical numerical model is performed for evaluating the temperature dependency of the elasto-plastic solution. Both temperature distributions and ground deformation are evaluated by solving an axi-symmetric problem to estimate the effects of elasto-plastic response of the medium. The inclusion of elasto-plastic material around the magmatic source, which is geologically expected, considerably reduces, with respect to elastic models, the pressure necessary to produce the observed surface deformation.

Keywords: Numerical solution, Etna volcano, rheology, plastic deformation.

1. Introduction

Ground deformation is a phenomenon commonly observed in connection with volcanic activity. Theoretical models have been developed to investigate the surface deformation patterns observed in volcanic areas. Most deformation models approximate the volcanic areas as linearly elastic, homogeneous half-spaces. While elastic half space models fit a variety of crustal deformation data, the elastic rheology in case of volcanic regions is an oversimplification. Also, the thermal state of the volcanoes can greatly influences the surface deformation field, making the elastic approximation inappropriate to model the observed ground deformation.

In the present work, a mechanical numerical model is performed for evaluating, in terms of displacement field, the effects of a heating pressurized magma chamber in a volcanic edifice. We applied the 3D axi-symmetric Finite Element Method (FEM) to asses the role that rheology may play in computing deformation field. Particularly, we performed different simulations considering elastic and elasto-plastic rheologies. In a second step, we introduced the thermal problem and we modified the properties of the rocks using the computed thermal profile to evaluate the deformation field for different value of the brittle-ductile transition.

Finally, we performed a 3D finite element modeling to analyze the ground deformation accompanying the 1993-1997 inflation period on Mt Etna, in which we used the real topography of volcanic edifice and the crustal heterogeneities inferred from seismic tomography.

2. Numerical model

2.1 Deformation model

We developed a 3D axi-symmetric Finite Element (FE) model to asses the role that rheology may play in computing deformation field. Numerical computations are carried out using the FE software COMSOL 3.4. Our approach assumes radial symmetry about the source centre. To approximate an half-space, the axi-symmetric FEM is composed of ~200000 triangular elements covering a region that extends 25 km horizontally from the source centre and 35 km below the surface. The source is located at 4 km depth, and the radius of the spherical magma chamber is 0.7 km. We assumed free displacement values at the upper surface and zero displacement values at bottom and lateral boundaries. A step-like increase in pressure (ΔP=320 MPa) is applied on the source wall.

We performed two different simulations considering: (i) elastic and (ii) elasto-plastic rheology. In the first case we considered a fully elastic half-space with Poisson ratio ν=0.25 and rigidity modulus μ=30 GPa. The second model was realized introducing a spherical shell (radius 1.7 km) surrounding the magmatic source. It was supposed an elasto-plastic behavior inside the shell and an elastic behavior outside it. We implemented the yield stress/strain laws considering an ideal plastic flow; the yield strength of surrounding rocks is assumed to σ_y=15 MPa while the elastic parameters of the medium are those of model previously described. Figure 1 shows the comparison of ground uplift due to a pressure source of 320 MPa considering elastic (black line) and elasto-plastic rheology (red line).

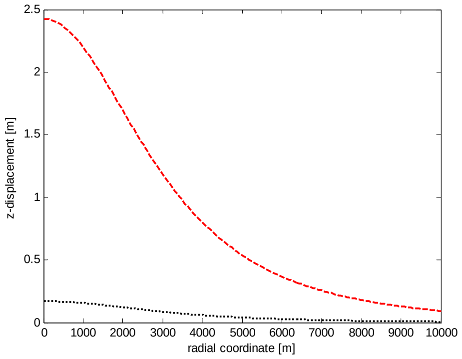


Figure 1. Comparison of ground uplift due to a pressure source of 320 MPa considering elastic (black line) and elasto-plastic rheology (red line). The radius of the source, located at a depth of 4km, is 0.7km.

The maximum uplift reaches 0.17 m in the case of a fully elastic half-space and 2.4 m considering an elasto-plastic medium. Therefore, the numerical model that includes an anelastic rheology enables to produce deformation comparable with those obtained from elastic model, requiring a significantly lower pressure. Particularly, in the case of elasto-plastic model, the deformation field is about 14 times higher than those obtained from the fully elastic model. A non-linear dependence between pressure source and deformation field can be found (Figure 2). Figure 2 shows that an uplift of 0.17 m (elastic deformation) in the case of elasto-plastic model can be reached with a pressure changes of only ~47 MPa that is near to crustal strength (~45 MPa).

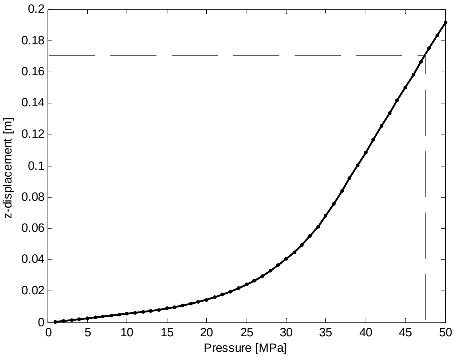


Figure 2. Non linear dependence between pressure source and ground uplift. Simulations were computed considering an ideal plastic medium with yield strength of σ_y =15 MPa.

2.2 Thermal model

The mechanical properties of the medium depend on the temperatures of the magma chamber and of the surrounding rocks. If we assume a magma chamber maintained at a constant temperature, the temperature distribution around the magmatic source can be computed by solving a heat conduction equation.

We developed the model in two steps solving separately: (i) the heat conduction equation to compute the temperature profile, and (ii) the mechanical problem to obtain the numerical solution of the deformation field.

To derive the temperature profile, we numerically solved the equations for heat conduction in a axial symmetric formulation:

$$\nabla \cdot (k \nabla T) = -A \qquad (1)$$

where T=T(r,z) is the temperature field, r is the radial coordinate, z is the vertical coordinate, k is thermal conductivity, and $A(z)=A_S\exp(-z/b)$ is the volumetric crustal volumetric heat production, where A_s is the volumetric rate of heat production, and b is a characteristic depth of the order of 10^5 km. As boundary condition at the ground surface, we assumed that the surface is kept constant at atmospheric temperature, since the thermal conductivity of the air is much smaller than that of the ground. At bottom and lateral boundaries we assigned the geothermal temperature values, because they are far enough to not be affected by the magmatic source. We used the steady-state geothermal profile given by (Ranalli, 1995; Turcotte and Schubert, 1982):

$$T(z)=T_s+\left(\frac{q_m z}{k}\right)+\left(\frac{A_s b^2}{k}\right)\left(1-e^{-z/b}\right) \qquad (2)$$

where T_s is the surface temperature, q_m is the heat flow coming from the mantle. The temperature on the magma wall was set to T_0=1500 K. Physically, this boundary condition is equivalent to stating that the magma walls act as heat sources, simulating a continuous refilling of the magma chamber (Dragoni et al., 1997; Civetta et al., 2004).

The thermal profile obtained from simulation is shown in Figure 3.

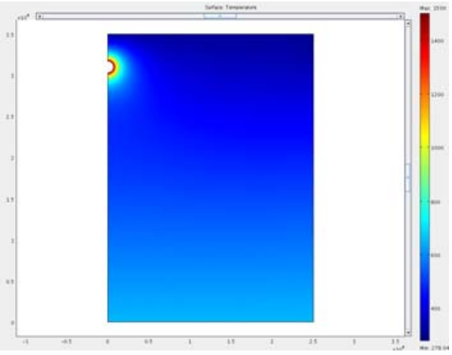


Figure 3. Temperature profile assuming a temperature of the magma wall of 1500 K

Hence, we solved the mechanical problem modifying the properties of the rocks using the computed thermal profile.

In the elasto-plastic model it was supposed an elasto-plastic behaviour where the temperature is higher than the brittle-ductile transition value, an elastic behaviour where it is lower. Figure 4 shows the ground uplift setting the threshold to 600 K (black line) and 700 K (red line). As the temperature threshold decreases, the volume participating to the elasto-plastic flow increases, giving more contribution to the deformation field.

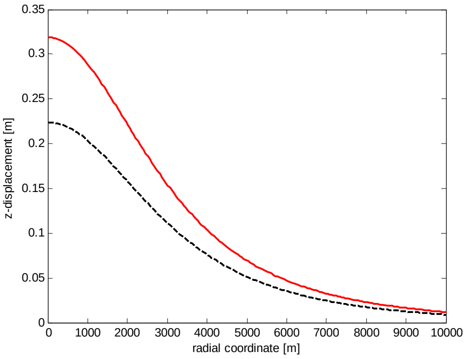


Figure 4. Ground uplift for the elasto-plastic model when the brittle-ductile transition is 700 K (black line) and 600 K (red line).

4. An application to Etna volcano

We extended the finite element method, used for the axi-symmetric model, to a fully 3D formulation to study the long-term deformation observed at Mt. Etna in 1993-1997 period. Mt. Etna is the biggest active volcano in Europe and one of the best monitored in the world. The continuous development of monitoring systems and the frequent eruptions in the last decades have improved the knowledge of the volcano during its different phases of activity (Bonaccorso et al. 2004). Since 1993, different geodetic techniques (EDM, GPS, SAR and leveling data) identified an inflation phase characterized by a uniform expansion of the overall volcano edifice. The beginning of the inflation phase was detected from the comparison of SAR images covering the 1993-1995 time intervals. The inversion of interferograms required the inflation from a spheroidal magmatic source located at about 5 km bsl (Lundgren al., 2003). Also levelling data supported the presence of a pressurized spherical source beneath the summit craters at 4.5 km bsl (Obrizzo et al., 2004). Recently, Bonaccorso et al. (2005) modeled the 1993-1997 GPS and EDM data by a pressurized ellipsoidal source with an extremely high pressure change of about 320 MPa. Since no eruption occurred during this period, the estimated pressure change should remain below the crustal strength. Following the conclusions of the studies cited above, Currenti et al. (2008) estimated the ground deformation expected to accompany the 1993-1997 inflation phase on Mt Etna in terms of viscoelastic response of the medium using the same ellipsoidal source model by Bonaccorso et al. (2005). Despite of the contribution of the viscous flow, the pressure change required to justify the observed deformation cannot be lowered than 170 MPa, which is still higher than the crustal strength.

The simulations performed in the previous section highlighted the strong influence of the elasto-plastic flow on the deformation field. With the aim to further decrease the value of the magma pressure, we reviewed the 1993-1997 inflation phase on Mt Etna considering the elasto-plastic rheology. The elliptical source is located 4.2 km bsl beneath the central craters (latitude 4177.9 UTM km and longitude 500.7 UTM km). The ellipsoid has a semi-major axis of 1854 m and the other two semi-axes of 725 m and 544 m, respectively with an orientation angle of 124° and a dip angle of 77°. In the numerical model we also included the rheological heterogeneities of the medium. We used P-wave and S-wave seismic velocities, inferred from recent seismic tomography studies (Chiarabba et al., 2000), in order to derive the elastic medium parameters. Particularly, the Young modulus was estimated by using the following equation (Kearey and Brooks, 1991):

$$E = 5/6\rho V_p^2 \qquad (3)$$

where V_p is the seismic P-wave propagation velocity, and is the density of the medium which was fixed to 2500 kg/m^3. Instead, the values of Poisson ratio were obtained using the equation (Kearey and Brooks, 1991)

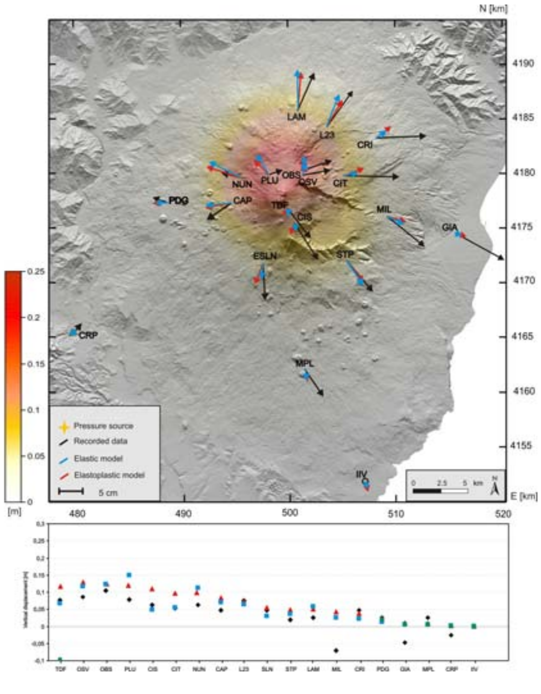
$$\nu = [(V_p/V_s)^2 - 2]/[2(V_p/V_s)^2 - 2] \qquad (4)$$

where V_s is the seismic shear wave propagation velocity. On the basis of Eqs. (3) and (4), the Young modulus varies from 11.5 GPa to 133 GPa, while the Poisson ratio is in the range 0.12-0.32.

The computational domain is set up to a large volume extending 100x100x100 km in order to avoid artifacts in the numerical solution because of the proximity of the boundary. The mesh of the ground surface was generated using a digital elevation model of Etna volcano from the 90 m Shuttle Radar Topography Mission (SRTM) data. The computational domain was represented by 53848 arbitrarily distorted tetrahedral elements connected by 9716 nodes.

Firstly, we solved the conductive heat transfer equation. As thermal boundary condition, a steady-state geothermal profile was set up along the bottom and lateral boundaries. A vertical geothermal gradient of 22 m°C/m was assumed for the areas surrounding the volcano edifice in agreement with temperature measurements carried out in deep AGIP boreholes (AGIP, 1977). A continuous refilling of the magma chamber was simulated by setting the temperature on the ellipsoidal source wall to 1500 K. Then, we solved the mechanical elasto-plastic model using the computational scheme described in the previous section.

The comparison between the elastic solutions obtained applying a pressure of 320 MPa and the elasto-plastic model in which we applied a pressure of 47 MPa are shown in Figure 4. Both the horizontal and vertical deformation in the elasto-plastic solution are comparable with those of the elastic solution. Therefore, the elasto-plastic model requires a lower pressure changes (~47 MPa) that is nearer to the crustal strength (~45 MPa).



5. Conclusions

Finite element models have been carried out to investigate the elasto-plastic deformation caused by pressure changes within a magmatic source. The definition of elastic\anelastic rock properties strongly affect the solution and, especially in volcanic region, cannot disregard the thermal regime of the crust. The thermo-mechanical model evidences that the thermal state of the crust can play an important role in the deformation field. The rheology assumption strongly affects the estimate of the magmatic pressure changes. The inclusion of a elasto-plastic shell in modeling the 1993-1997 inflation period on Mt Etna allows for lowering the pressure changes from 320 MPa for the elastic model to ~47 MPa for the elasto-plastic model. Since the estimated pressures are significantly different from each other, the picture of the volcano state is completely altered. Therefore, the definition of the rheology becomes a key parameter in hazard assessment.

8. References

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