



Mechanical Damage Models for Concrete

From Classical Mazars' model to fully integrated multiaxial regularized methods

24th October 2018

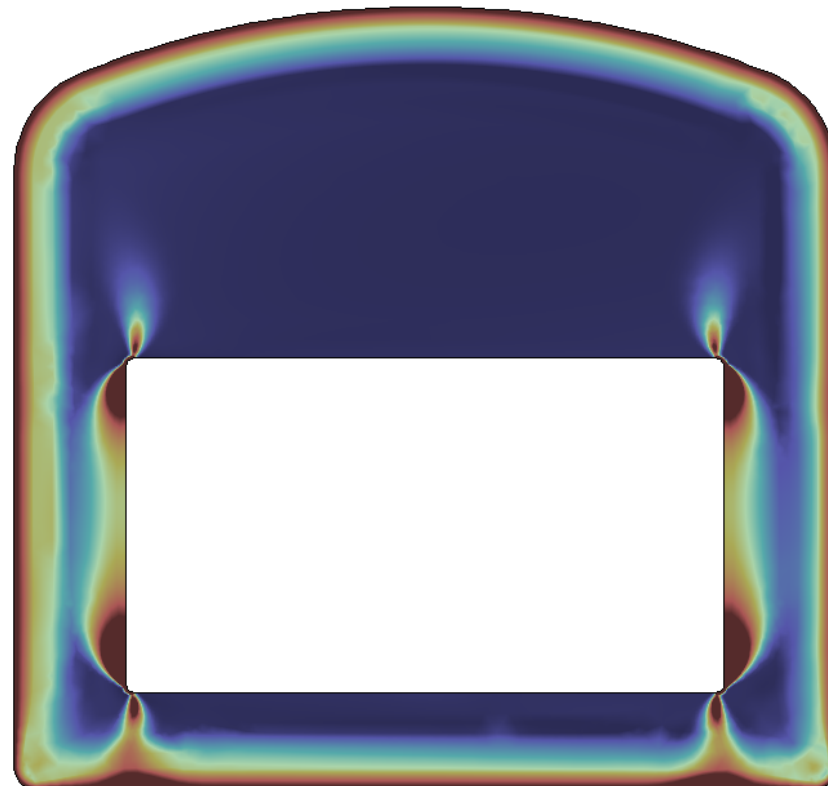
marcelo.lavina@amphos21.com

www.amphos21.com

A²¹

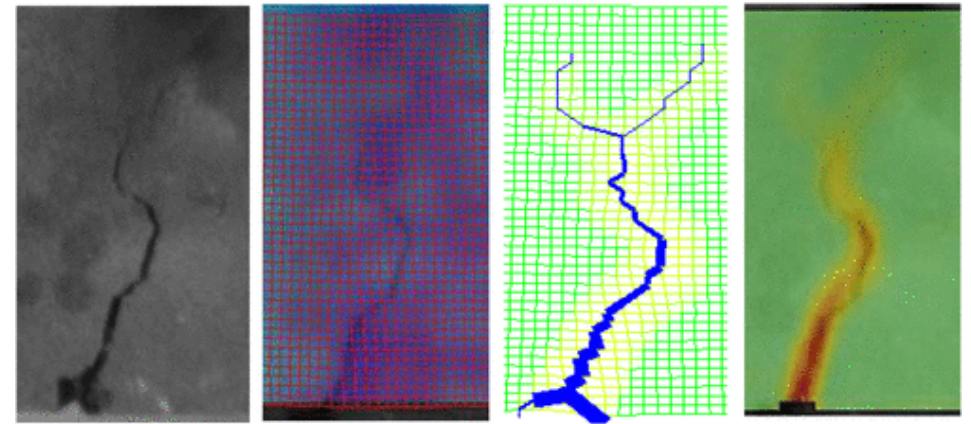
Outline

- Introduction
- Mazars' damage model
- External material vs built-in implementation
- Regularization
- μ damage model
- Conclusions
- Application case



Introduction

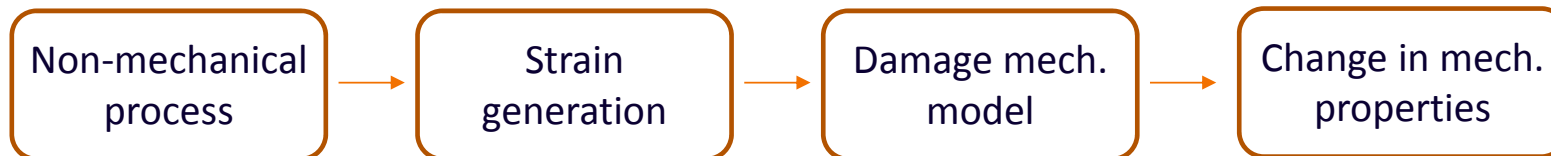
- Concrete structures
 - Damage mechanics
 - Quasi-brittle behaviour
 - Cracking representation
 - Various loading conditions
 - Load bearing capacity and post-peak behaviour
 - Accurate representation through a simple isotropic model



Concrete cracking representation with different approaches

Final goal:

Develop concrete mechanics model in the Comsol interface that allows its coupling with other processes, such as chemical degradation for durability assessment, or moisture and heat transport.



Mazars' damage model

- Damage mechanics theory [1]
 - Based on scalar damage variable affecting directly the stiffness tensor

$$E = E_0 \cdot (1 - d)$$

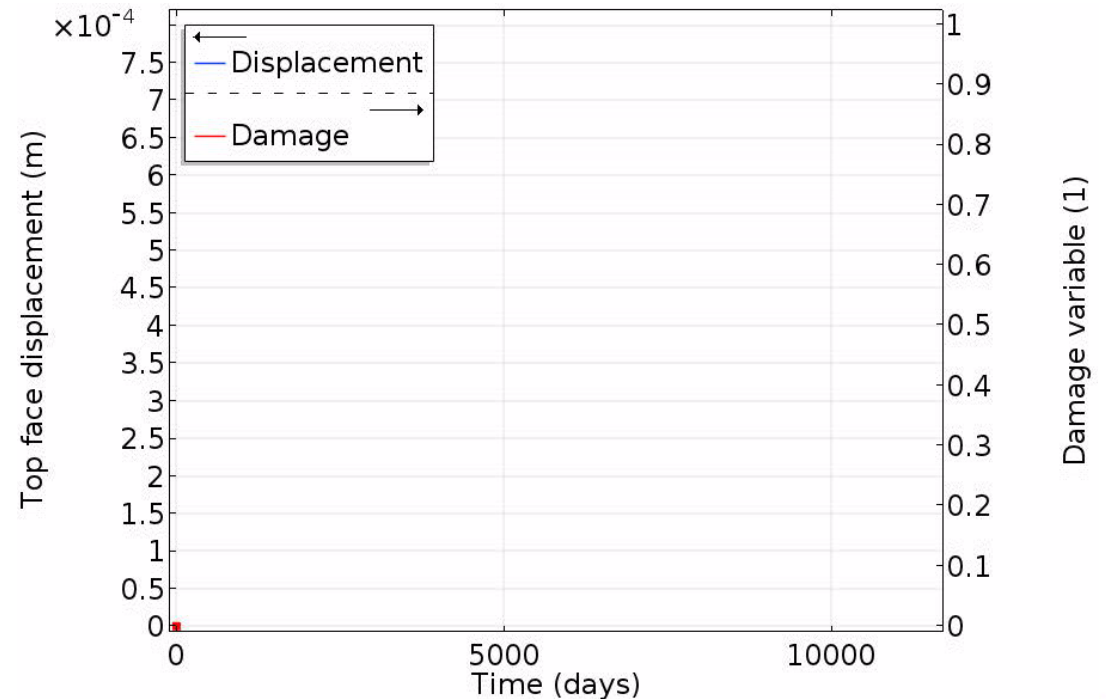
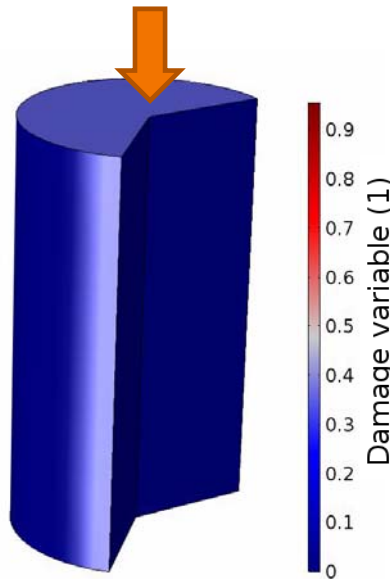
- Stress – strain non-linear relation

$$\sigma = f(d) \cdot \varepsilon$$

$$d = f(\varepsilon)$$

- Mazars' formulation [2]
 - d is isotropic and scalar
 - Different laws for compression or tension stress state
 - Strain maximum values

History variable



Uniaxial cyclic loading test. Test representation, damage variable evolution and deformation (left); time evolution of the top face displacement in m (cyclic) and damage variable (monotonous increasing) on the right.

[1] Kachanov LM, 1958. *Isv. Akad. Nauk. SSR*, 8, 26–31.
 [2] Mazars J, 1986. *Engineering Fracture Mech.*, 25(5–6), 729-737.

External material vs. built-in implementation

- External material model
 - Comsol post by Ed Gonzalez (2015) [1]
 - Any constitutive model can be programmed
- Built-in implementation
 - 2 History variables storage
 - Domain Ordinary Differential Equations
 - Specific solver configuration

2 additional degrees of freedom

Segregated step + Previous solution node

The image shows a screenshot of the COMSOL Multiphysics software interface. On the left, the 'Model Builder' tree is visible, showing a hierarchy of definitions: Global Definitions (Parameters, Materials), Component 1 (Definitions, Geometry, Materials, Molal Solute Transport, Phreeqc Results), and Solid Mechanics (Linear Elastic Material 1, Free, Initial Values, External Stress-Strain Relation 1, Symmetry, Rigid Domain, Prescribed Displacement). On the right, a code editor displays C++ code for an external material model. The code includes headers for math, stdlib, string, and stdio, and defines macros for MIN and MAX. It also defines a function 'eval' that takes a strain tensor 'e' and returns a stress tensor 's' and its Jacobian 'D'. The code is commented with the Mazars' damage model interface.

[1] Gonzalez E, 2015. Accessing External Material Models for Structural Mechanics. COMSOL Blog December 2015.

External material vs. built-in implementation

- External material model
 - Comsol post by Ed Gonzalez (2015) [1]
 - Any constitutive model can be programmed
- Built-in implementation
 - 2 History variables storage
 - Domain Ordinary Differential Equations
 - Specific solver configuration

2 additional degrees of freedom

Segregated step + Previous solution node

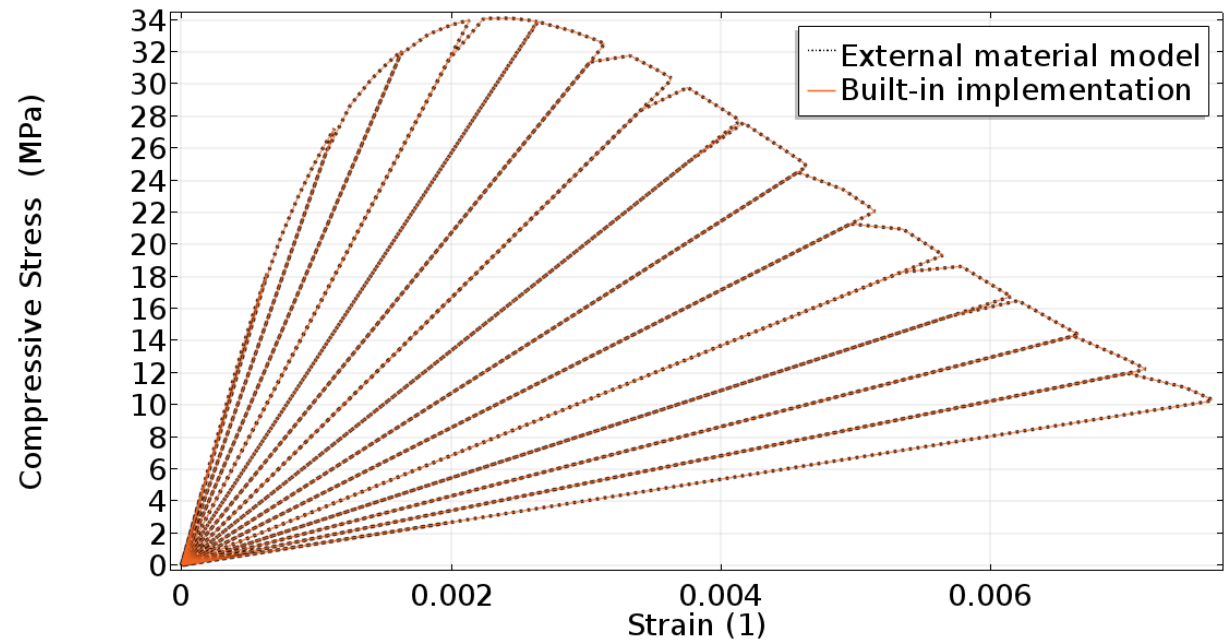
| Name | Expression | Unit | Description |
|-------------|--|------|-------------|
| eef2 | $\max(et1 * et1 + et2 * et2 + et3 * et3, 1e-10)$ | | |
| eef | $\text{sqrt}(eef2)$ | | |
| ets1 | $(1.0 + nu0) / E0 * st1 - nu0 / E0 * stsum$ | | |
| ets2 | $(1.0 + nu0) / E0 * st2 - nu0 / E0 * stsum$ | | |
| ets3 | $(1.0 + nu0) / E0 * st3 - nu0 / E0 * stsum$ | | |
| ecs1 | $ep1 - ets1$ | | |
| ecs2 | $ep2 - ets2$ | | |
| ecs3 | $ep3 - ets3$ | | |
| alphat | $(ets1 * \max(ep1, 0.0) / eef2) + (ets2 * \max(ep2, 0.0) / eef2) + (ets3 * \max(ep3, 0.0) / eef2)$ | | |
| alphac | $(ecs1 * \max(ep1, 0.0) / eef2) + (ecs2 * \max(ep2, 0.0) / eef2) + (ecs3 * \max(ep3, 0.0) / eef2)$ | | |
| kappa_max | $\max(kappa, \max(eef, kappa0))$ | | |
| damt | $1.0 - kappa0 * (1.0 - At) / kappa_max - At * \exp(-kappa_max * t)$ | | |
| damc | $1.0 - kappa0 * (1.0 - Ac) / kappa_max - Ac * \exp(-kappa_max * t)$ | | |
| mechdam | $\text{alphat} * \text{damt} + \text{alphac} * \text{damc}$ | | |
| MechDa... | $\max(\text{Mech_Dam}, \max(\text{mechdam}, 0))$ | | |
| mechanic... | $\min(\max(\text{MechDamMax}, 0), 0.999)$ | | |

[1] Gonzalez E, 2015. Accessing External Material Models for Structural Mechanics. COMSOL Blog December 2015.



External material vs built-in implementation

- External material model
 - Comsol post by Ed Gonzalez (2015) [1]
 - Any constitutive model can be programmed
- Built-in implementation
 - 2 History variables storage
 - Domain Ordinary Differential Equations
 - Specific solver configuration
- Advantages
 - Fully coupling with other constitutive models
 - Fully coupling with other physics
 - Variables availability (pre/post-process)
 - Easier adjustment or reformulation (compilation avoided)
- Drawback
 - Increased model complexity (DOF's and solvers)



Results of the verification (uniaxial compression) test in terms of stress-strain curves using two different damage model implementations.

[1] Gonzalez E, 2015. Accessing External Material Models for Structural Mechanics. COMSOL Blog December 2015.

Regularization method – Gradient enhanced formulation

- Implicit gradient formulation [1 ; 2]:
 - Implemented as a Helmholtz differential equation

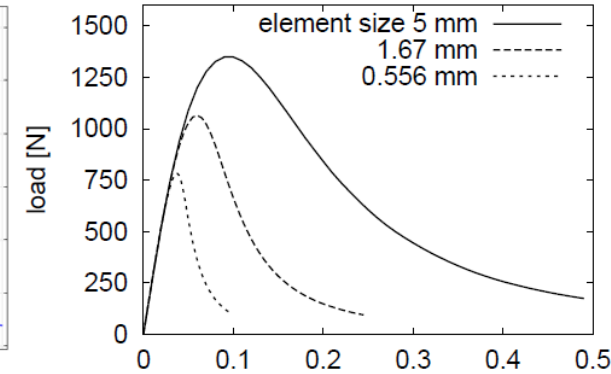
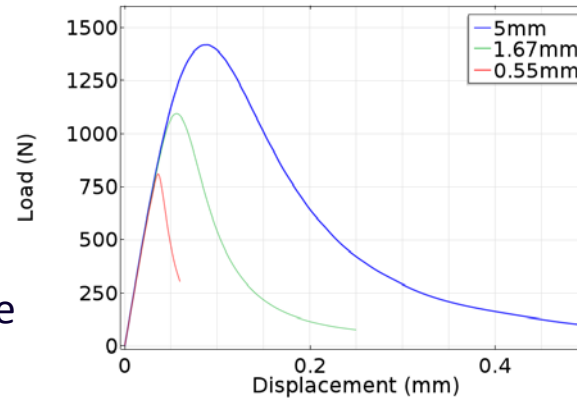
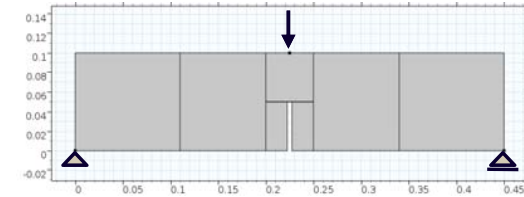
$$\bar{\varepsilon} - l^2 \nabla^2 \bar{\varepsilon} = \tilde{\varepsilon}$$

- Local equivalent strain $\tilde{\varepsilon}$
- Non-local equiv. strain $\bar{\varepsilon}$
- characteristic length l (m)

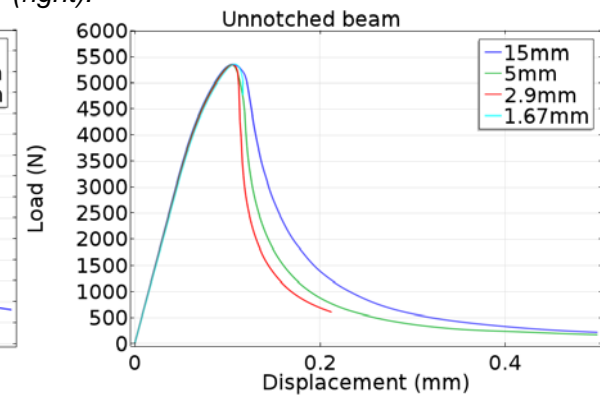
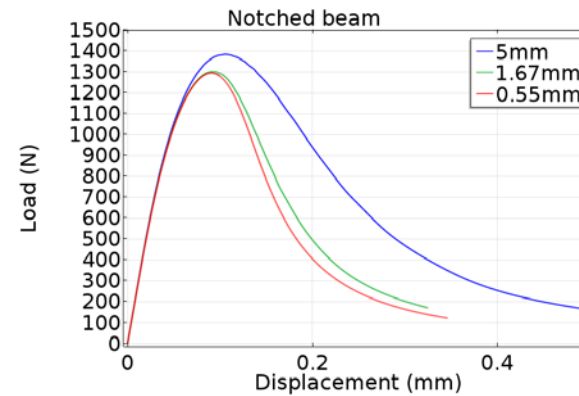
- Three-point bending tests of notched and unnotched concrete beams modelled with the regularized and non-regularized models

Mechanical parameters and damage model parameters

| Parameters | Notched | Unnotched |
|-------------------|---------------------|---------------------|
| E_0 (GPa) | 20 | |
| ν_0 | 0.2 | |
| ε_0 | $1.2 \cdot 10^{-4}$ | $0.9 \cdot 10^{-4}$ |
| ε_f^* | 0.007 | 0.003 |
| A_c | 1.09 | |
| B_c | 1500 | |
| l (mm) | 0.6 | 1.0 |



Results of the Comsol model for different mesh refinements (left), results from [3] (right).



Results of the Comsol regularized damage model for different mesh refinements.

[1] Peerlings R H J et al., 1996. *Int. J. for Num. Meth. Engng.*, 39, 3391-3403.

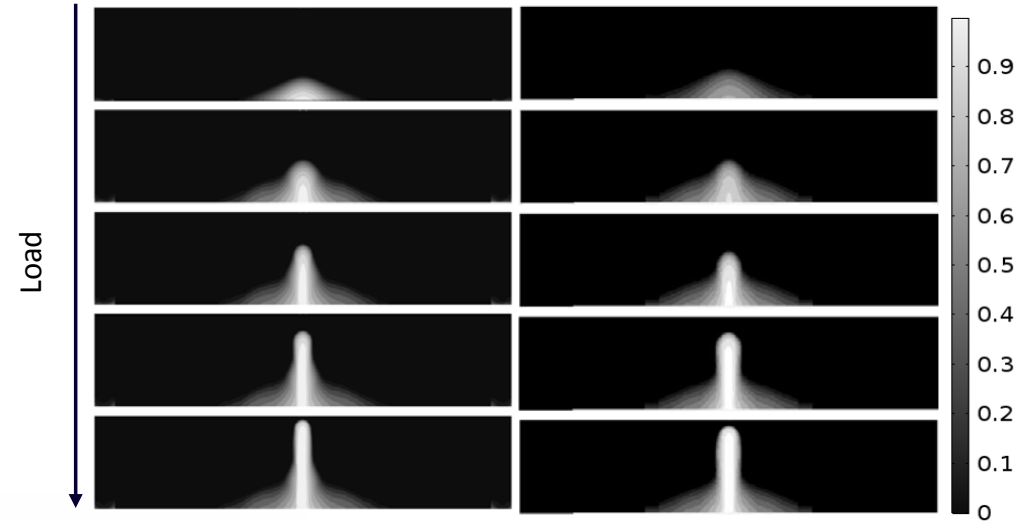
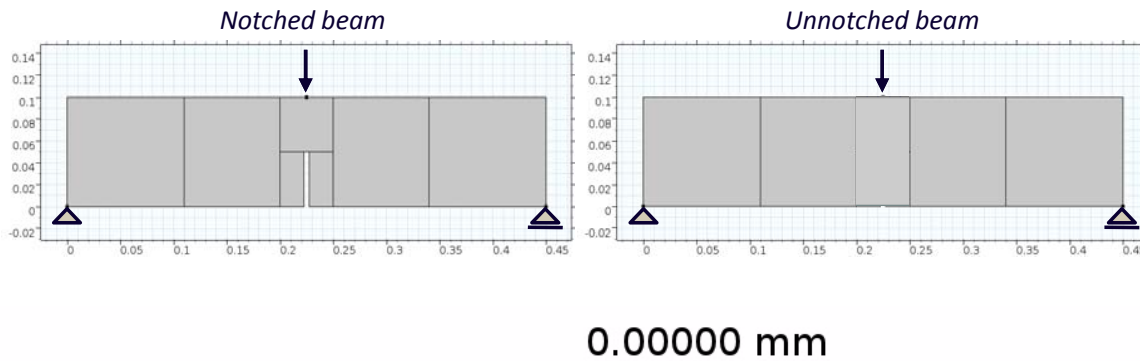
[2] Simone A, 2007. *Revue Européenne de Génie Civil*, 11(7-8), 1023-1044.

[3] Jirásek M, 2011. In *Numerical Modeling of Concrete Cracking*. Eds.: G Hofstetter, G Meschke, 1-49.

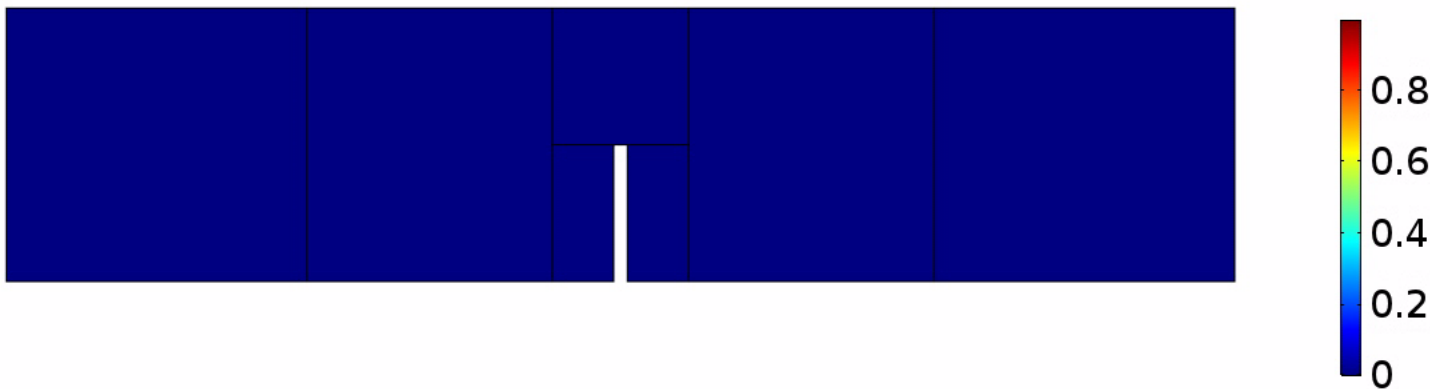
Regularization method – Gradient enhanced formulation

- Comparison with results presented in [1]

— Damage variable evolution



Mechanical damage evolution in Comsol (left) and results from [1] (right).

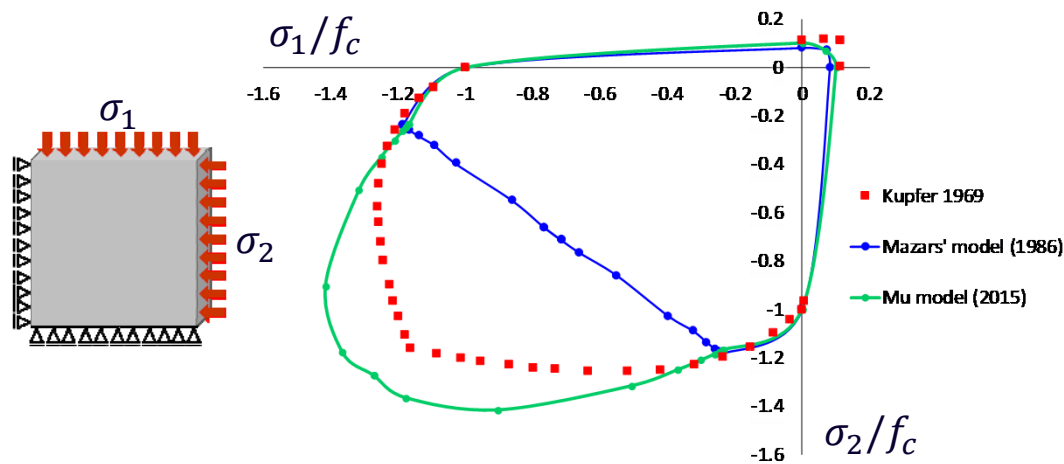


Evolution of mechanical damage as a function of the imposed displacement in the top face center point

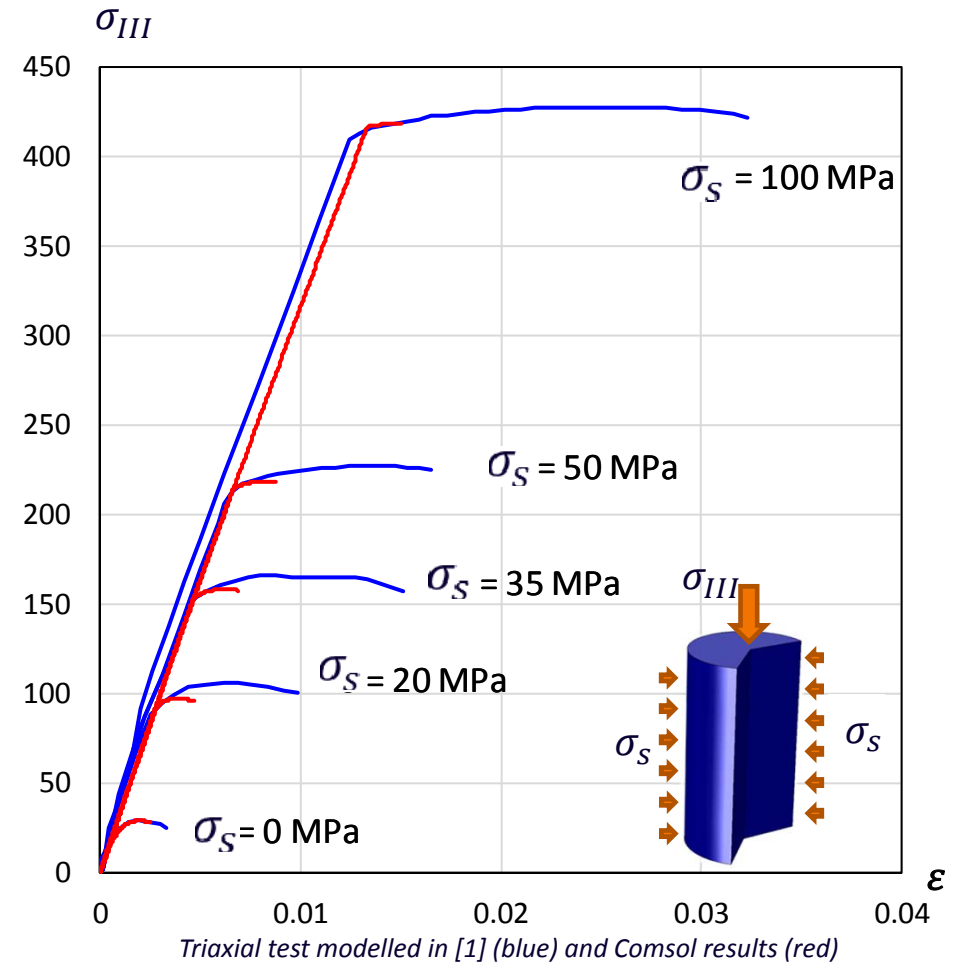
[1] Jirásek M, 2011. In Numerical Modeling of Concrete Cracking. Eds.: G Hofstetter, G Meschke, 1-49.

μ damage model

- Improvements of the formulation presented in [1]:
 - Two principal models:
 - Cracking in tensile state
 - Crushing in compressive state
 - Good representation of cyclic loading paths
 - Behaviour under biaxial compression
 - Behaviour under triaxial (EA) compression



Biaxial loading tests from [2], model results for classical and μ damage models



[1] Mazars J, Hamon F, Grange S, 2015. *Materials and Structures*, 48, 3779–3793.

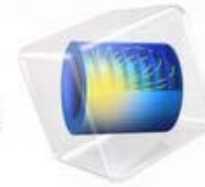
Conclusions

- The data
 - Are
 - Over
 - Rep
 - Are
 - Car

- ✓ The gc couple succes

5.4

COMSOL
MULTIPHYSICS®

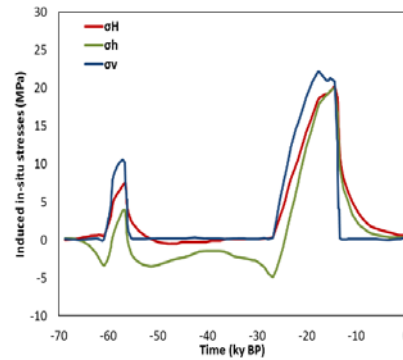
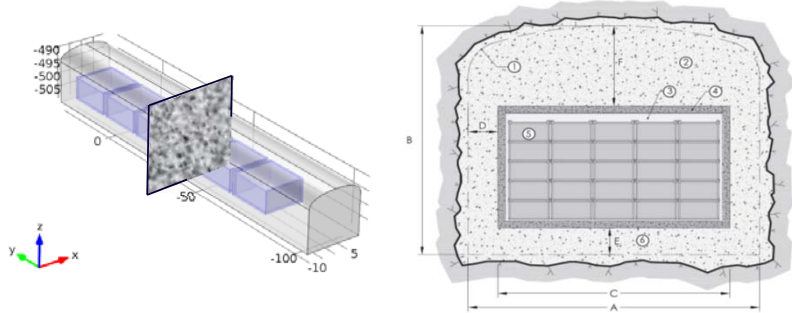


Acknowledgements

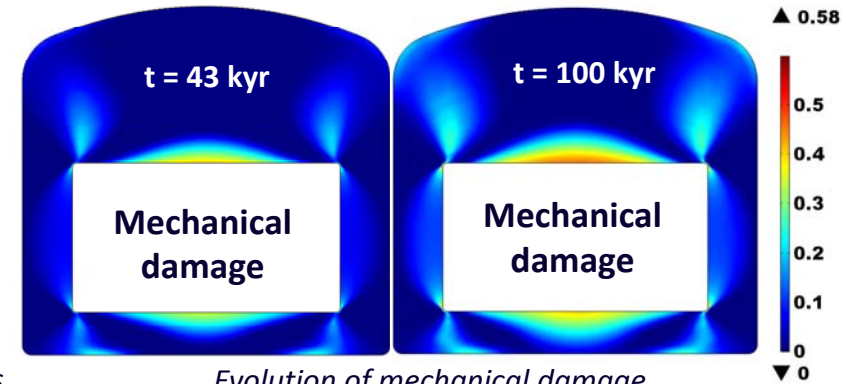
This work has been supported by the European Union (H2020-NFRP-2014/2015) under grant agreement n° 662147 (CEBAMA) and from the Swedish Nuclear Fuel and Waste Management Company (SKB), which are gratefully acknowledged.

Application case – Glaciation

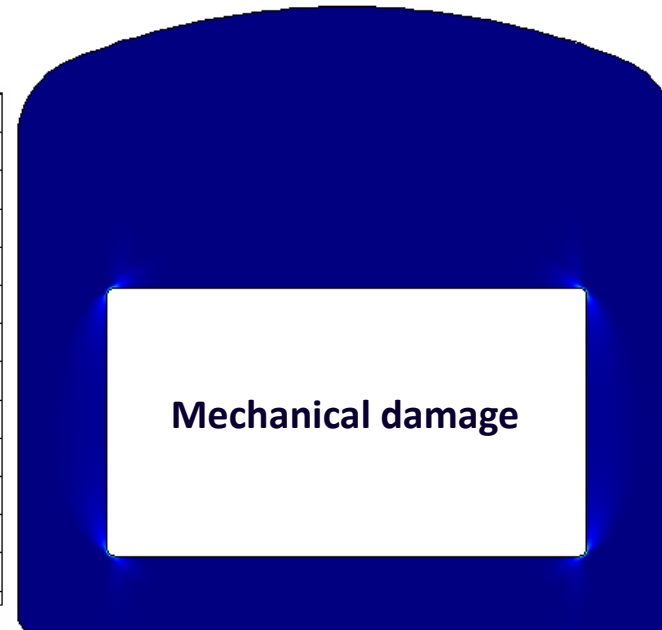
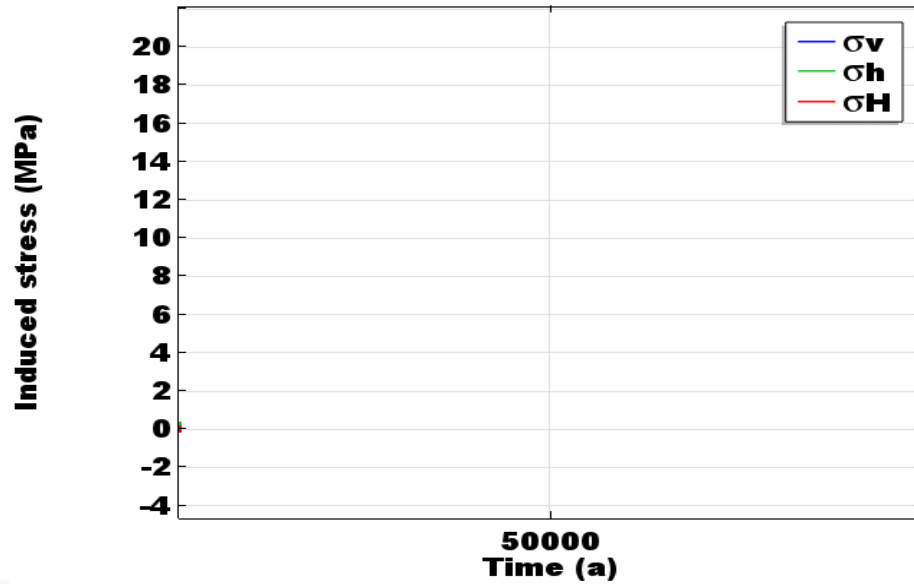
- Deep geological repository for nuclear waste



Evolution of in-situ stresses

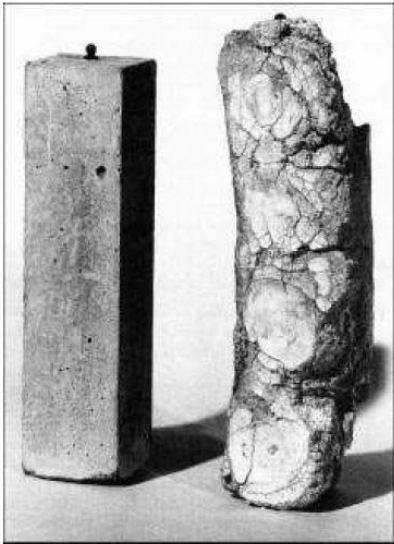
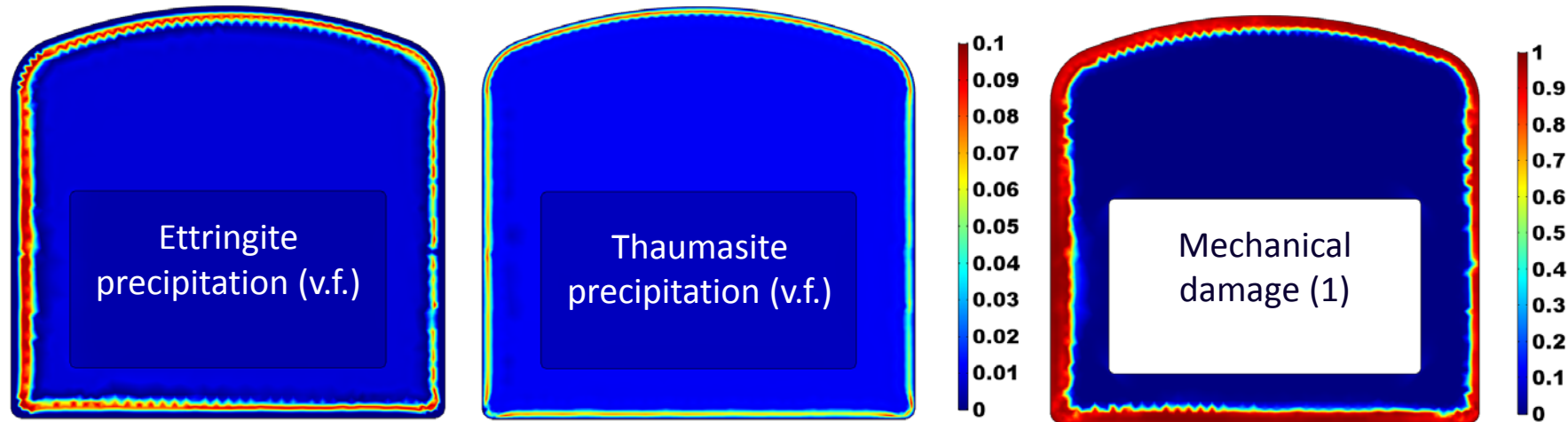
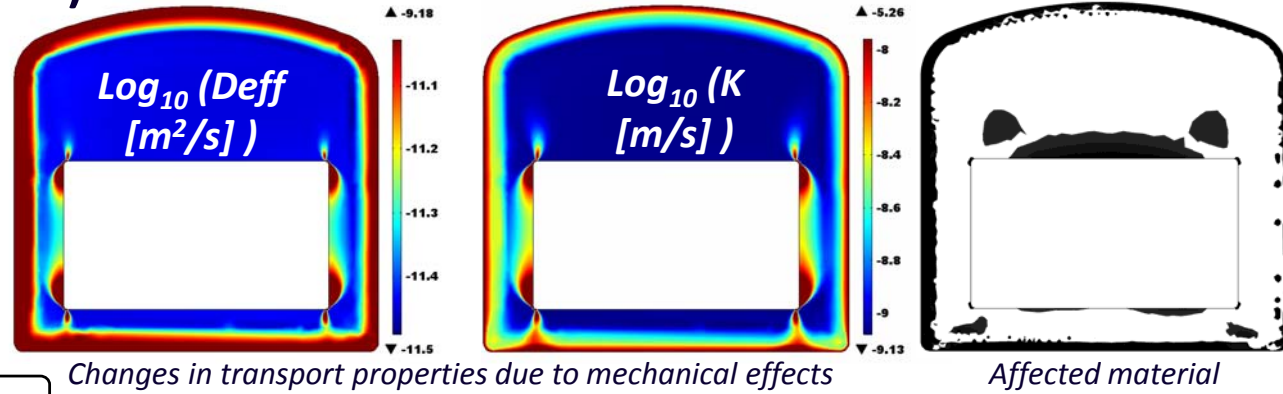
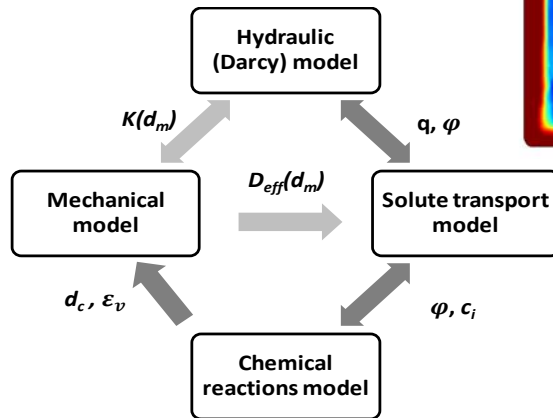


Evolution of mechanical damage



Application case – Sulphate attack (HCM)

- The concrete damage model presented:
 - Completely built-in Comsol interface



4.1 Conventional sulfate attack associated with expansive ettringite in a concrete prism (RHS) and non-degraded control prism (LHS). I reproduced from CEB Design Guide, *Durable Concrete Structure* Thomas Telford, 1989.



AMPHOS²¹

SCIENTIFIC AND STRATEGIC ENVIRONMENTAL CONSULTING

ESPAÑA

Paseo de García Faria, 49-51
08019 Barcelona
Tel.: +34 93 583 05 00

Paseo de la Castellana 40, 8ª Planta
28046 Madrid
Tel.: +34 620634729

CHILE

Avda. Nueva Tajarar, 481
WTC – Torre Sur – Of 1005
Las Condes, Santiago
Tel.: +562 2 7991630

PERÚ

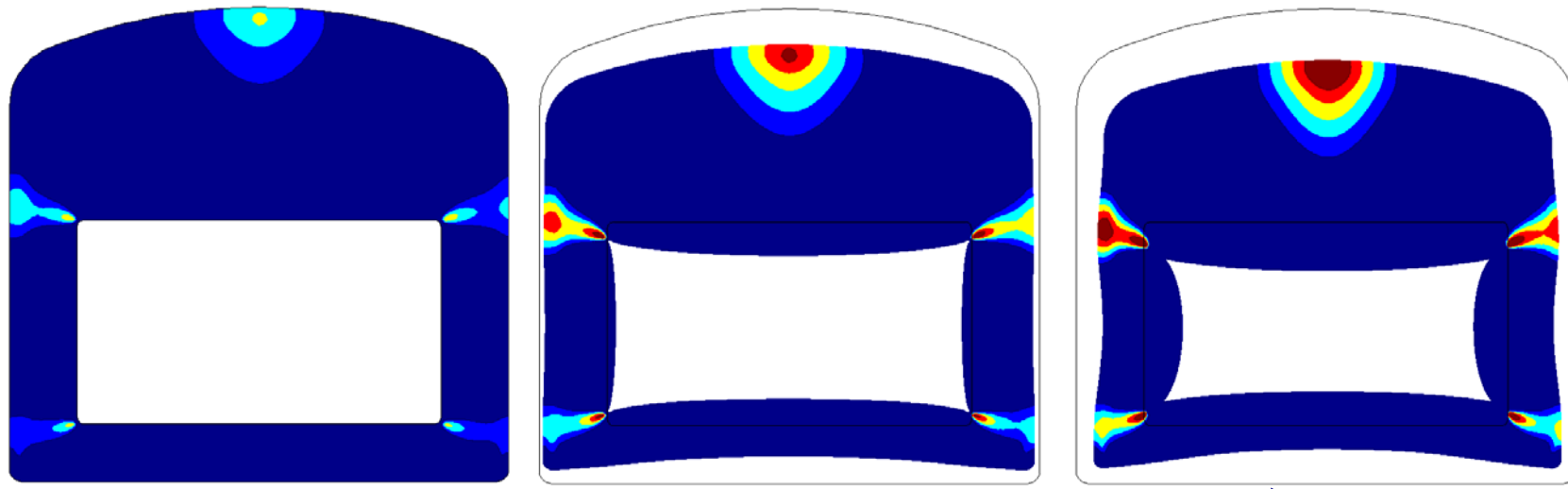
Jr. Pietro Torrigiano 396
San Borja, Lima 41
Tel.: +51 1 592 1275

FRANCE

14, Avenue de l'Opéra
75001 Paris
Tel.: +33 645 766 322



<http://www.amphos21.com> amphos21@amphos21.com



Time

