Free Surface Flow Stability Analysis

E. Favre[1], S. Mingozzi[2]

[1]Consultant for Renaissance Fusion, 14, rue Jean-Pierre Timbaud, 38000 Fontaine, France
[2]Renaissance Fusion, 14, rue Jean-Pierre Timbaud, 38000 Fontaine, France

Abstract

The stellarator concept developed by Renaissance Fusion makes use of a lithium free surface flow surrounding the plasma fusion core. Understanding the instability mechanism of this flow is of primary importance for controlling it. We use COMSOL Multiphysics® to solve the 1D linear stability analysis of a free surface liquid flow on an inclined plane, using the well-known Orr-Sommerfeld equations(1). This problem has been solved since the 70s by using spectral methods. Finite elements method is only mentioned in the literature by exception(2), most probably due to non-trivial boundary conditions (BC) that need special care for the free surface. COMSOL Multiphysics® flexible framework permits describing and solving those equations in a straightforward way, with careful attention to the rewriting of the boundary condition at the free surface and using weak constraints as implementation technique. Note that the interested reader is invited to refer to reference 3 for details about the notations used.

The framework used by COMSOL Multiphysics® allows to solve very efficiently the linear stability problem described in the literature, providing the writing of the problem as a quadratic eigenvalue problem, and using Lagrange multiplier to enforce the constraints describing the boundary conditions. This becomes a flexible basis for solving linear stability analysis of flows under magnetic field that are met in the stellarator under development at Renaissance Fusion.

Reference

1. F. Charru, Hydrodynamic Instabilities. Cambridge University Press, 2011.
2. G.D. McBain, The primitive Orr–Sommerfeld equation and its solution by finite elements, The ANZIAM Journal, vol. 63, C168-C181, 2022.
3. A.T.L. Horng, Chebyshev collocation method on solving stability of a liquid layer flowing down an inclined plane, The Mechanics of Thin Film Coatings, 95-106, 1996.

Figures used in the abstract

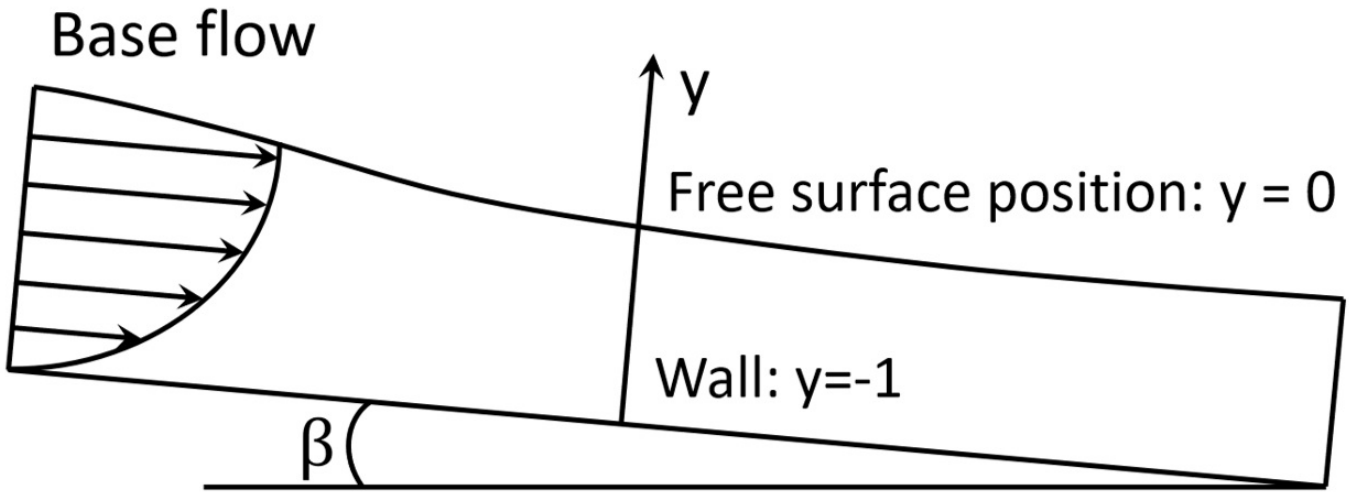

Figure 1 : FIGURE 1: Setup of the problem. The 1D analysis determines if the base flow is stable with respect to sinusoidal perturbations.

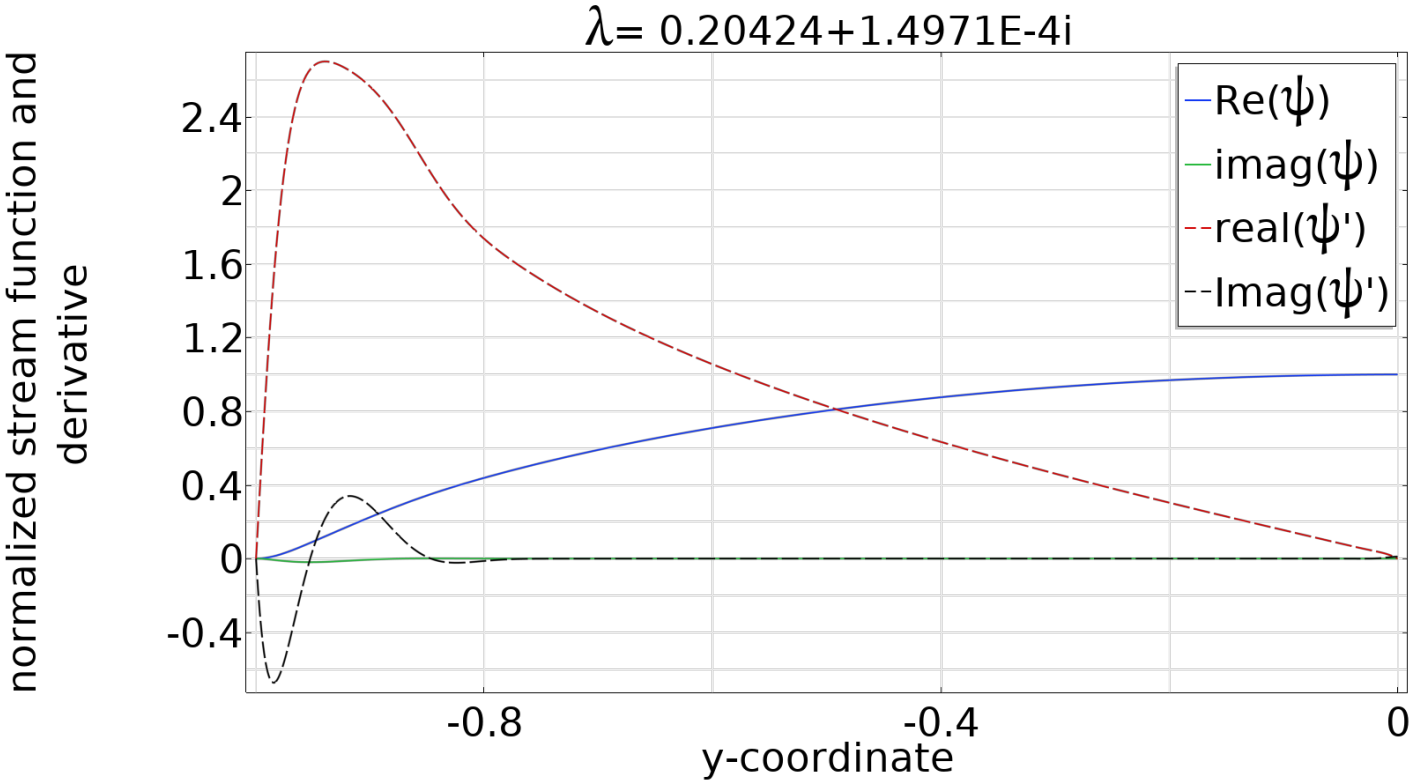

Figure 2 : FIGURE 2: normalized stream function and derivative for the first unstable mode, reproducing precisely the shape of the solution found in ref. 3, Fig 3.