COMSOL CONFERENCE
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UNLV

A COUPLED COMSOL-MESHLESS MODEL FOR MULTISCALE HEAT TRANSFER

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Outline

- Introduction to Meshless Method
- Computational Method
  - Conduction Problem set up
  - Hybrid Meshless-FE Model
  - Using MATLAB LiveLink
- Results and Discussion
- Conclusion

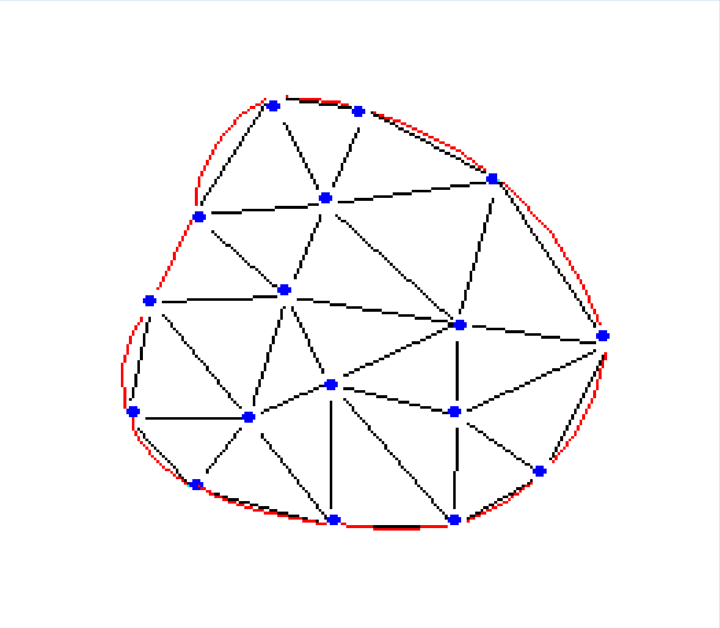
Meshless Method

- **Development of meshless methods began in the 1970's**

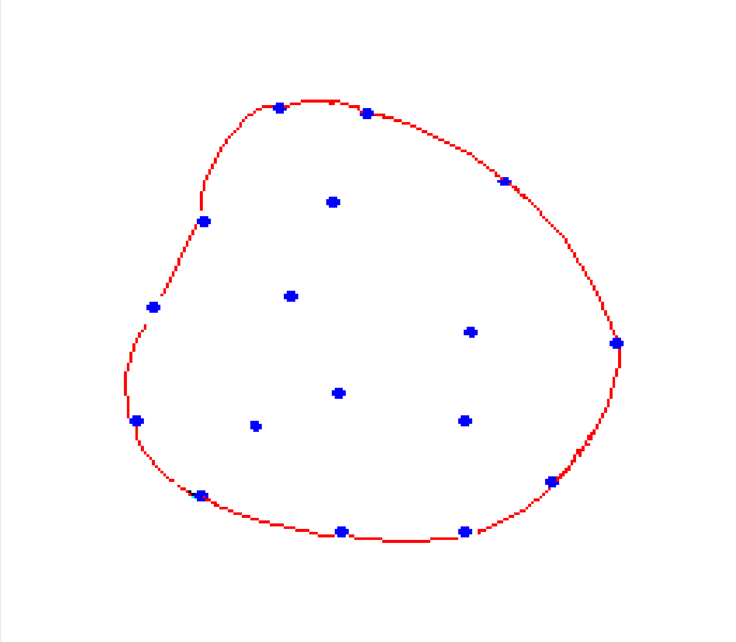
- **However, they are relatively newcomers to the field of computational methods**

- **Traditional techniques, such as FDM, FVM, and FEM rely on a structured mesh, or interconnected node points to calculate the values of interest.**

Mesh vs meshless



Element mesh by FEM



Nodal points by Meshless Method

Meshless Method - Advantages

- They do not rely on a structured mesh.
- Domain and boundary mesh discretization is not required;
- Domain integration is not required;
- Custom points (e.g. randomly generated or imported from a file) can be used as the domain;
- Exponential convergence for smooth boundary shapes and boundary data can be realized;
- Multi-dimensional problems are naturally handled
- Implementation is comparatively easy

Meshless Method con't

- One of the simplest implementations of a meshless method is to use RBF
- RBF functions depend only on the distance from some center point (node)
- Using distance functions, RBFs can be easily implemented to model variables in arbitrary dimensions
- Multiquadrics (MQ) where d is distance between radial position and Φ is shape function

$$d_i = \left[(r - r_i)^2\right]^{1/2}$$

$$\Phi_i(d) = (d_i^2 + c_i^2)^{\beta}$$

- The most commonly used basis function is the MQ as proposed by Hardy [1] with an exponent of $\beta = +0.5$.

[1] Hardy, R.L., "Multiquadric equations of topography and other irregular surfaces," Journal of Geophysical Res. 1971. 176: pp. 1905-1915

Radial Basis Functions

- **The distance between points (x,y) and point (x_i,y_i) is denoted as follows:**

$$r = \|(x,y)-(x_i,y_i)\|$$

- **The most commonly used RBFs**
  - **Multiquadrics (MQs):** $\phi(r) = \sqrt{r^2 + c^2}$
  - **Gaussian:** $\phi(r) = e^{-\alpha r^2}$
  - **Inverse MQs:** $\phi(r) = 1 / \sqrt{r^2 + c^2}$

where c is a shape parameter represented as a positive real number

Radial Basis Functions con't.

Example:

$$\Delta T = f(x,y), \qquad (x,y) \in \Omega$$
$$T = g(x,y), \qquad (x,y) \in \partial\Omega$$

We approximate *u* by *û* assuming

$$\hat{T}(x,y) = \sum_{j=1}^{N} c_j \varphi(r_j)$$

where $r_j = \sqrt{(x-x_j)^2 + (y-y_j)^2}$

$$\left[\text{For MQ: } \varphi(r_j) = \sqrt{r_j^{\,2} + c^2} = \sqrt{(x-x_j)^2 + (y-y_j)^2 + c^2} \right]$$

Radial Basis Functions con't.

$$\frac{\partial\varphi}{\partial x} = \frac{x - x_j}{\sqrt{r_j^{\ 2} + c^2}}, \qquad \frac{\partial\varphi}{\partial y} = \frac{y - y_j}{\sqrt{r_j^{\ 2} + c^2}},$$

$$\frac{\partial^2\varphi}{\partial x^2} = \frac{\left(y - y_j\right)^2 + c^2}{\sqrt[3]{r_j^2 + c^2}}, \qquad \frac{\partial^2\varphi}{\partial y^2} = \frac{\left(x - x_j\right)^2 + c^2}{\sqrt[3]{r_j^2 + c^2}}$$

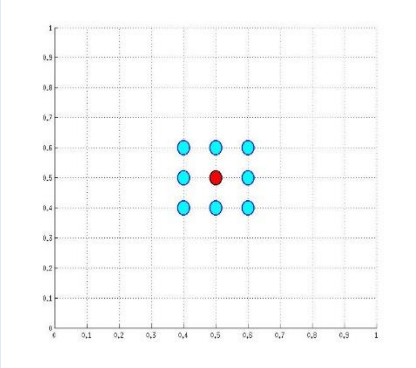
$$.\sum_{j=1}^{N}\left(\frac{\partial^2\varphi(r_j)}{\partial x^2} + \frac{\partial^2\varphi(r_j)}{\partial y^2}\right)\mathrm{T}_j = f(x_i, y_i), \quad i = 1, 2, \cdots N_I,$$

$$\sum_{j=1}^{N}\varphi(r_j)\,\mathrm{T}_j = g(x_j, y_j), \quad i = N_I + 1, N_I + 2, \cdots N$$

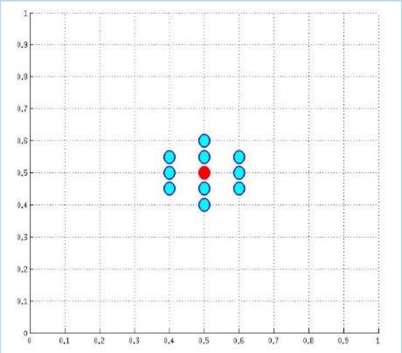
$\{T_j\}_{j=1}^{N}$ can be obtained by solving N×N system

Global RBF Meshless vs localized RBF Meshless

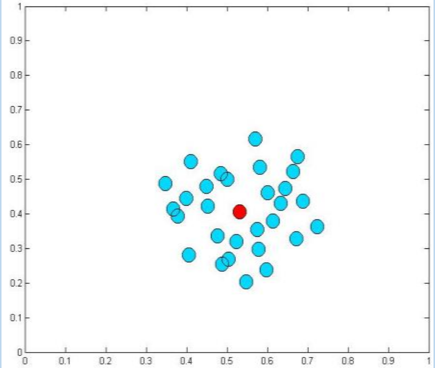
1. **Global RBF meshless: global interpolation on non-ordered spatial point distributions over the entire domain.**
2. **Localized RBF meshless: uses a local collocation defined over a set of overlapping domains of influence.**



a)



b)



c)

Localized a) 9-points stencil, b) 11-points stencil and c) random 30-point stencil

Conduction Test Case

A simple 2-D heat conduction equation was discretized using both Meshless and COMSOL:

$$\nabla .(K(\nabla T) = S$$

Two runs were conducted:

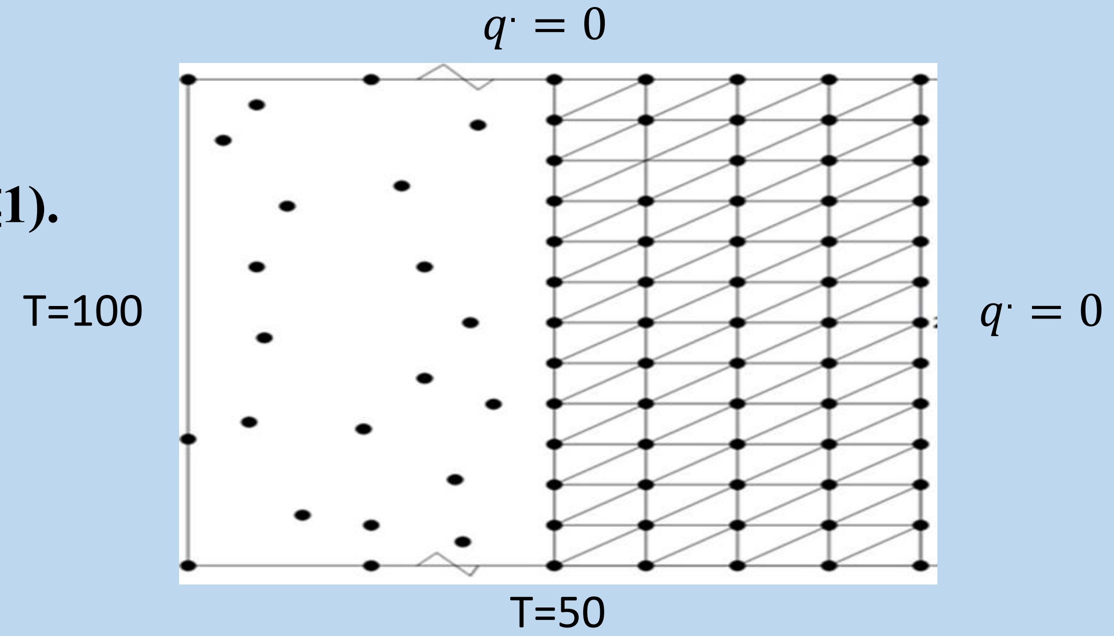
- ❖ **1st run: COMSOL was run and results implemented in MATLAB Meshless**

- ❖ **2nd run: COMSOL with MATLAB Livelink used to implement MATLAB Meshless with COMSOL**

Meshless – COMSOL Link

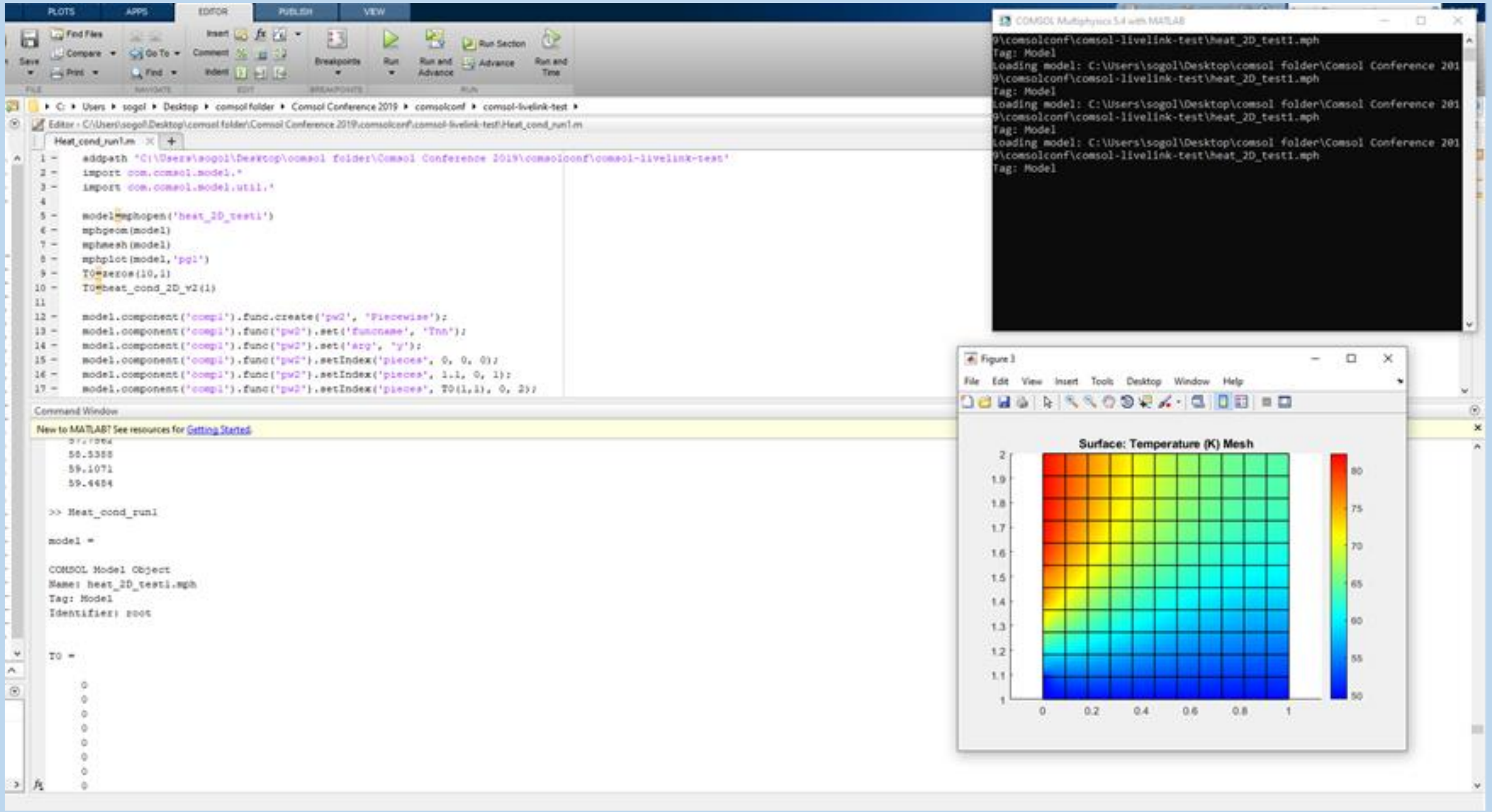
- **The problem geometry is represented with nodes scattered across the domain and boundary for meshless method. In COMSOL linear elements were considered.**
- **Results of the meshless method via the COMSOL livelink with MATLAB implemented into the COMSOL model and vice versa.**
- **Shape functions are generally referred to as the support domain for the node of interest and the best possible shape function is considered for meshless code.**

The computational domain is a square of (0≤x≤2, 0≤y≤1).



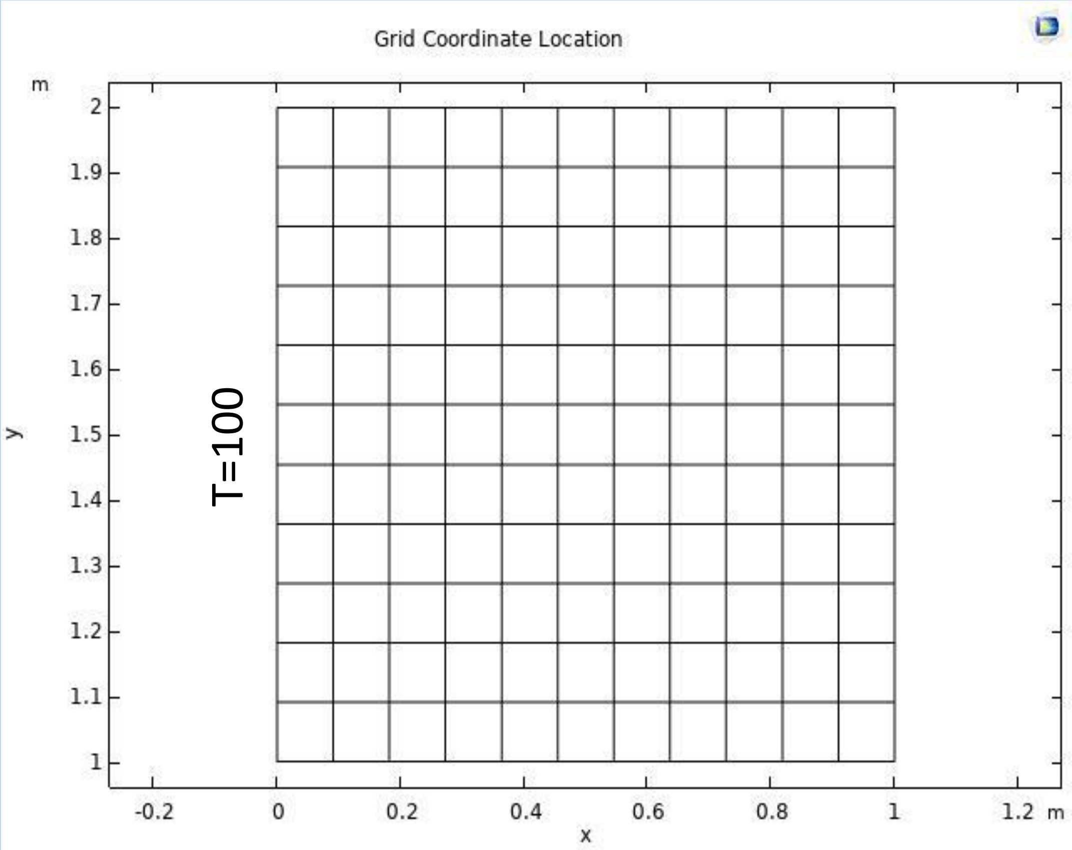
Livelink with MATLAB

- **Livelink with MATLAB enables the user to integrate COMSOL Multiphysics with MATLAB scripts for preprocessing, manipulation of model and postprocessing or calling MATLAB function from the COMSOL desktop.**



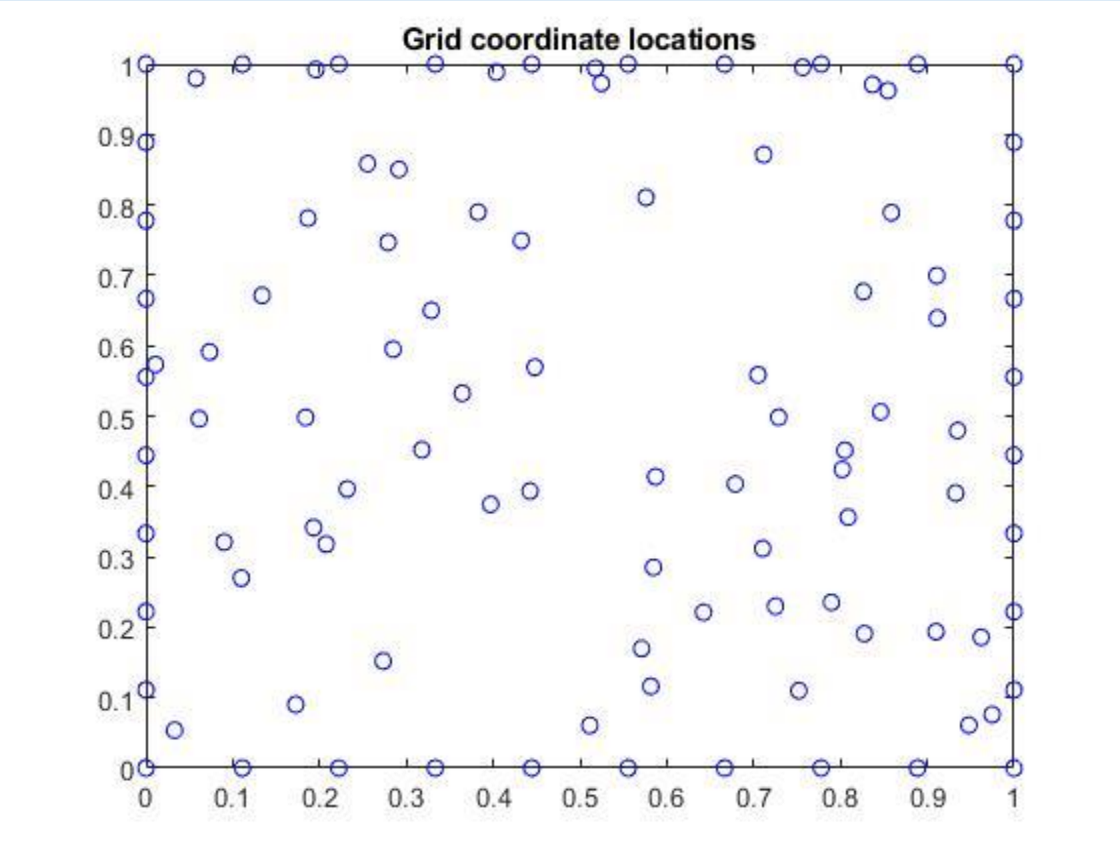
Results and Discussion

Grid generation of hybrid COMSOL-MESHLESS



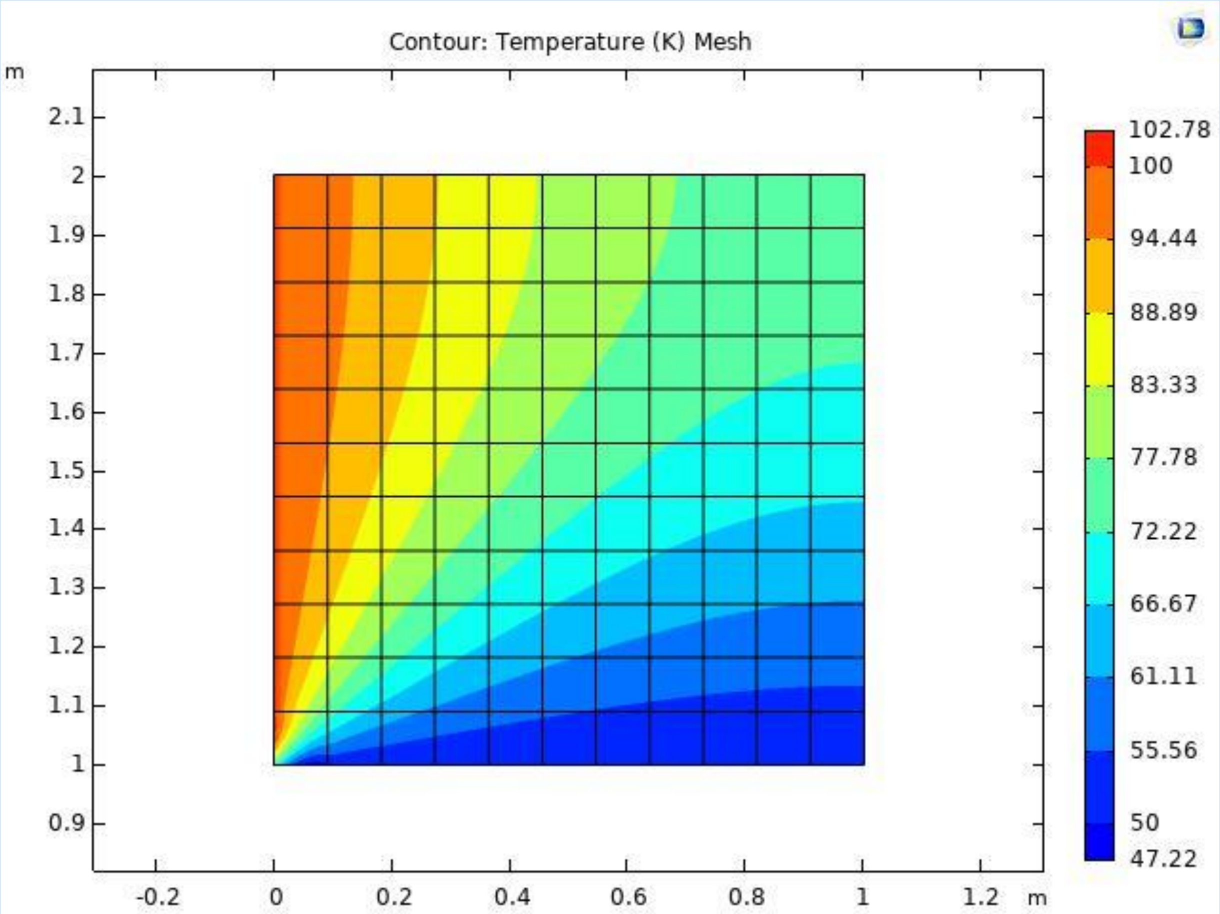
T=100

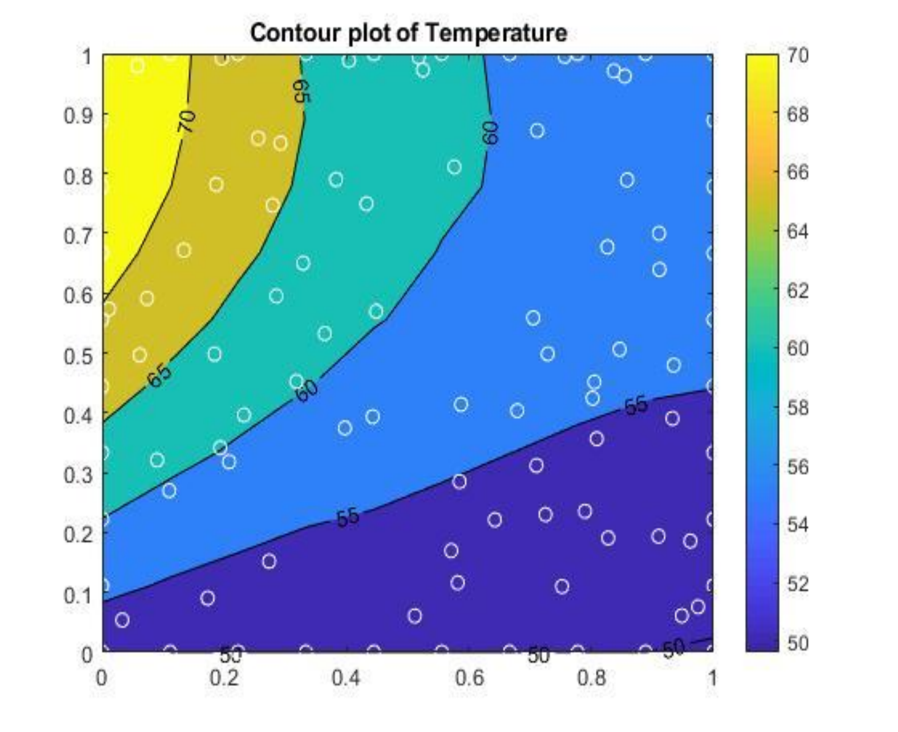
T=50



Results and Discussion con’t

Temperature distribution of hybrid COMSOL-MESHLESS

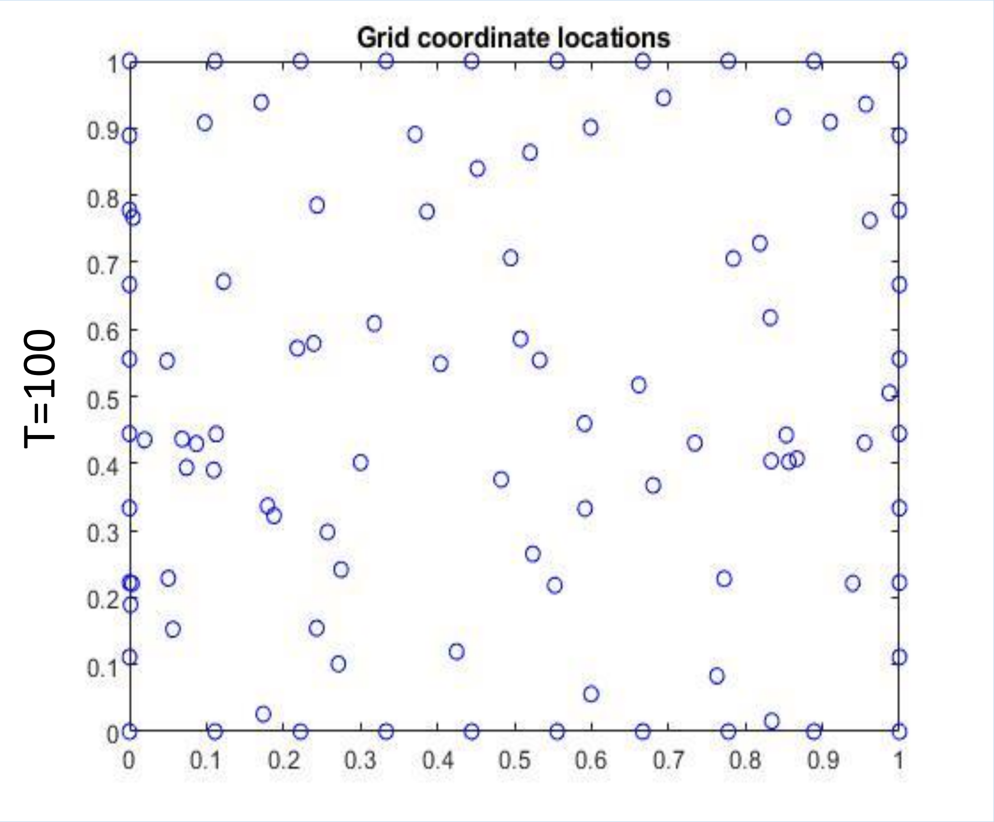




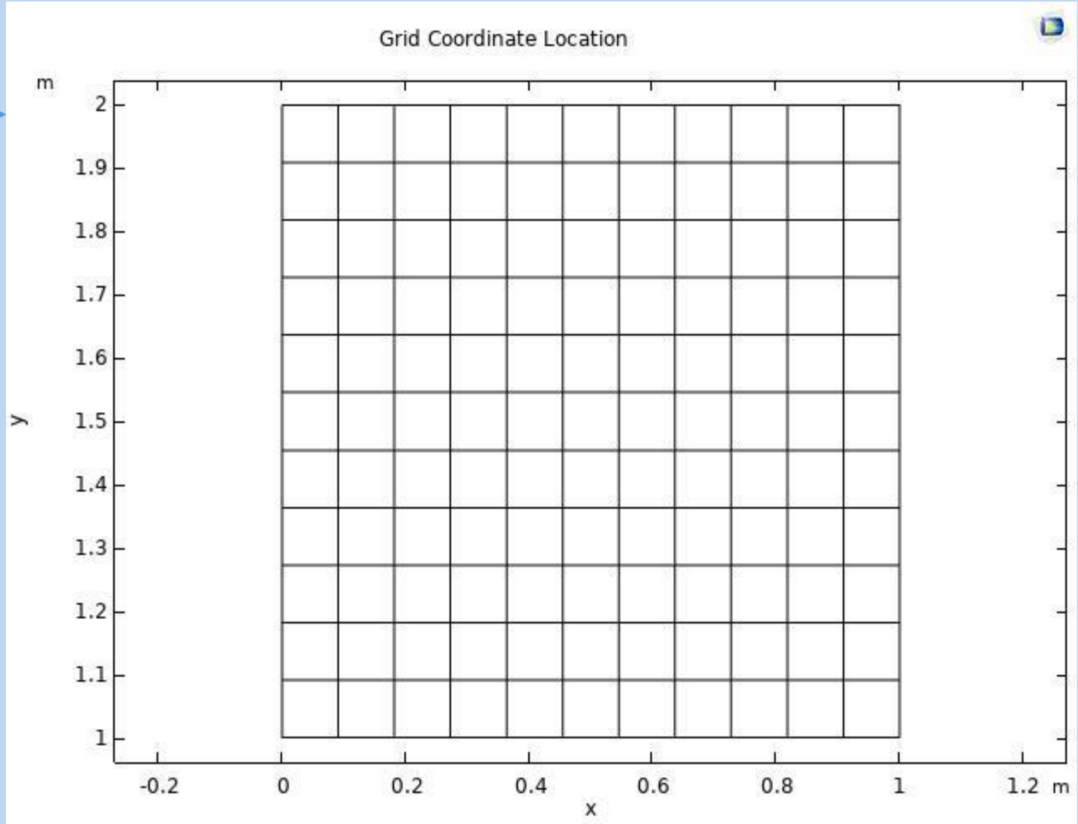
Results and Discussion con't

Grid generation of hybrid MESHLESS-COMSOL MATLAB Livelink

T=100



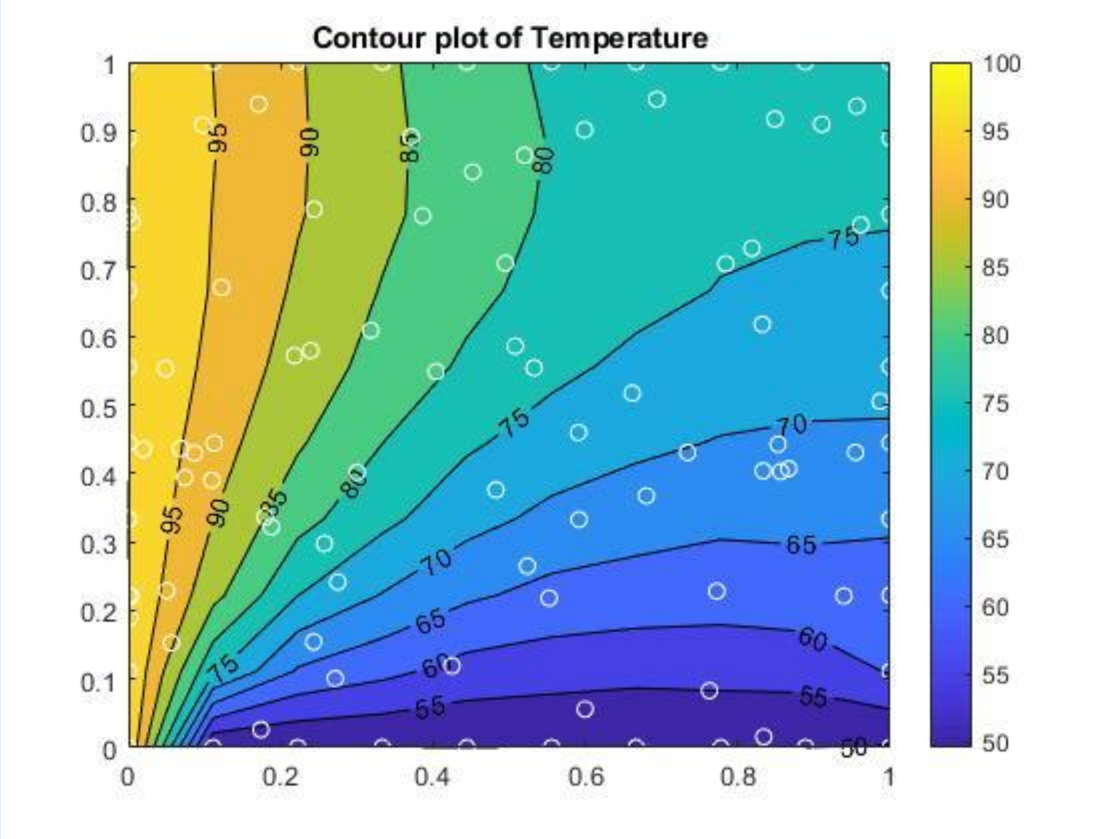
T=50

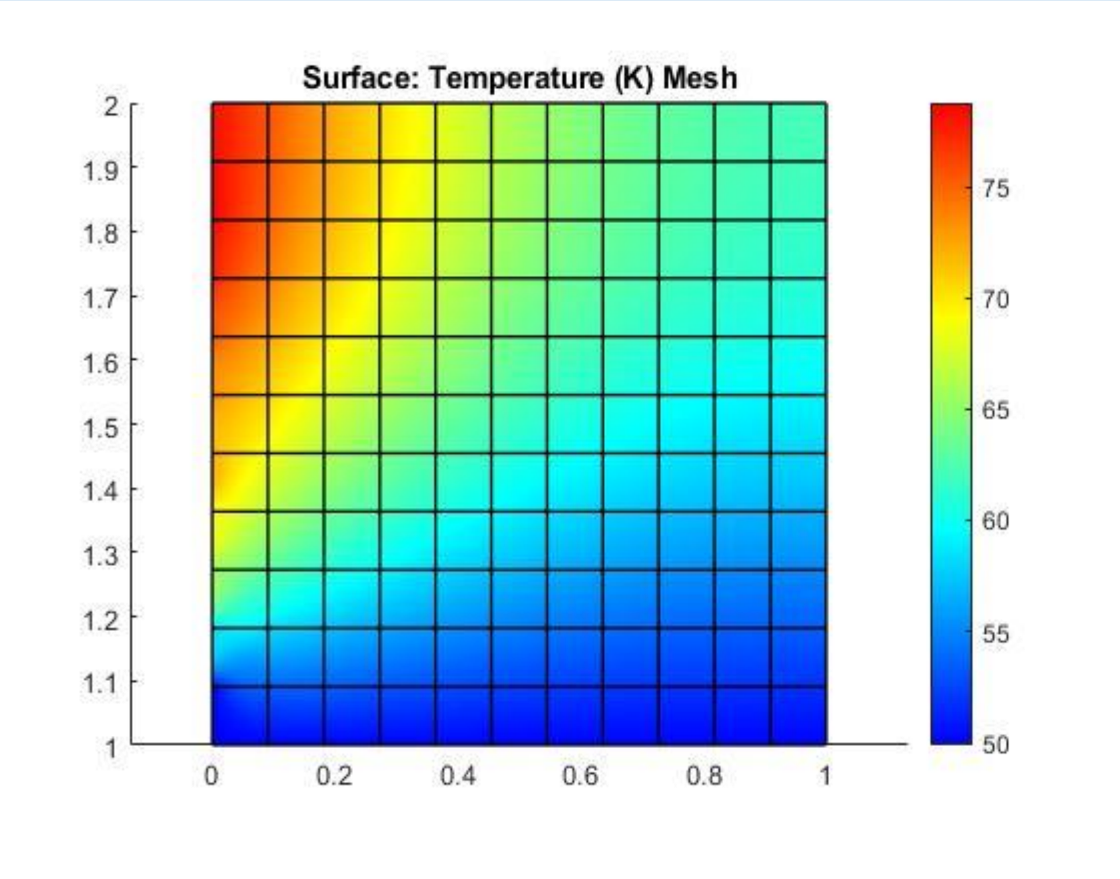


T=50

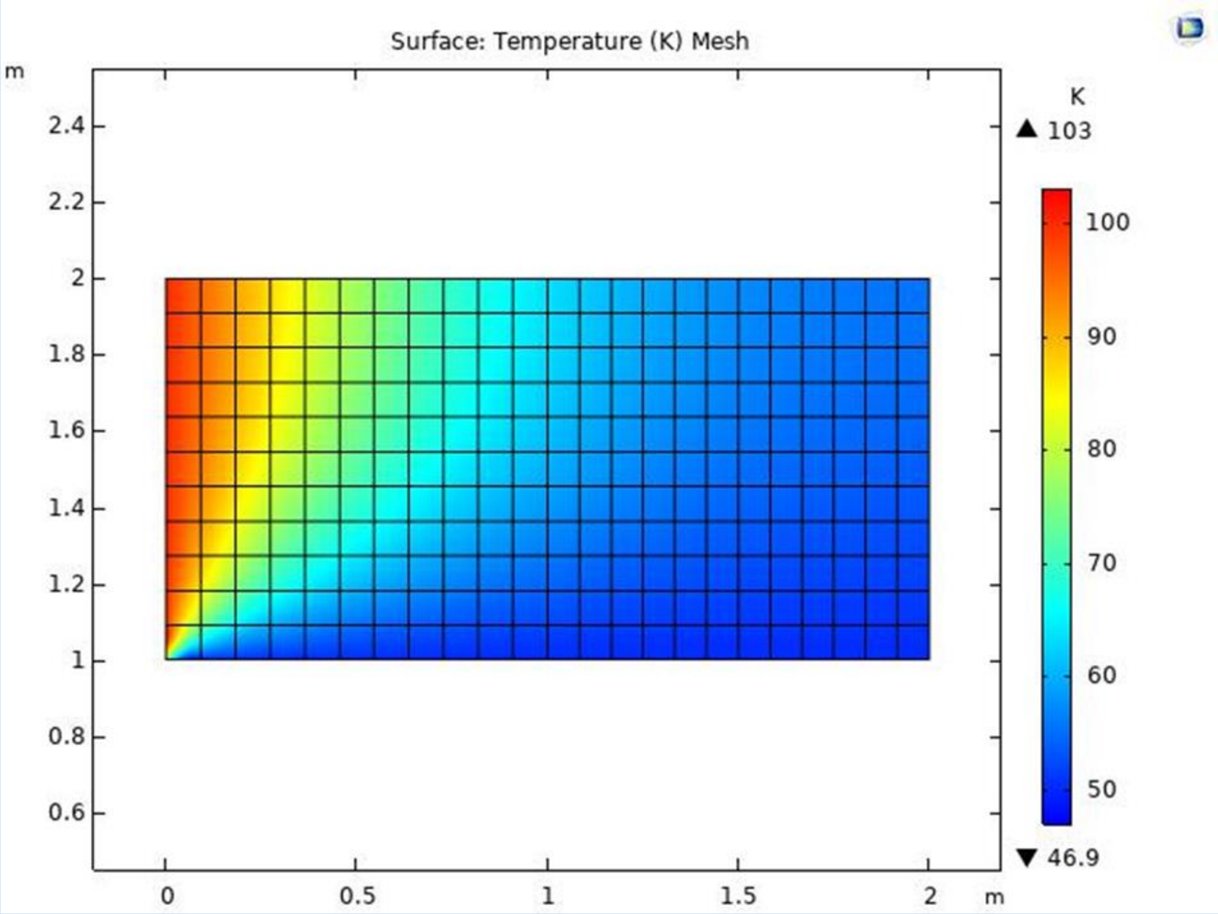
Results and Discussion con't

Temperature distribution of MESHLESS-COMSOL MATLAB Livelink



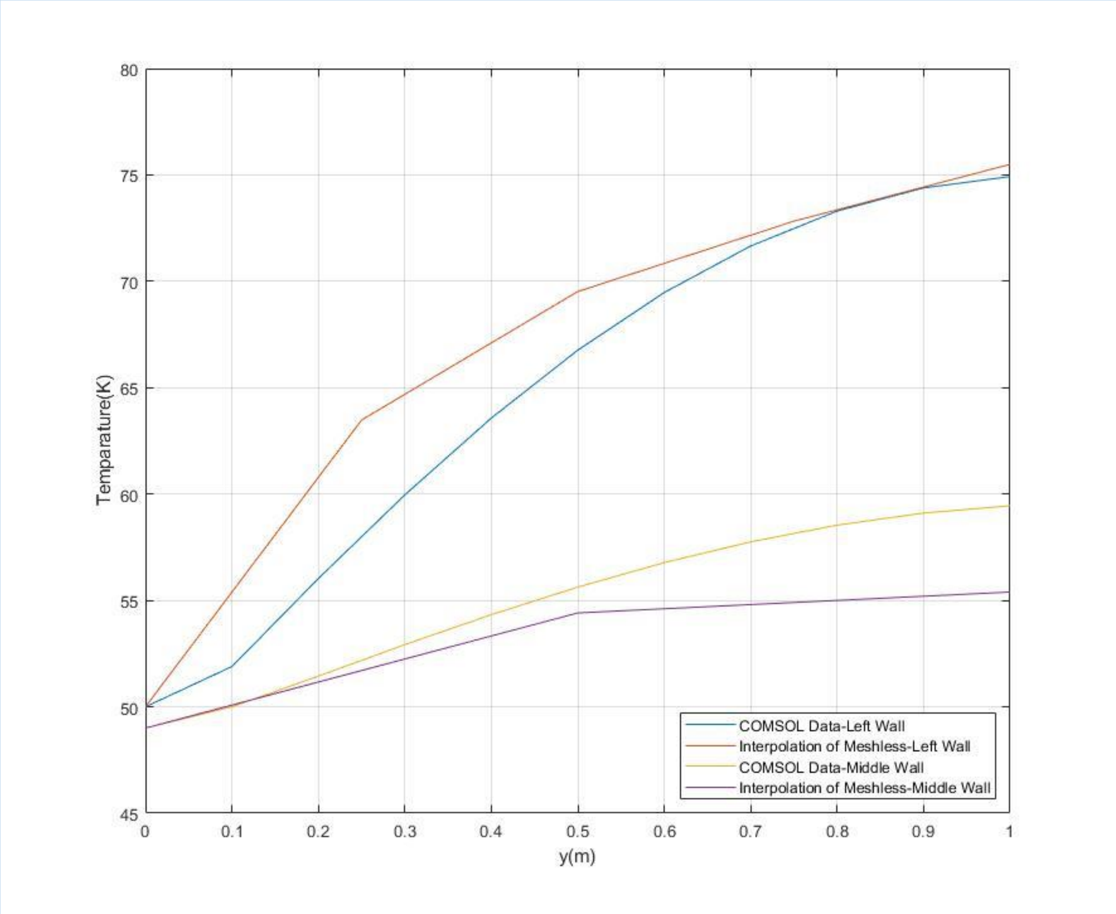


Complete geometry run in COMSOL



Results and Discussion con’t

Temperature plot comparison of two runs for MESHLESS-COMSOL MATLAB Livelink and COMSOL-MESHLESS



Conclusion

- **Employing multiquadric functions, the method permits easy coupling to COMSOL**
- **No need to evaluate complicated functions or discretizing large domain boundaries at every step of an iteration or time-marching scheme**
- **Memory demands are minimal**
- **Coupled procedure permits large regions bordering refined configurations to be linked where detailed information is required**
- **Applications include fluid flow, species transport, and structural mechanics**