Modeling of Electrodynamics in High Temperature Superconducting Magnets with COMSOL Multiphysics®

L. Bortot[1,2], B.Auchmann[2,3], M. Mentink[2], S. Schöps[1], A. Verweij[2],

[1] Technische Universität Darmstadt, Germany, [2] CERN, Geneva, Switzerland [3] Paul Scherrer Institute, Villigen, Switzerland

Contact: lorenzo.bortot@cern.ch

COMSOL CONFERENCE 2018 LAUSANNE

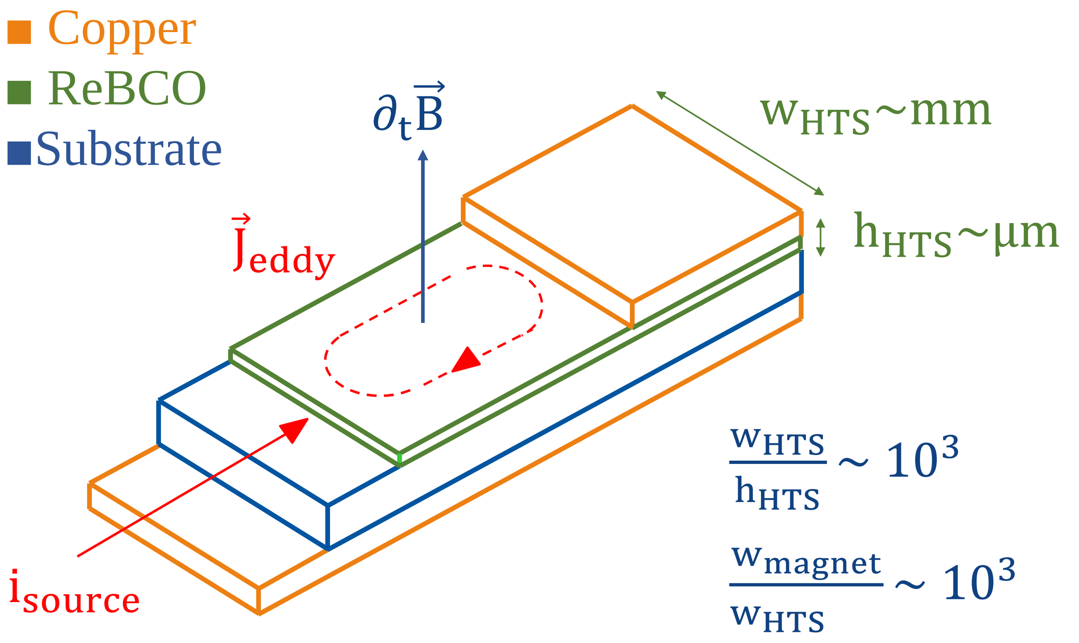
Motivation

NEW MAGNETS	20+ Tesla dipoles for future high-energy particle accelerators	
NEW ACCELERATORS	Upgrading existing accelerators, exploring new physics	⇨ **NEW SIMULATIONS!**
NEW TECHNOLOGY	Upgrade via High Temperature Superconductors (HTS)	

1) Challenge: Current-driven Eddy Current Problem, in a Thin Shell

HTS Tape structure

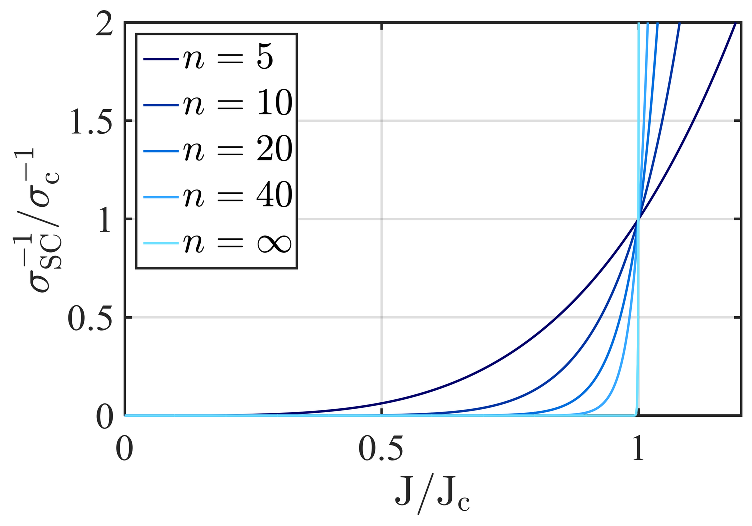
Rare-Earth Bismuth-Copper-Oxide layer (ReBCO)



Multiscale

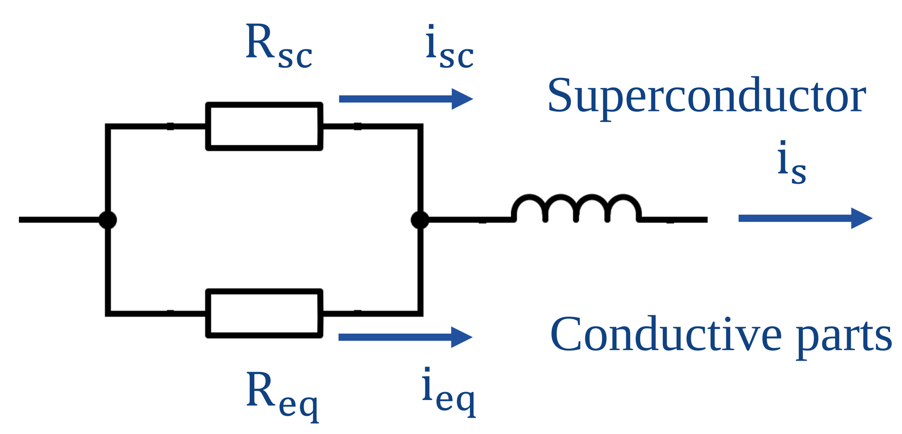
HTS resistivity

$$\sigma_{SC}^{-1} = \sigma_c^{-1}(\vec{B}) \left(\frac{J}{J_c(\vec{B})}\right)^{n-1} \qquad \sigma_{c,\,B=0}^{-1} \approx 10^{-15}\ \Omega\text{m} \qquad n_{ReBCO} \approx 20$$



Nonlinear

Current sharing regime in tape



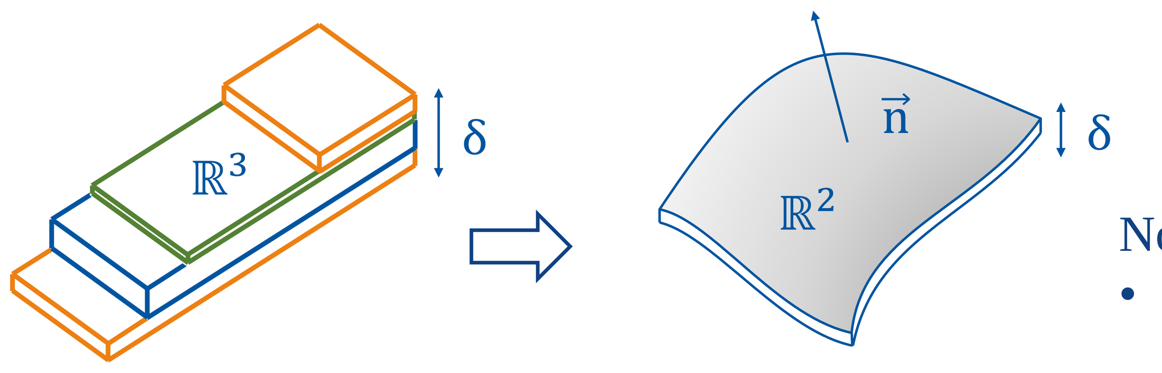
Algebraic constraints:

$$i_s = i_{sc} + i_{eq} \qquad i_{sc} = \frac{R_{eq}}{R_{sc}(i_{sc}) + R_{eq}} i_s$$

Implicit

2) Method: Field Formulation

Model order reduction

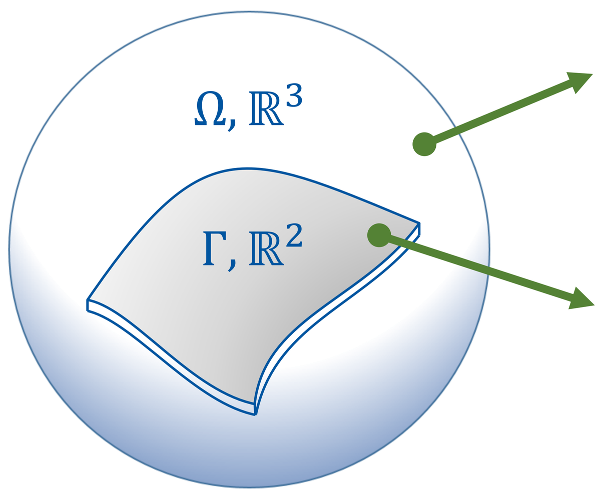


No dynamics in δ:

- $\partial_{\vec{n}}\,(\dots) = 0$

Domain decomposition

Combination of current and magnetic vector potentials [1]



Ω: Static field problem

$$\vec{B} = \nabla \times \vec{A}$$
$$\nabla \times \mu^{-1} \nabla \times \vec{A} = \vec{0}$$

Γ: Eddy current problem on boundary

$$\vec{J} = \nabla \times \vec{T}$$
$$\nabla \times \sigma^{-1}(\nabla \times \vec{T}) + \mu \partial_t \vec{T} = \vec{0}$$

Fully coupled weak formulation

$\vec{J} = \nabla T \times \vec{n}$ stream function [2] , Galerkin method

$$\int_\Omega \mu^{-1} \nabla \times \vec{A} \cdot \nabla \times \vec{W} + \delta \int_\Gamma \vec{W} \cdot \nabla T \times \vec{n} = 0 \qquad \text{Mf module}$$

$$\int_\Gamma \sigma_{SC}^{-1}(\nabla T \cdot \nabla W) - \int_\Gamma W(\partial_t(\nabla \times \vec{A}) \cdot \vec{n}) = 0 \qquad \text{Weak form b-PDE module}$$

$$\int_{\partial\Gamma} \delta\, T_0\ d(\partial\Gamma) - i_{source} = 0 \quad \text{Dirichlet}$$

Alternatives

A-Formulation

$$\nabla \times \mu^{-1} \nabla \times \vec{A} - \sigma \partial_t \vec{A} = \vec{0}$$
$$\nabla \cdot \sigma \partial_t \vec{A} = 0 \longrightarrow \text{Instability}$$

H-Formulation

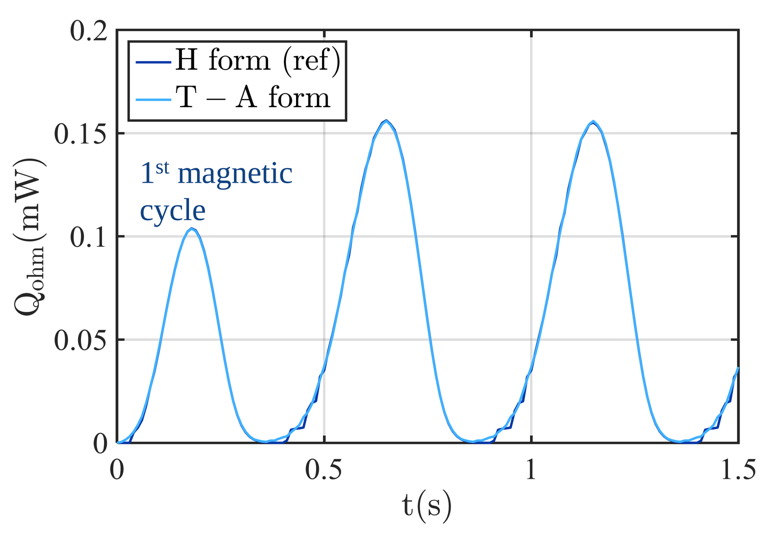
$$\nabla \times \sigma \nabla \times \vec{H} - \mu \partial_t \vec{H} = \vec{0}$$
$$\vec{J}\,\delta \longrightarrow \text{No surface currents}$$

[1] Biro, O., and Preis, K. "Finite element analysis of 3-D eddy currents." *IEEE Trans. Mag.* (1990).
[2] Rodger, D., and Atkinson N. "Finite element method for 3D eddy current flow in thin conducting sheets." *IEE Proc.* (1988).

3) Validation: 2D Crosscheck and Benchmark

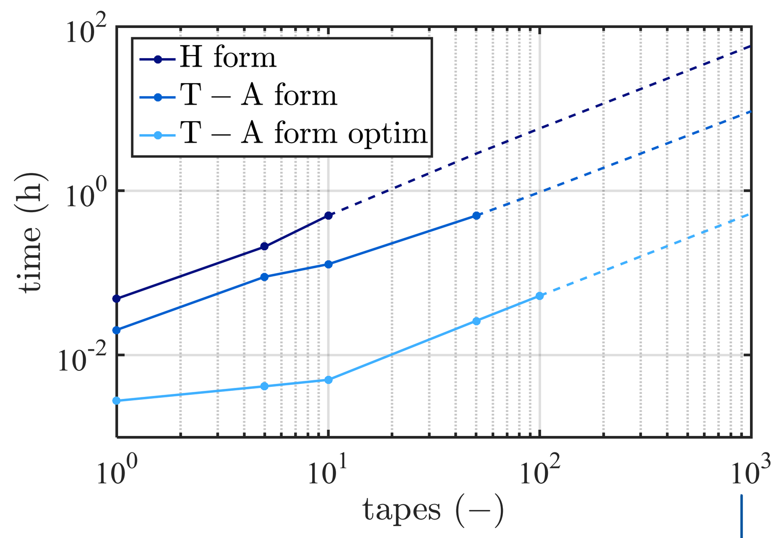
Crosscheck: Single tape

$i_{source} = 0.5 I_{crit} \sin(2\pi t)$



Benchmark: H vs T-A form

Linear extrapolation



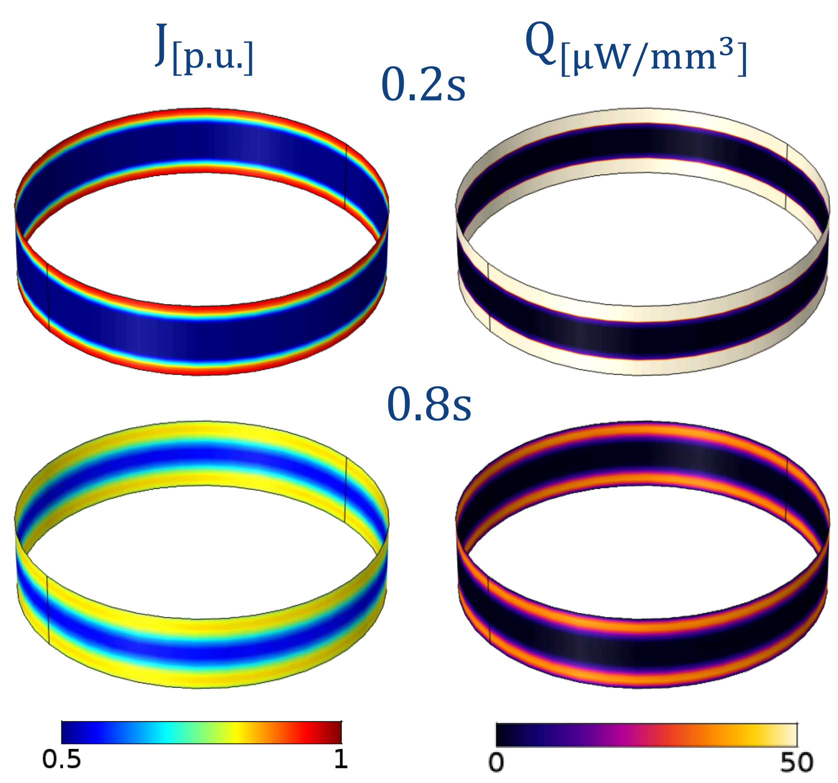
10^3 tapes, 0.5 hours

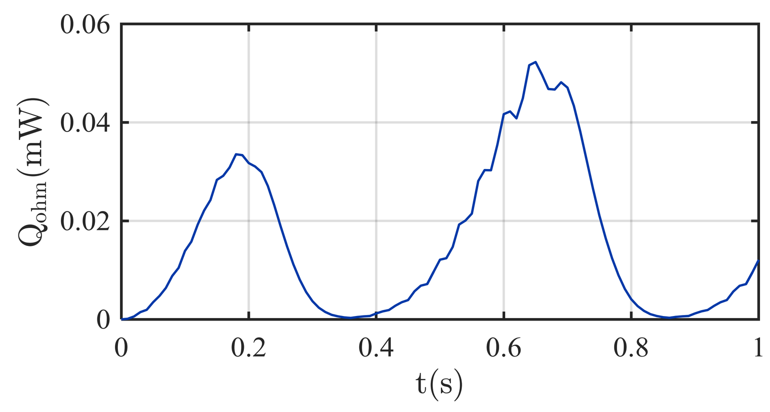
Reference: HTS MODELLING WORKGROUP

2D: stable, efficient, simple

4) Numerical Results: 3D Flat Ring

CPU	:	Intel Xeon E5-2667 3.2 GHz
DoF	:	220k
$\mathbf{t_{comp}}$	:	0.6 h
$\mathbf{i_{source}}$	:	$0.75 I_{crit} \sin(2\pi t)$





Outlook

3D encouraging, but open questions:

- Accuracy in $Q_{ohm} \propto J^{n+1}$
- Efficient $\vec{A}$ gauge
- Optimized mesh and solver

Next Steps

- Thermal physics
- Complex cables (e.g. Roebel)
- Full HTS magnets

Lausanne, Switzerland, October 22 -24, 2018

This work has been sponsored by the Wolfgang Gentner Programme of the German Federal Ministry of Education and Research (grant no. 05E12CHA)

Special Thanks: F. Grilli, M. Maciejewski, M. Prioli, E. Ravaioli, J. Van Nugteren