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Rapid Control Prototyping for the Production of Functionally Graded Materials with Tailored Microstructural Properties utilizing COMSOL Multiphysics

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Abstract: Within the field of hot metal bulk forming the demand arises for fully three-dimensionally tailored properties at the microstructural level, nevertheless, reaching a predefined geometry with such tailored properties puts high requirements on the control mechanisms utilized in the process chain for combined heating, metal forming, and cooling processes. Novel control strategies need to be implemented within an underlying control architecture being freely configurable with respect to a predefined database and fully extendible to new geometries, microstructural distributions, and materials suiting this precision manufacturing process with simultaneous applicability to industrial mass production processes. A simulation based rapid control prototyping ansatz with the help of COMSOL Multiphysics was followed utilizing both complex optimization algorithms and self adjusting fuzzy logic controllers leading to a significant reduction in controller implementation time.

Keywords: rapid control prototyping, thermo-mechanical processes, fuzzy logic controller, induction heating

1. Introduction

Increasing economical competitiveness and simultaneous improvements in product quality are two foremost challenges for modern industrial mass production and can only be encountered through utilization of novel production processes which guarantee the required tailored mechanical properties within the workpiece, on the one hand, and reduce cycle time and production costs, on the other hand [1,2]. Moreover, the capability of achieving complex geometrical structures incorporating the aforementioned fully three-dimensionally tailored microstructural properties, being the governing parameter in terms of functionally graded materials, is emphasized in modern production in order to mass customize use and load case-specific products [3-5]. Within the framework of an ongoing research project an integrated thermo-mechanical process is utilized for the production of functionally graded flanged shafts entailing inductively heating a steel billet to a predefined temperature distribution and simultaneously cooling and deforming the shaft in a second step. Achieving locally varying microstructural properties is a complex interrelation of coupled thermo-mechanical phenomena, the local cooling rate of fully austenized material being the governing factor for transformations on the microstructural level. A high precision serial production processes for geometrically and microstructurally complex products requires an exact process control in all process stages in order to achieve the desired quality despite changing boundary conditions in serial production. Moreover, mass customization requires changing process routes and process parameters to be implemented within the control algorithms specifically tailored to the production goals and, hence, the control architecture is required to be self adapting with respect to these set-values. The aforementioned reasoning is underlined by the fact that conventional control algorithm development involves knowledge regarding the underlying differential equations and involves time consuming implementation trials requiring a high degree of process knowledge being not readily available in industrial production. An ansatz in order to circumvent the necessity of an accurate mathematical description constitutes the utilization of a Fuzzy Logic Controller (FLC) with integrated self-learning capabilities and online parameter adjustment which is implemented within the control architecture. A finite element rapid control prototyping approach using COMSOL Multiphysics 3.5a, by COMSOL Multiphysics GmbH, Göttingen, was chosen for the purpose of controller algorithm evaluation utilizing a fully non-linear model being controlled by a Matlab interface

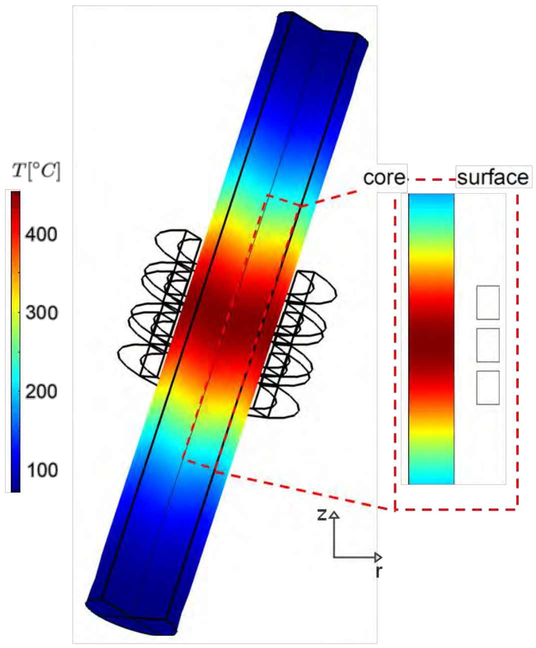

Figure 1. Revolved model geometry with a tailored temperature field of 700K in the coil vicinity.

incorporating the underlying control algorithm. Subsequent implementation effort for the integration of the controller into the programmable logic controller (PLC) was significantly reduced due to the chosen simulation based rapid prototyping approach.

2. Coupled Electro-Thermal Phenomena

2.1 Model Geometry

The model used within this context was created according to an existing test facility at the Chair of Metal Forming at the University of Kassel and modeled with Comsol 3.5a utilizing the *heat transfer* as well as the *AC/DC* module. The geometrical features of the cylindrical and thus rotationally symmetric billet are given through a radius of 15mm and height of 200mm. The billet is placed in the center of a three turn copper coil with inner and outer radius of 22.5mm and 30mm, respectively. Due to the symmetry of the setup a 2D model was created in order to reduce the computational complexity. Moreover, due to constant rotation of the billet during induction heating the coil inclination was neglected constituting merely a minimal approximation error. A triangular mesh was utilized with a mesh size of at least 2 mesh elements per penetration depth δ calculated at room temperature and a frequency of 8kHz according to equation (5).

Pertinent temperature dependent material properties are provided through interpolated lookup tables. Merely the magnetic permeability is implemented as a continuous function of temperature through utilization of the Gombertz function as discontinuities in the vicinity of the Curie temperature, being the temperature for which non-magnetic material properties result, caused undesired effects and significant decreases in solution speed.

2.2 Induction Heating Theory

The underlying mathematical model for induction heating for functionally graded flanged shafts is derived from a coupled multiphysics approach involving both electromagnetic phenomena and thermo-mechanical transfer phenomena. Within this framework we will implement the electro-thermo-mechanical continuous functions utilizing the commercially available software Comsol Multiphysics 3.5a and apply appropriate boundary conditions. The aim of this approach is to verify whether simulation-based optimization can be utilized in order to accomplish predefined temperature distributions in the actual workpiece. Furthermore, computer-based simulation allows the identification of pertinent impact factors regarding the spatial microstructure distribution, e.g. impact of the temporal evolution of the three-dimensional temperature field on the microstructure or demands regarding soaking time prior to metal-forming and transfer conditions. The underlying simulation model is validated using data gained from temperature measurements in a reference process.

A mathematical description of induction processes is given through Maxwell's equations, viz. for general time-varying electromagnetic fields the four governing equations are

$$\begin{aligned} \nabla \times \mathbf{H} &= \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} \\ \nabla \times \mathbf{E} &= -\frac{\partial \mathbf{B}}{\partial t} \\ \nabla \cdot \mathbf{B} &= 0 \\ \nabla \cdot \mathbf{D} &= \rho_{elec} \end{aligned} \tag{1}$$

with **H** being the magnetic field, **B** the magnetic flux density, **E** represents the electric field, **D** the electric flux, $\mathbf{J}$ the conduction current density, and ρ_{elec} the electrical charge density. In this context bold letters represent vectors and ∇ represents the Nabla operator with $\nabla \times \mathbf{A}$ and $\nabla \cdot \mathbf{A}$ being the curl and divergence of a vector field, respectively.

The completion of the aforementioned Maxwell equations can be achieved by embracing relations derived from the material parameters, being

$$\begin{aligned}\mathbf{B} &= \mu_0 \mu_r (T, \|\mathbf{H}\|)\mathbf{H} \\ \mathbf{D} &= \epsilon_0 \epsilon_r \mathbf{E} \\ \mathbf{J} &= \sigma(T)\mathbf{E}\end{aligned} \tag{2}$$

with μ, ε, and σ being the magnetic permeability, dielectric constant, and electrical conductivity, respectively. Moreover, $\mathbf{J}$ accounts to zero in the air. Within the considered induction heating process of this paper we will encounter frequencies in the low kHz regime, thus, we can utilize a magneto-quasi-static model with $\mathbf{J} \gg \partial \mathbf{D} / \partial t \approx 0$. As outlined within the previous section the workpiece subject to the electro-magnetic field is an axisymmetric cylindrical steel billet. Therefore, both the electric field $\mathbf{E}$ and the source current density $\mathbf{J}_S$ contain non-zero elements only in θ-direction. By introduction of a magnetic vector potential $\mathbf{A}$ the magnetic flux density can be expressed as

$$\mathbf{B} = \nabla \times \mathbf{A} \tag{3}$$

and thus our equation results as

$$\frac{1}{\mu}\left(\frac{\partial^2 \mathbf{A}}{\partial r^2} + \frac{\partial \mathbf{A}}{r \partial r} + \frac{\partial^2 \mathbf{A}}{\partial z^2} - \frac{\mathbf{A}}{r^2}\right) = -\mathbf{J}_s + \jmath \omega \sigma \mathbf{A}. \tag{4}$$

For the coupled electro-thermal implementation the consideration of temperature dependent material properties is of paramount importance. Within the context of induction heating two phenomena are emphasized at this point. First, the penetration depth δ of the induced eddy currents is given by the equation

$$\delta = \frac{1}{\sqrt{\pi f \mu_0 \mu_r (T, \|\mathbf{H}\|) \sigma(T)}} \tag{5}$$

with f being the applied induction frequency. Thus, δ is a strongly temperature dependent function as both the relative magnetic permeability and the electrical conductivity are temperature dependent. Second, as the lattice structure changes from a body-centered cubic (bcc) to a face-centerd cubic (fcc) during phase transformation from ferrite to austenite (α-γ transformation) it is of paramount importance to include temperature dependent values for material density (ρ), thermal conductivity (k), and heat capacity (C_p). In order to assure a flexible control architecture which is capable of handling a multitude of different materials an extendible material database is indispensable.

In order to implement these phenomena in an FE environment a thorough understanding of the underlying differential equations is necessary. This fact requires the use of Bessel functions resulting from the solution of the Bessel differential equation, given in the general form as [6]

$$x^2 y'' + x y' + (x^2 - n^2) y = 0 \tag{6}$$

with a solution of n-th order of the first kind given by

$$J_n(x) = \sum_{k=0}^{\infty} \frac{(-1)^k (\frac{x}{2})^{n+2k}}{k! \Gamma(n+k+1)} \tag{7}$$

with $\mathbf{\Gamma}$ being the Gamma-function described by

$$\Gamma(x) = \int_0^{\infty} t^{x-1} e^{-1} \mathrm{d}t \qquad (x > 0). \tag{8}$$

The utilization of Bessel functions originates from the fact that for a conducting material each solution of the differential equations in one point depends on the solution in every other point within the solid conductor. In case that an alternating current is present in a cylindrical conductor with components in z-direction only (considering the cylindrical coordinate system) we consider $\mathbf{H}$ merely in θ-direction and utilizing the Maxwell equations we derive [7]

$$\frac{\mathrm{d}^2 J}{\mathrm{d}r^2} + \frac{\mathrm{d}J}{\mathrm{d}r}\frac{1}{r} = \jmath \frac{\omega \mu J}{\rho_{elec}} \tag{9}$$

being a zero order Bessel function of the form

$$\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} + \frac{\mathrm{d}y}{\mathrm{d}x}\frac{1}{x} - k^2 y = 0 \tag{10}$$

with a solution being a linear combination of Bessel functions $I_0(kx)$ and $K_0(kx)$ given as [7]

$$\begin{aligned} I_0(x) &= \sum_{\ell=0}^{\infty} \frac{(\frac{x}{2})^{2\ell}}{\ell!^2} \\ K_0(x) &= I_0(x)(\ln 2 - \gamma - \ln x) + \sum_{\ell=1}^{\infty} \left(\sum_{m=1}^{\ell} \frac{1}{m}\right) \frac{(\frac{x}{2})^{2\ell}}{\ell!^2} \end{aligned} \tag{11}$$

with γ being Euler's constant. These complex valued functions can be separated into an imaginary and real part utilizing the fact that according to equations (9) and (10) $k^2 = \jmath\omega\mu / \rho_{elec}$ and thus

$$I_0(x\sqrt{\jmath}) = ber(x) + \jmath\, bei(x) \qquad (12)$$

with ber, bei denoting Bessel real and Bessel imaginary, respectively [8]. Following [9] we simplify

$$\begin{aligned} ber(x) &= M_0(x)\cos(\Theta_0(x)) \\ bei(x) &= M_0(x)\sin(\Theta_0(x)) \end{aligned} \qquad (13)$$

with $M_0(x) = \sqrt{(ber^2(x) + bei^2(x))}$ and $\Theta_0(x) = \arctan(bei(x)/ber(x))$. The surface current density $\mathbf{J_S}$ at $r=R$ is in phase with the applied voltage described by E/ρ_{elec} and thus we get for the current density

$$J = J_s \frac{M_0(\sqrt{\frac{\omega\mu}{\rho_{elec}}}r)\exp(\jmath\Theta(\sqrt{\frac{\omega\mu}{\rho_{elec}}}r))}{M_0(\sqrt{\frac{\omega\mu}{\rho_{elec}}}R)\exp(\jmath\Theta(\sqrt{\frac{\omega\mu}{\rho_{elec}}}R))} \qquad (14)$$

which by integration over the radius and simplification through utilization of the fact that in our case $R \gg \delta$ yields the total current

$$I = \sqrt{2}\pi R\delta J_s \exp(-\jmath\frac{\pi}{4}). \qquad (15)$$

For induction heating the first Maxwell equation yields the relation between current density and magnetic field, with respect to simplifications due to the symmetries and the cylindrical coordinate system, as

$$\mathbf{J} = \nabla \times \mathbf{H} = -\mathbf{e}_\varphi \frac{\partial H_z}{\partial r} \qquad (16)$$

which results with equation (14) to

$$J = -\frac{\partial}{\partial r}\frac{ber(\sqrt{2}\frac{r}{\delta}) + \jmath\, bei(\sqrt{2}\frac{r}{\delta})}{ber(\sqrt{2}\frac{R}{\delta}) + \jmath\, bei(\sqrt{2}\frac{R}{\delta})} H_s. \qquad (17)$$

2.3 Thermo-Mechanical Theory

For the consideration of induction heating processes one has to include a coupled electro-thermal model. For the thermal modelling one has to embrace the heat generated through the eddy currents in the workpiece as well as transport phenomena including heat flux within the billet, convection, and radiation boundary phenomena. The temperature flux within the workpiece is described by the heat transfer equation

$$\rho C_p \frac{\partial T}{\partial t} - k\nabla^2 T = Q_{source} \qquad (18)$$

with ρ being the density of the workpiece material, T the temperature, t a time variable, and Q_{source} a heat source term. The heat source Q results from the eddy currents induced in the workpiece surface and is given by [10]

$$Q_{source} = \overline{\sigma E^2} \qquad (19)$$

taken as the average over one period of the induction current source. Thermal boundary conditions for both radiation and convection were implemented through the equation

$$-k\nabla\mathbf{T} = h(T - T_{ambient}) + \epsilon\sigma_{rad}(T^4 - T^4_{ambient}) \qquad (20)$$

with $\mathbf{T}=\mathrm{T}\cdot\mathbf{n}$, h denotes the temperature dependent convection coefficient, ε the temperature dependent emissivity, σ_{rad} the Stefan Boltzmann constant, and $T_{ambient}$ the room temperature.

3. Temperature Field Optimization

In order to utilize complex optimization algorithms, as well as complex control algorithms, the Matlab interface is facilitated through creation of the Comsol model as a *Model-m* file.

In order to optimize the continuous pulse shape *I(t)*, defining the current distribution as a function of time during the induction heating process, we have to implement a search algorithm which minimizes a cost function ε_{opt} such that

$$\epsilon_{opt}(I_{opt}(t)) = min\, \epsilon_{opt}(I(t)) \qquad (21)$$

where I_{opt} is the resulting optimal pulse shape after the completion of the search algorithm.

For the given case only a discretized pulse shape is feasible and thus the discretized pulse *I[m]* is given through a sampling of *I(t)* at a sampling rate $1/\Delta T$, with ΔT being the sampling interval, and $m=0,1,...,M$ being the sample number corresponding to the sampling time $t_{sample,m} = m\cdot\Delta T$. The vector is handed over to the *Model m-file* by the optimization routine, describing the current iteration step k for the induction current I_k and is consecutively

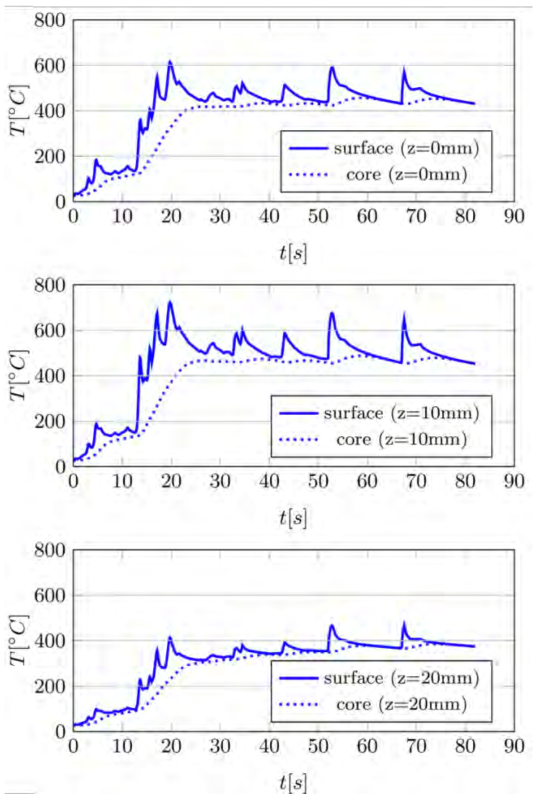

Figure 2. Optimization of heating strategy for the evolution of the billet core and surface temperature in order to reach a uniform pre-heating to 700 K, as depicted in fig. 1.

converted to the appropriate Comsol m-file string through the function `num2str(I_k(m))` and the model is solved for a predefined time interval utilizing the current iteration step. In order to yield full non-linearity of the underlying model over a large temperature interval the solution of the next time interval step $I_k[m+1]$ of the optimization vector is initialized utilizing the solution of the previous interval $I_k[m]$ by leveraging the command lines

```
fem1.sol = asseminit(femmodel,
'init',femmodel.sol);
fem.sol=femtime(fem,'init',fem1.
sol, ... ,[…]);
```

with `fem1.sol` being the solution object of the previous iteration saved as a the initial starting point for the next time interval. This routine is repeated until the end of the desired simulation time.

A thorough investigation and description of the utilized model and optimization algorithms as well as analyses regarding model verification can be found in [11].

4. Controller Implementation

The given control problem is rather convoluted due to the fact that, as emphasized in the coupled differential equations describing the underlying thermo-mechanical effects, a strong non-linear temperature dependence of energy influx impacts the required controller output. This embraces both boundary conditions as convection and ration as well as internal effects as penetration depth or heat transfer coefficient. Therefore, the given system experiences a highly dynamic behavior within a single heating cycle, which needs to be considered in the control algorithm. Conventional control algorithms, as PID controllers, were disqualified through simulations with COMSOL, as a fine-tuning of controller parameters k_p, k_i, and k_d for the PID controller is required for every new set-point, material as well as coil geometry. As a result, a fuzzy controller was implemented utilizing fuzzy membership functions based on the absolute error and the temporal derivative of the error and a geometric distribution of skewed triangular membership functions. The max and min T norm were chosen as possible fuzzy inference methods and a center of gravity (COG) or center of area (COA) defuzzification strategy was leveraged. The initial rule base is a zero matrix, with no a priori information regarding the process and online adaptation is based on a weighted steepest gradient descend optimization algorithm, directly adapting the previously active rules with respect to the degree of activation.

The implementation of the controller strategy is based upon a similar approach as outlined for the implementation of complex optimizations algorithms. Thus, no linearization of the underlying differential equations is necessary and pertinent impact parameters for the control algorithm can be optimized using the simulation model, rendering the time consuming controller tuning utilizing the actual induction facility obsolete and leading to a significant reduction of controller development and implementation time.

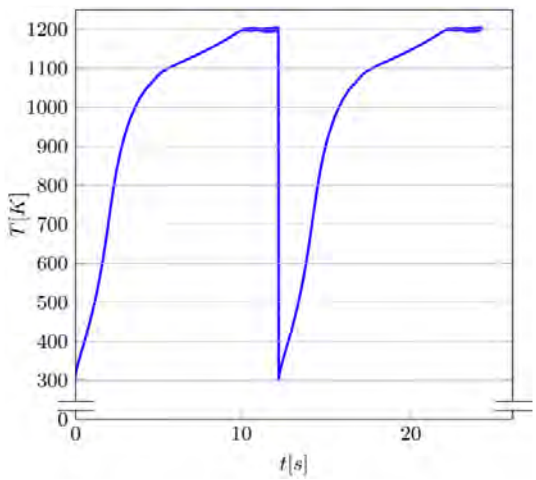

Figure 3. 9th and 10th run of the self learning fuzzy algorithm for a cycle time of 20ms and set point of 1200 K. The concatenation of simulation results was performed in order to enhance readability.

5. Results

The controller model as outlined in the previous section enables the rapid prototyping of our controller with respect to all pertinent controller parameters, being the determination of the admissible cycle time of the programmable logic controller (PLC) as well as distribution of error and error derivative membership functions or learning factor in rulebase adaption and defuzzification strategy considering the limiting factor that arises through the PLC being the cycle time and therefore the sampling interval of both temperature measurements and induction power adaption through the controller.
Automated control trials with the aforementioned Comsol model yielded the min T norm as the optimal fuzzy inference method, as this approach yields a higher stability in the vicinity of the set-point, and a COG defuzzification strategy. Figures 3 through 5 exemplarily depict the influence of the cycle time on the controller performance. As expected low cycle times (20-50ms) are preferable to high cycle times (>100ms). Moreover, one can derive that a small number of repetition steps for the self-adapting controller are sufficient in order to obtain an adequate accuracy regarding the desired set-point for the given thermo-mechanical process.

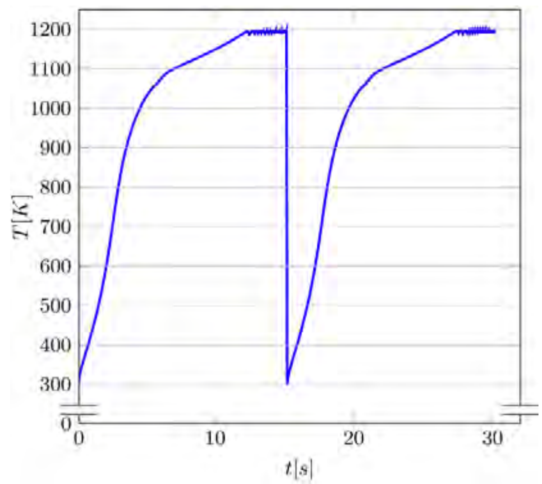

Figure 4. 4th and 5th run of the self learning fuzzy algorithm for a cycle time of 50ms and set point of 1200 K. The concatenation of simulation results was performed in order to enhance readability.

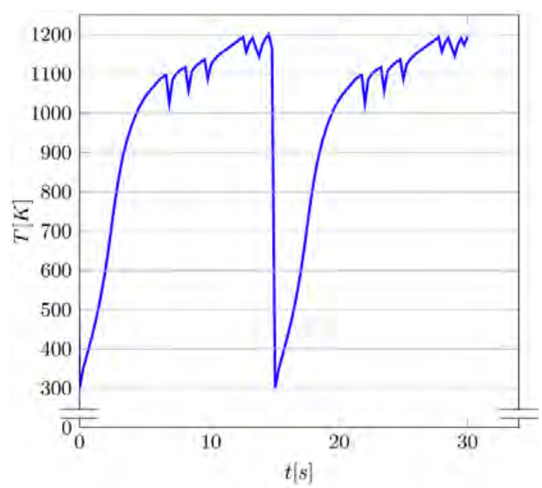

Figure 5. 4th and 5th run of the self learning fuzzy algorithm for a cycle time of 250ms and set point of 1200 K. The concatenation of simulation results was performed in order to enhance readability.

The implementation of the controller on a PLC of the company B&R Automation, Eggelsberg (Austria), was performed in the programming language C and the pertinent control parameters for the spacing of membership functions, cycle time, and defuzzification strategy were directly transferred into the PLC code as optimized through the COMSOL induction control model.

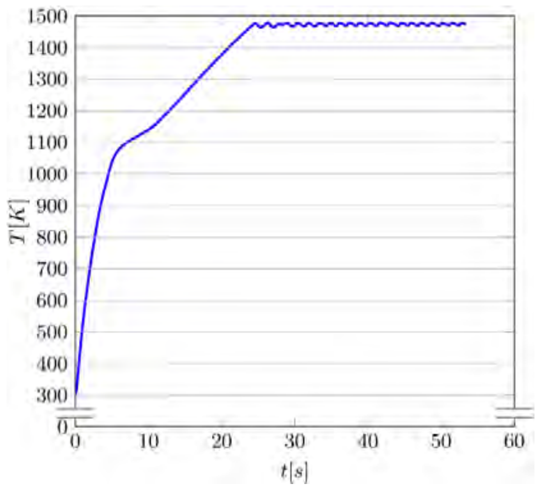

Figure 6. 5th run of the self learning fuzzy algorithm for a cycle time of 52ms and set point of 1473 K measured with a type K thermocouple connected to the PLC.

Results are depicted in fig. 6 showing the fifth self-optimization step of the implemented C code for a PLC cycle time of 52ms and measured with a type K thermocouple applied to the billet surface. The achieved maximum error is in the order of 8 K and therefore within the thermo-couple error margin being 9 K at T=1500 K.

6. Conclusions and Future Work

By utilizing the commercially available software COMSOL an electro-magnetic-thermo-mechanical and highly non-linear simulation model could be obtained allowing the implementation of both complex optimization routines as well as self-adjusting controller routines though the Matlab interface without the need for linearization of the underlying coupled differential equations. The optimization of pertinent controller parameters using the simulation model enabled a rapid control prototyping and, therefore, a significant reduction of controller implementation time into the PLC, which merely embraced a translation of the Matlab code to the programming language C. Future work will embrace an extension of the given self-adjusting controller in order to yield the capability of achieving complex temporally and spatially varying temperature profiles as given in fig. 1 and fig. 2 independent of the billet material.

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