On the Non-coiled Spring Optimization of Utilizing FEM Assisted by an Analytical Model

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Abstract

The non-coiled spring, the most ancient and crucial mechanical component, has been widely used in different industry at different scales. During the spring design process, performance, robustness, tolerances, and cost always need to be considered and certain trade-off have to be made. As a result, spring design will always benefit from the advancement of theories and numerical tools that can answer the question of what makes a spring design optimal.

Non-coiled spring problem has large number of multi-variables, non-linear equations, and sophisticated geometry. Traditional optimization techniques either over-simplifying the spring shape such that they can apply beam theory or directly apply a numerical method which can be time-consuming and trapped in some local minima.

The purpose of this work is to look into the fundamental issues regarding spring design and develop a new approach which take both the merits of analytical solution and numerical solution. With our treatment, engineer can start with a curved non-coiled spring in FEM/Comsol environments to identify the spring's effective spring rate and maximum stress at any thickness and width. With both those identified values and target values, we can plug them into our derived analytical equations and get the predicted dimensions. The performance of new dimensions will be verified in FEM/Comsol environments. Only two iterations of simulation are needed to get the ultimate spring performance.

Keywords: Beam theory, FEA, optimization, non-coiled spring

Introduction

Non-coiled spring applications are usually challenging due to the space limitation in engineering practice. Engineers consistently struggle come up with the right dimensions to satisfy its mechanical performance requirements. Among those requirements, the most common ones are spring rate and maximum stress. These springs, usually made out of stamped or laser cut sheet metal, have a uniform thickness. In the axial direction, their geometry and dimensions are limited to the space they reside in. Thus, in most scenarios, the two design variables the engineer are left with are just: thickness and width. When engineers use those knobs to tune the spring mechanical performance, they are faced with the optimization problem [1, 2, 3]: when the spring is tuned to be compliant, the strength (maximum stress under the same load) is compromised.

In this paper, we have proposed a time efficient approach to optimize the characteristics of a non-coiled spring to the desired spring rate and maximum strength. We perform structure simulations in COMSOL with the analytical model built-in. validated it with experiments. In this approach, we start the simulation with a guess thickness and width. Then, we applied the classical beam theory [4, 5, 6] to the first round simulation results to acquire the thickness and width to satisfy our spring rate and maximum strength target. Those geometry dimensions are updated to the simulation model to run the second round simulation for validation. We showed one application of this approach to illustrate how this approach benefits our engineering practice.

Problem Statement

Though the approach we proposed here is not limited to a specific application, for a lucid illustration, we adopt a rotary non-coiled spring as an example.

With this rotary non-coiled spring design, its primary design objectivity is that the spring can exert a consistent force over different units and their use time.

Over the span of different units, the main variation contributor is deflection variation which are composed of the tolerance of the spring itself, the tolerance of the part the spring press against and the tolerance of the assembly. In practices, engineers are reluctant to call out a tight tolerance as it would mean more cost. To mitigate this, a common practice is to design a relative soft spring. Base on Hoke's law,

$$\Delta F = k\Delta d$$

a small spring rate would decrease the force variability.

Over the span of time, the main variation contributor is either creep or fatigue. Those behavior are always correlated to the maximum stress the spring experiences. And under most circumstance, maximum stress and spring rate are conflicting attributes to a spring design.

When the circumference of the space where the rotary wiper resides in is determined, the shape in the axial direction is determined. The two design variables we are left with are thickness and width.

Combining all the objectivities, condition, and assumption above, for a typical spring design problem, it can be formulated to the following optimization problem:

$$Min_{\mathrm{w,th}}\text{: } k$$
$$s.t.\text{: } \sigma_{max} < c\%\sigma_{yield} \quad \text{at max. load}$$

Classical Rod Theory

Definitions:

k	Spring rate
D	Deflection
I	Area moment of inertia
W	Work/energy
E	Energy
V	Volume
σ	Stress
F	Load
w	Width
t	Thickness
s	Distance along the longitude axis
sf	Safety factor

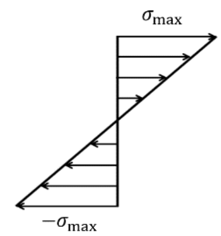

Figure 1. The stress distribution in the beam cross section

Based on beam theory, the energy stored in a deformed beam is

$$E = \iint \frac{\sigma^2}{2E} dAds\,.$$

With work-energy theorem, the work to generate the above energy is

$$W = \frac{Fd}{2} = \frac{F^2}{2k}.$$

With two equation above and the assumption that the axial geometry and load is not changed,

$$\frac{1}{2k} \propto \sigma_{max}{}^2 wt\,.$$

With the beam constitutive equation,

$$k \propto I \propto wt^3\,.$$

With the equations above, we can derive

$$t_{target} = \frac{sf\sigma_{yield}k_{target}}{\sigma_{max\_initial}\,k_{initial}} t_{initial}\,,$$

$$w_{target} = \frac{\sigma_{max\_initial}{}^3 k_{initial}{}^2}{(\sigma_{target})^3 k_{target}{}^2} w_{initial}\,.$$

Therefore, as long as we have an initial point where the spring rate and maximum stress are known with respect to certain thickness and width, we can obtain the thickness and width for our targeted spring rate and maximum stress in one iteration.

Comsol Simulation Workflow

Due to the complexity of most non-coiled spring's shape in the axial direction, deriving the effective spring rate and maximum stress for the initial point through beam theory can be challenging. Thus, Comsol Structure Module's FEM solver which subdivides the non-coiled spring shape into smaller domain over which a set of equations are solved is utilized here. Together with the analytical model we derived in previous section, we come up with the following workflow which is illustrated in Figure 2.

Here both FEM's merit of solving complicate geometry and analytical model's merit of fast iterating speed are combined.

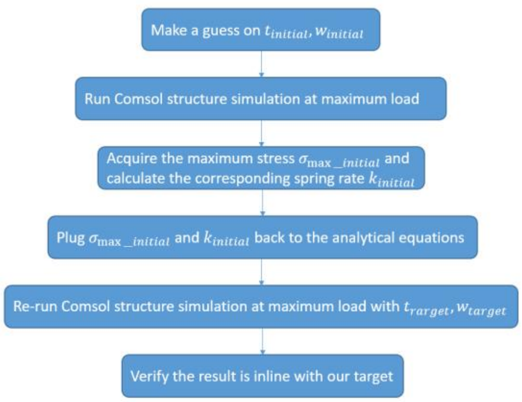

Figure 2. The flowchart of non-coiled spring optimization of utilizing FEM model assisted by an analytical model

Application of the approach

Again, for better illustrating our approach, we adopt the rotary non-coiled spring design as an example. However, this approach can be used in more application where spring rate and maximum stress is a concern.

In this application, the spring which is made of stainless steel need to reside in a 20mm hemisphere and the targeted spring rate and maximum stress are: 5N/mm and 200MPa under 3N.

First, we made a guess of thickness and width as:

$$t_{initial} = 0.5mm, w_{initial} = 2mm.$$

We set one side of the spring as fixed condition with the other side loaded with the maximum load 3N as show in Figure 3.

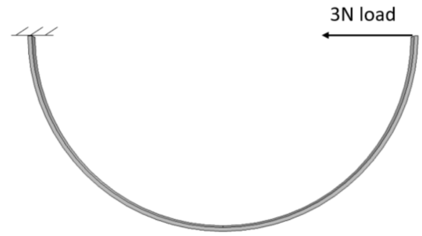

Figure 3. The geometry and boundary conditions of the rotary spring model

Model is based on linear elastic constitutive equation throughout the domain:

$$\varepsilon = \frac{1}{2}(\nabla u + \nabla u^T),$$

with the following governing equation:

$$\nabla \bullet \sigma + F = 0\,.$$

In the first/initial iteration of simulation, with our initial thickness and width, we identify the spring rate and maximum stress as 2.936N/mm and 366.25MPa, respectively.

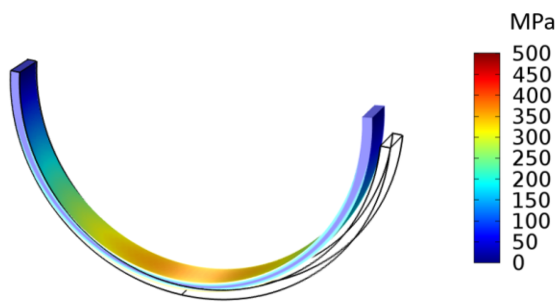

Figure 4. Stress distribution of the spring with initial guessed design variable value: $t_{initial} = 0.5mm, w_{initial} = 2mm.$

Update the two identify value to our analytical model, we get

$$t_{target} = 1.163mm, w_{target} = 0.271mm.$$

With the updated thickness and width above, we identify the spring rate and maximum stress as 5.045N/mm and 212MPa, respectively. The simulated performance with our predicted dimensions are only ~5% off from our targeted value: 5N/mm spring rate and 200MPa maximum stress. In reality, those error are negligible when the dimension are rounded to a manufactured dimensions. Results are also summarized in Table 1.

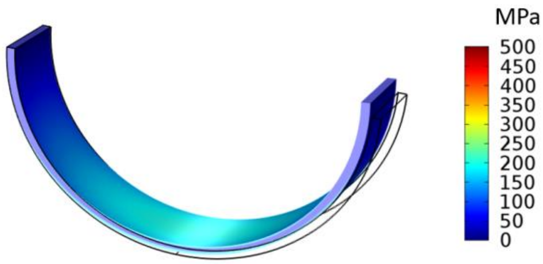

Figure 5. Stress distribution of the spring with assisted analytical model predicted design variable value: $t_{target} = 0.465mm, w_{target} = 4.234mm$.

	Spring rate (N/mm)	Max. stress (MPa)	Thickness (mm)	Width (mm)
Target	5	200		
Iteration 1	2.936	366	0.5	2
Iteration 2	5.045	212	0.465	4.234

Table 1. Summaries of the simulation results at each iteration.

Beyond directly applying our approach, we can inverse the analytical model and get the maximum stress and spring rate predicted w.r.t. to each set of spring thickness and width:

$$\sigma_{max\_target} = \frac{w_{initial} {t_{initial}}^2}{w_{target} {t_{target}}^2} \sigma_{max\_initial} ,$$

$$k_{target} = \frac{w_{target} {t_{target}}^3}{w_{initial} {t_{initial}}^3} k_{initial} .$$

Interested engineer or practitioner can further explore the performance of the spring within the design parameter space. Again, we plotted the space using our rotary spring as an example where the arrows represents the trajectory of our two iterations approach.

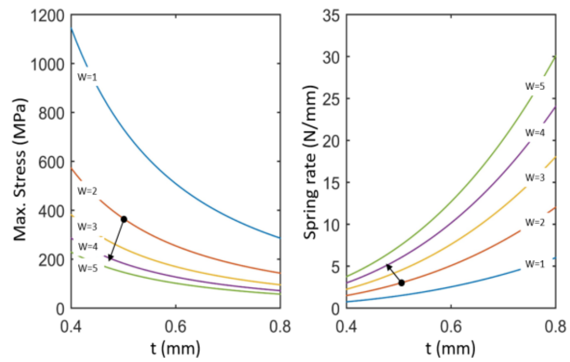

Figure 6. Spring performance w.r.t. design variable space. The arrow represents the optimization trajectory in Table 1.

Conclusions

In this paper, we have proposed a time efficient approach to optimize the characteristics of a non-coiled spring to the desired spring rate and maximum strength. We perform structure simulations in COMSOL with the analytical model assisting the optimization. In this approach, we started the simulation with a guess thickness and width. Then, we applied the classical beam theory to the first iteration simulation results to acquire the thickness and width to satisfy our spring rate and maximum strength target. Those geometry dimensions are updated to the simulation model to run the second round simulation for validation. We showed one application of this approach to illustrate how this approach benefits our engineering practice.

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