Chaotic Behavior of the Airflow in a Ventilated Room

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Abstract: Chaotic systems may lead to instability, extreme sensitivity and performance reduction. Therefore it is unwanted in many cases. Due to these undesirable characteristics of chaos in practical systems, it is important to recognize such a chaotic behavior. The existence of chaos has been discovered in several areas during the last 30 years. However, there is a lack of studies in relation with buildings that also can be regarded as complex dynamic systems. In this paper the chaotic behavior of the airflow in case of an ordinary ventilated room is researched. Chaotic behavior is already observed in the simulations by changing the supply air temperature from 22 ^{o}C into 21.9 ^{o}C. In the case when no buoyancy is taking into account, minor chaotic behavior is observed by a small change in the air supply control parameters.

Keywords: Airflow, chaos, system, Comsol

1. Introduction

Chaotic systems may lead to instability, extreme sensitivity and performance reduction. Therefore it is unwanted in many cases. Due to these undesirable characteristics of chaos in practical systems, it is important to recognize such a chaotic behavior. The existence of chaos has been discovered in several areas during the last 30 years. However, there is a lack of studies in relation with buildings that also can be regarded as complex dynamic systems as well. In this paper we research the chaotic behavior of the airflow in case of an ordinary ventilated room with an on/off controller. The main approach to detect chaos in a given system is to investigate numerical results. There are several universal indicators for chaos. In this research we use the property that all chaotic systems are extreme sensitive for initial conditions and/or system parameters, i.e. small differences in them can lead to extraordinary differences in the system states.

Our research methodology was as follows:

(1) Literature review on the application of chaos in the built environment. References [1-5] provide in good introduction to the subject.

(2) Selection of a specific promising case study and reproduce simulation results using Comsol;

(3) Investigate chaotic behavior by small changes in system parameters.

(4) Extend the model with an on/off controller.

The paper is organized in the same way.

2. Airflow in a room using Comsol

A case study was selected based on the work of Sinha et al [6]. The case comprehends airflow in a ventilated room. Figure 1 shows the geometry.

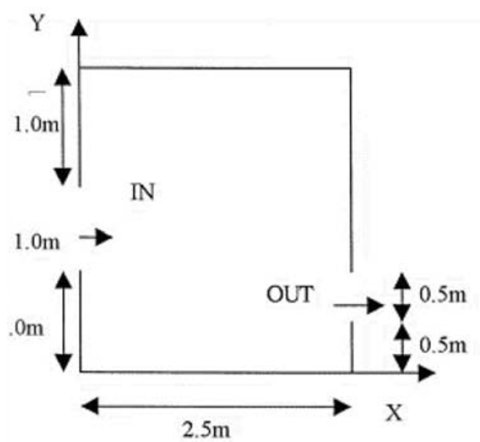


Figure 1. The geometry of the airflow problem.

The PDEs are based on the well-known Navier-Stokes equations :

$$\frac{\partial u}{\partial t} = -\frac{\partial(uu)}{\partial x} - \frac{\partial(vu)}{\partial y} - \frac{\partial p}{\partial x} + \frac{1}{\mathrm{Re}}\nabla^2 u$$

$$\frac{\partial v}{\partial t} = -\frac{\partial(uv)}{\partial x} - \frac{\partial(vv)}{\partial y} - \frac{\partial p}{\partial y} + \frac{1}{\mathrm{Re}}\nabla^2 v + \frac{Gr}{\mathrm{Re}^2}T$$

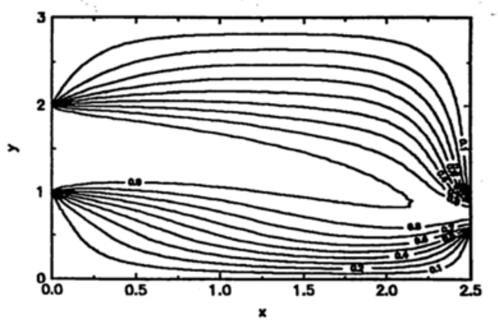
$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\frac{\partial T}{\partial t} = -\frac{\partial(uT)}{\partial x} - \frac{\partial(vT)}{\partial y} + \frac{1}{\mathrm{Re}\,\mathrm{Pr}}\nabla^2 T$$

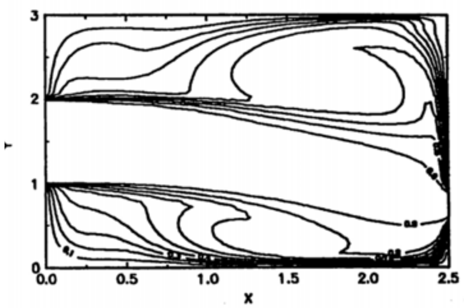
The boundary conditions are:

At the left, right, top and bottom walls:
u=0, v=0, T=0.
At the inlet:
u=1, v=0, T=1.
At the outlet :
Neuman conditions for u,v and T

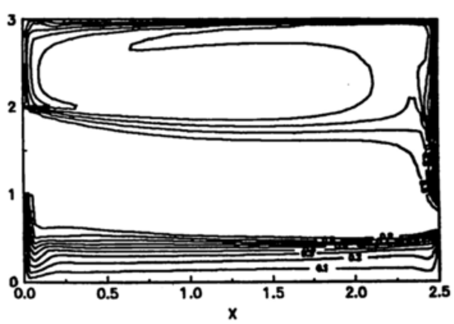
The temperature solutions for several Re and Gr numbers from [6] are shown in figure 2.



a) Re = 50 , Gr=0



b) Re =1000 , Gr =0



c) Re = 1000 , Gr =2.5 10^7

Figure 2. Temperature inside the room for several Re and Gr according [6]

The model of the previous Section was implemented in Comsol [7;8]. Figure 3 shows the (satisfactory) verification results:

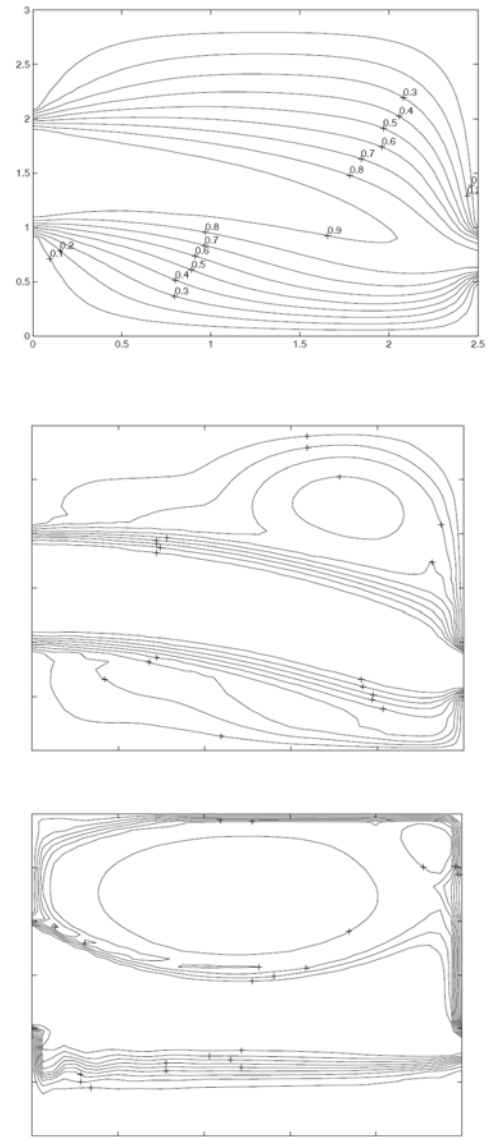


Figure 3. Temperature inside the room for several Re and Gr using Comsol

The most interesting behavior of the airflow in the ventilated room is observed when buoyancy is taken into account (i.e. Gr = 2.5 10^7 and Re = 1000). Figure 4 shows the temperature distributions after respectively 20 and 40 time steps in case of the constant inlet air temperature boundary condition.

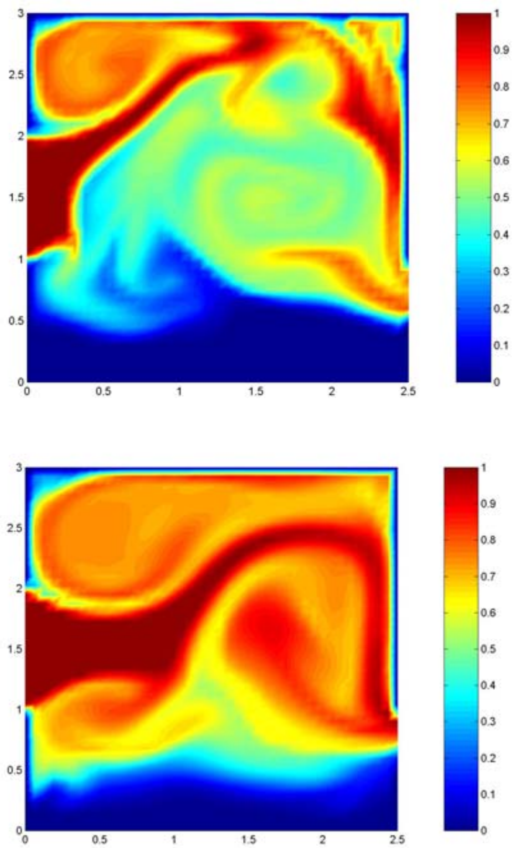


Figure 4. Temperature distribution (scaled between 0 and 1) after 20 (top) and 40 time steps (bottom).

4. Chaotic behavior

In order to detect chaotic behavior, the airflow was simulated again with the only difference that the supply air temperature was slightly changed from 22 ^{o}C into 21.9 ^{o}C (i.e. 1 into 0.98 for the scaled temperature). In Appendix A through C the results are presented. Each appendix figure shows the temperature distribution after respectively 10, 40 and 60 time steps. The top left figures have air supply temperatures equal to 22 ^{o}C. The top right figures have air supply temperatures equal to 21.9 ^{o}C. The bottom (left) figures represent the air temperature difference distribution between its top figures. The results of the appendix A through C show that temperature distribution seems to be very sensitive for the air supply temperature: A small change of 0.1 ^{o}C in the air supply temperature (i.e. 1 into 0.98 for the scaled temperature) may temporary and locally lead to opposite temperatures i.e. hot instead of cold and visa versa. This can be best observed in the bottom (left) figure of Appendix C. It is clear that chaotic behavior is observed, even without an on/off controller which is common in ventilated rooms. The next Section proceeds with adding an on/off controller.

5. Extension with on/off controller

Figure 5 shows the SimuLink model including the on/off controller (Relay) and a so-called S-Function with the Comsol model. Modeling details can be found in references [7;8].

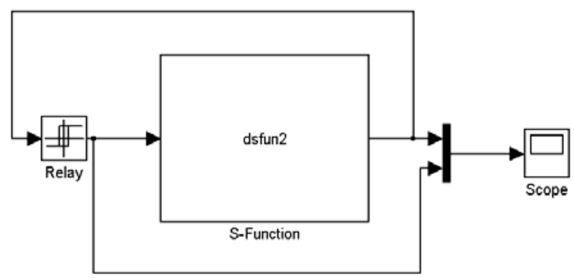


Figure 5. The SimuLink model with the Comsol model implemented into an S-Function

In this case no buoyancy is taken into account i.e. Gr = 0. The ‘sensor’ of the Relay provides input for this SimuLink block and is located at position x=2; y=2.5 in the room (see also figure 6 at the marked circle). The Relay switches the supply temperature (scaled) between 1 (hot, 22 ^{o}C) and cold (cold 17 ^{o}C) if the ‘sensor’ temperature is respectively below 0.3 (i.e. 19.2 ^{o}C) and above 0.5 (i.e. 20 ^{o}C). Figure 6 presents the temperature distributions after respectively 8 and 10 time steps in case of alternate switching the inlet air temperature boundary between hot and cold air respectively.

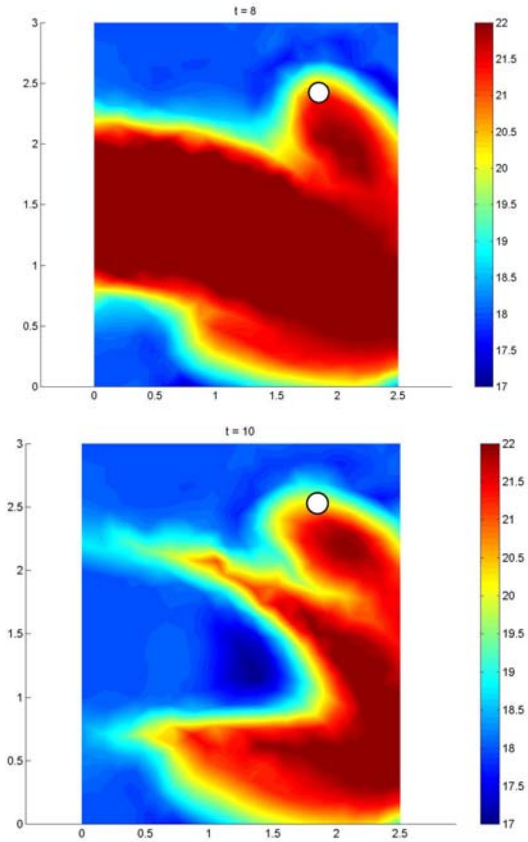


Figure 6. Temperature distribution (Gr=0; Re=1000) in case of supplying hot (top) and cold (bottom) air. The circle marks the location of the 'sensor' used as input for the Relay i.e. switch.

In order to investigate the occurrence of chaotic behavior analog to Section 4, a slightly change of a system parameter is made and differences in airflow patterns is studied. In this case the effect of changing the 'sensor' temperature threshold for switching cold air from 0.3 into 0.32 and hot air from 0.5 into 0.48, is studied. Appendix D shows the results that are similar presented as the previous graphs at the appendices. In contrast with Section 4, no permanent difference between the two temperature distributions is observed. However, temporality differences between the just mentioned distributions are observed due to an occurring time lag between the on/off switching. This can be clearly seen from the figure in appendix D.

6. Conclusions

In this paper the chaotic behavior of the airflow in case of an ordinary ventilated room is researched. Chaotic behavior is already observed in the simulations by changing the supply air temperature from 22 ^{o}C into 21.9 ^{o}C. In the case when no buoyancy is taking into account, minor chaotic behavior is observed by a small change in the air supply control parameters.

References

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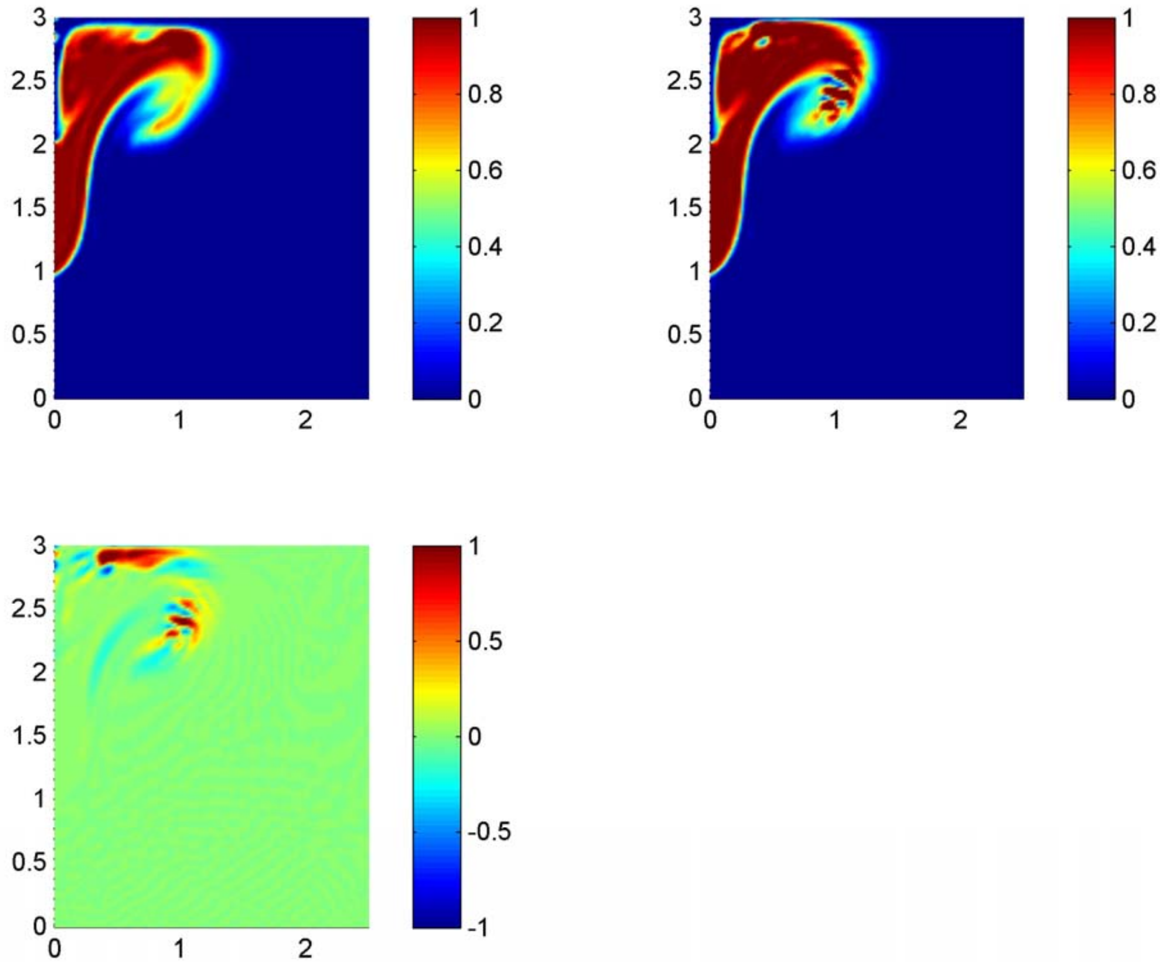
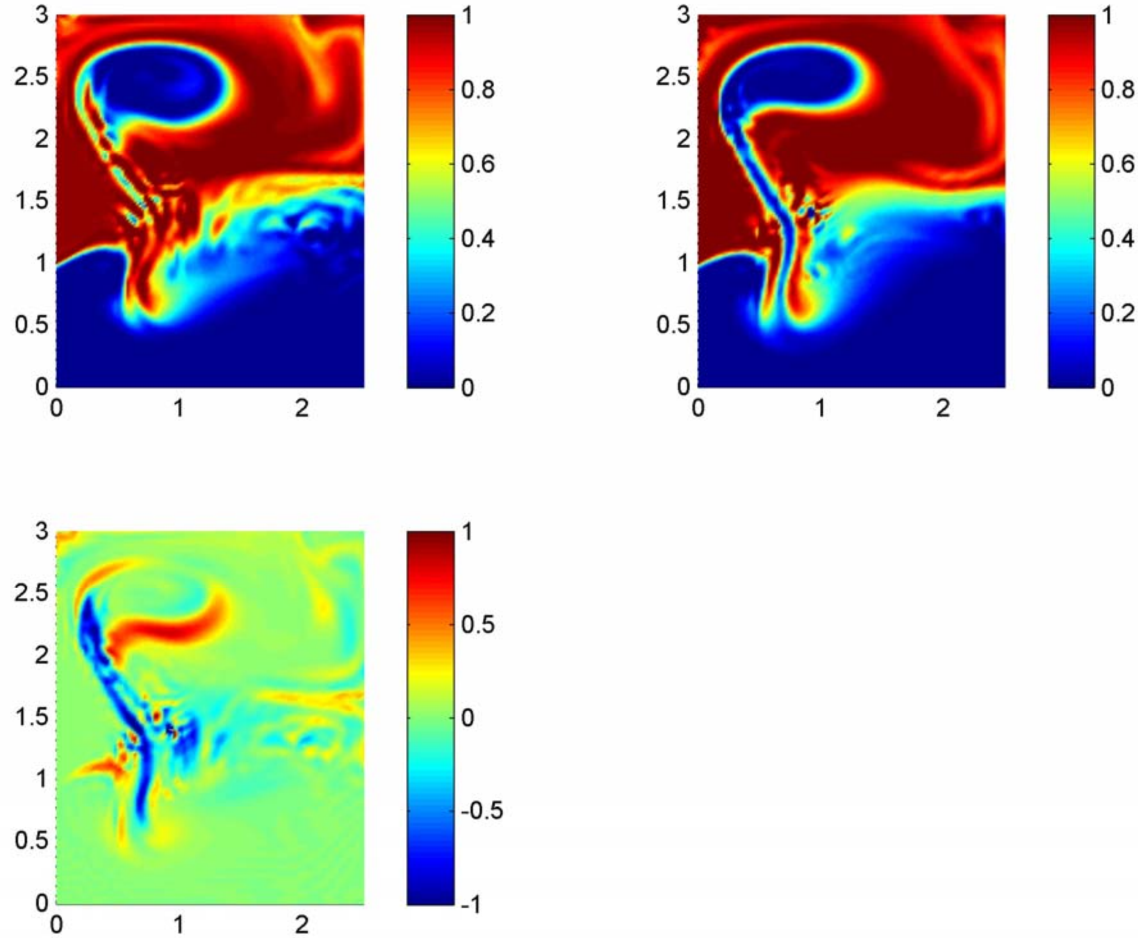
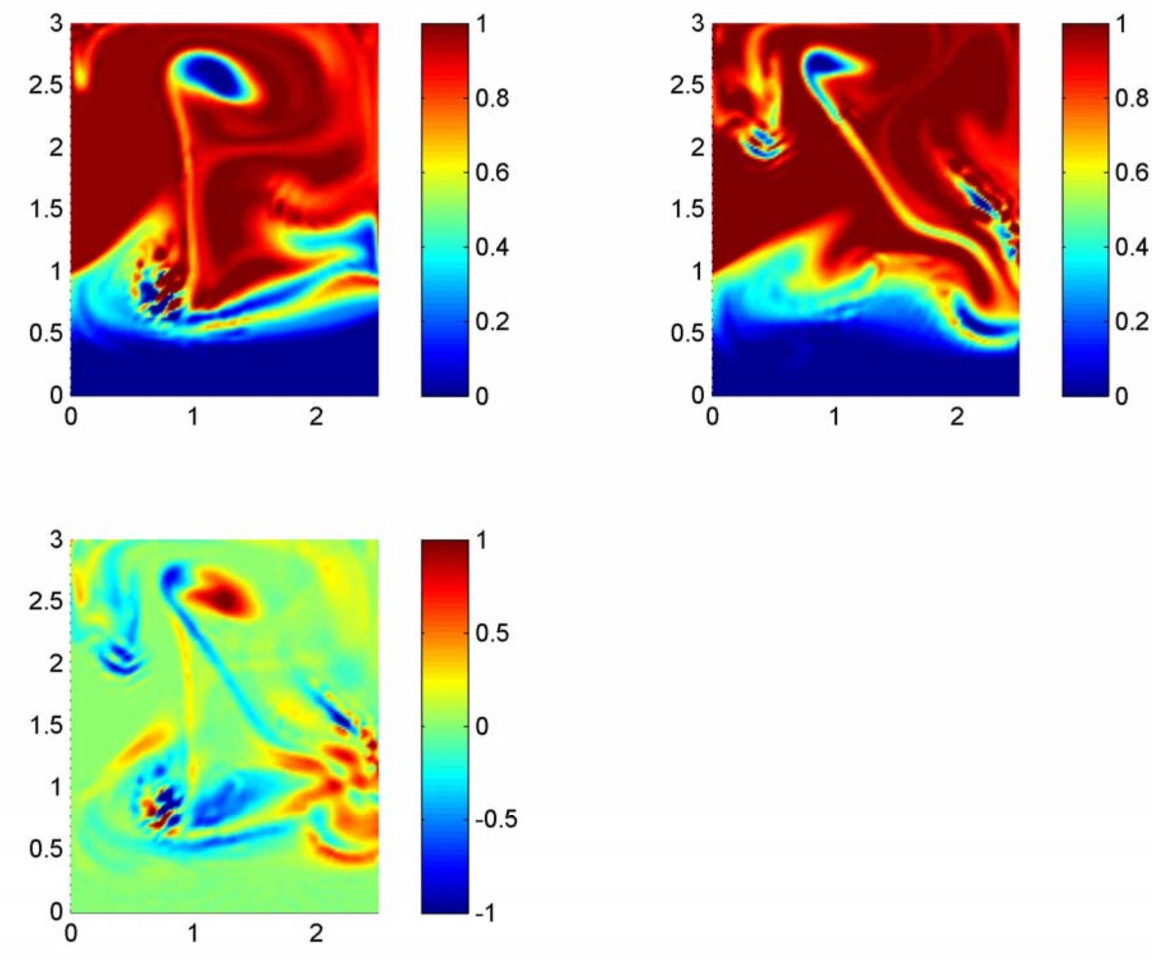
Appendix



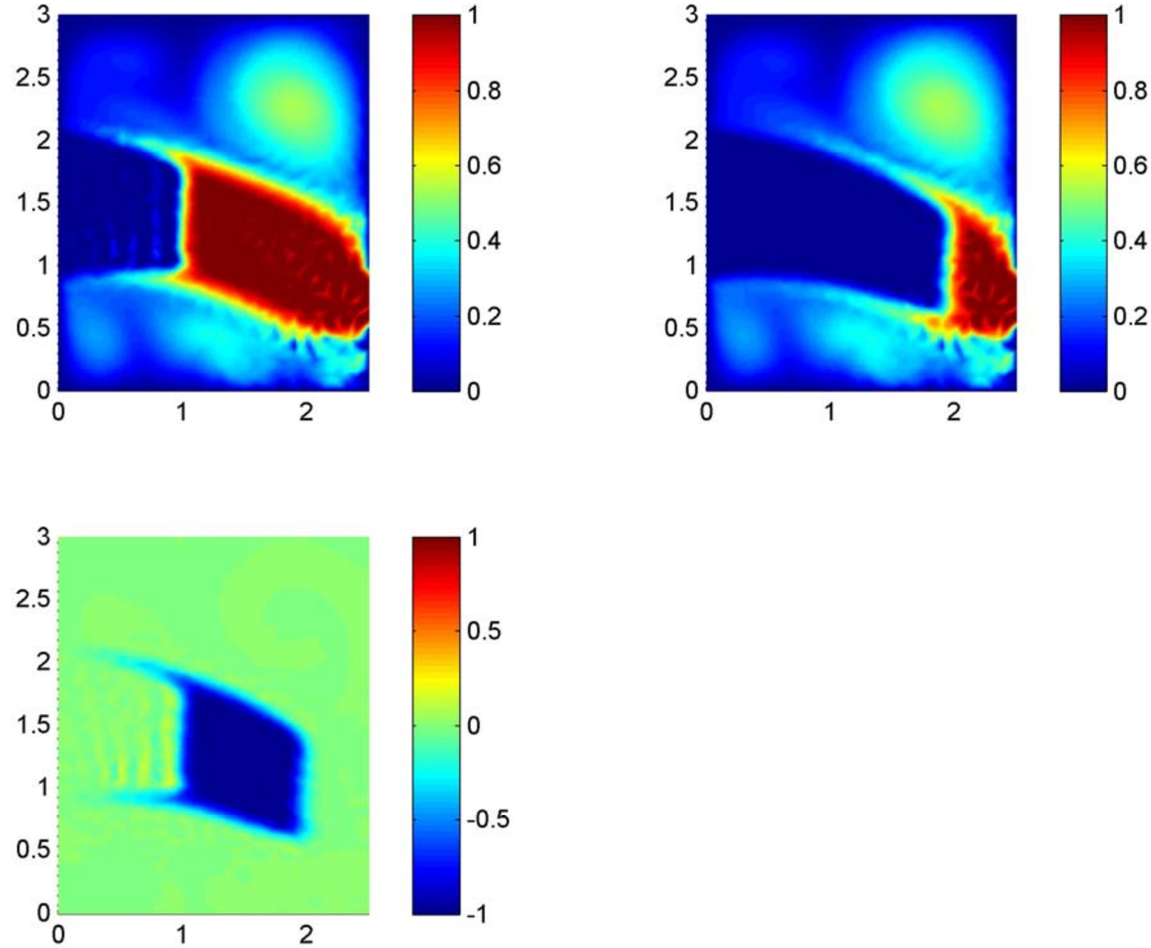
Appendix A: Scaled temperature distribution after 10 time steps (case: Gr = 2.5 10^7 and Re = 1000); Top Left: Air supply temperature equals 1 (22 ^{o}C); Top Right: Air supply temperature equals 0.975 (21.9 ^{o}C); Bottom: Air temperature difference between the top figures where 0 means no difference; +1 means 4 ^{o}C hotter; -1 means 4 ^{o}C colder



Appendix B: Scaled temperature distribution after 40 time steps (case: Gr = 2.5 10^7 and Re = 1000); Top Left: Air supply temperature equals 1 (22 ^{o}C); Top Right: Air supply temperature equals 0.975 (21.9 ^{o}C); Bottom: Air temperature difference between the top figures where 0 means no difference; +1 means 4 ^{o}C hotter; -1 means 4 ^{o}C colder



Appendix C: Scaled temperature distribution after 60 time steps (case: Gr = 2.5 10^7 and Re = 1000); Top Left: Air supply temperature equals 1 (22 ^{o}C); Top Right: Air supply temperature equals 0.975 (21.9 ^{o}C); Bottom: Air temperature difference between the top figures where 0 means no difference; +1 means 4 ^{o}C hotter; -1 means 4 ^{o}C colder



Appendix D: Scaled temperature distribution after 30 time steps with on/off controller case Gr = 0 and Re = 1000); Top Left: The ‘sensor’ temperature thresholds are respectively below 0.30 and above 0.50. Top Right: The ‘sensor’ temperature thresholds are respectively below 0.32 and above 0.48. Bottom: Air temperature difference between the top figures where 0 means no difference; +1 means 4 ^{o}C hotter; -1 means 4 ^{o}C colder