Navier-Stokes

$$\frac{\partial \vec{u}}{\partial t} + \vec{u} \cdot \nabla \vec{u} = -\frac{1}{\rho} \nabla p + \nu \nabla^2 \vec{u}$$

ρ = density = ( ) kg/m^3

ν = kinematic viscosity = ( ) m^2/s

Define *R* as some characteristic lengths of the system, such as pipe diameter. New dimensionless variables:

$$\vec{U} = \frac{R\vec{u}}{\nu} \quad ; \quad P = \frac{R^2 p}{\rho \nu^2} \quad ; \quad T = \frac{\nu t}{R^2} \quad ; \quad X = \frac{x}{R},\ Y = \frac{y}{R},\ Z = \frac{z}{R}$$

Now, the dimensionless Navier-Stokes is

$$\frac{\partial \vec{U}}{\partial T} + \vec{U} \cdot \nabla' \vec{U} = -\nabla P + \nabla'^2 \vec{U}$$

$$\nabla' = \frac{1}{R}\left(\frac{\partial}{\partial X} + \frac{\partial}{\partial Y} + \frac{\partial}{\partial Z}\right) \quad ; \quad \nabla'^2 = \frac{1}{R^2}\left(\frac{\partial^2}{\partial^2 X} + \frac{\partial^2}{\partial^2 Y} + \frac{\partial^2}{\partial^2 Z}\right)$$