## Turbulent Flow Over a Backward Facing Step

## Introduction

The backward facing step has long been a central benchmark case in computational fluid dynamics. The geometry is shown in Figure 1.


Figure 1: Backstep geometry. Dimensions in SI units.
Fully developed channel flow enters at the domain from the left. When the flow reaches the step, it detaches and a recirculation zone is formed behind the step. Because of the expansion of the channel, the flow slows down and eventually
reattaches. The flow field is displayed in Figure 2.


Figure 2: Resulting flow field.
Though seemingly simple, the flow field is a challenge for turbulence models that utilize wall functions. The reason is that wall functions are derived by invoking equilibrium assumptions. Separation and reattachment do not adhere to these assumptions and it must therefore be asserted by numerical experiments that the wall functions can give accurate results even if the underlying theoretical assumptions are not strictly satisfied. The experiment is motivated by the fact that flow with separation and subsequent reattachment are of central importance in many engineering applications.

## Model Definition

The model data is taken from Ref. 1. The parameters are given in Table 1. The Reynolds number based on $V_{\text {inl }}$ and the step height, $S$, is $4.8 \cdot 10^{4}$ and the flow is therefore clearly turbulent.

TABLE I: MODEL PARAMETERS

| PROPERTY | VALUE | DESCRIPTION |
| :--- | :--- | :--- |
| S | 0.038 I m | Step height |
| $\mathrm{h}_{\mathrm{c}}$ | $2 \cdot \mathrm{~S}$ | Inlet channel height |
| H | $3 \cdot \mathrm{~S}$ | Outlet channel height |
| LI | 0.3048 m | Inlet channel length |
| L 2 | 1.3335 m | Outlet channel length |
| $\mathrm{V}_{\mathrm{inl}}$ | $18.2 \mathrm{~m} / \mathrm{s}$ | Velocity at centre of upstream channel |
| $\rho$ | $\mathrm{I} .23 \mathrm{~kg} / \mathrm{m}^{3}$ | Density |
| $\mu$ | $I .79 \cdot 10^{-5}$ | Dynamic viscosity |

You build the model in two steps:
I Simulate flow in a long channel of the same height as the inlet to give inlet boundary conditions for the actual geometry.
2 Simulate the flow over the backward facing step using the inlet boundary condition from Step 1.

## THE INLET CHANNEL

Ref. 1 suggests to simulate a channel that is $100 \cdot h_{\mathrm{c}}$ in length. Because the channel is symmetric around the midplane, the geometry is taken to be a rectangle with lower left corner at $(x, y)=(0,0)$ and upper right corner at $(x, y)=\left(100 \cdot h_{\mathrm{c}}, 0.5 \cdot h_{\mathrm{c}}\right)$. The upper boundary at $y=0.5 \cdot h_{\mathrm{c}}$ is a symmetry plane and the lower boundary at $y=0$ is the wall.

## Inlet Boundary Conditions

At the inlet $x=0$, a plug flow boundary condition with $3 \%$ turbulent intensity and a turbulent length scale according to Table 3-4 in the section Theory for the Turbulent Flow User Interfaces in the CFD Module User's Guide is prescribed. The inflow velocity cannot be set directly to $18.2 \mathrm{~m} / \mathrm{s}$ since the resulting centerline velocity at the outlet then becomes too high. While it is possible to set up an ODE that automatically computes the appropriate inlet velocity, it is far easier for small models like this one to find it by trial and error. A few iterations reveal that an inlet velocity of $16.58 \mathrm{~m} / \mathrm{s}$ gives a centerline value at the outlet very close to $18.2 \mathrm{~m} / \mathrm{s}$.

## Outlet Boundary Conditions

Outlet boundary conditions can give local artifacts at the outlet. One possible strategy is to elongate the channel and extract data some distance before the outlet. That is however not necessary since fully developed flow in a channel only has a velocity component tangential to the wall, that is normal to the outflow. By prescribing that the outflow must have no tangential component, the outlet artifacts can be removed.

## THE BACKWARD FACING STEP

There are two aspects of the backward facing step that need special consideration.

## Mesh Generation

It is important to apply a fine enough mesh at the separation point to accurately capture the creation of the shear layer. It must also be remembered that both the flow field and turbulence variables can feature strong gradients close to the walls and that the mesh must be fine enough there to represent these gradients.

## Solver Settings

The balance between the turbulence transport equations and the Navier-Stokes equations is rather delicate. If an iteration brings the flow into a state with unphysically large gradients, there is a considerable risk that the simulation will diverge. It is therefore advisable to use the parametric solver to gradually increase the Reynolds number of the flow. The most robust way is to decrease the viscosity which will be done in this model.

## Results and Discussion

As shown in Figure 3, the recirculation length normalized by the step height becomes 6.93. Ref. 2 gives an experimental result of 7.1. The result provided by COMSOL is well within the range shown by other investigations (see Ref. 1 and Ref. 3). The separation lengths in Ref. 1 ranges between 6.12 and 7.24. In Ref. 3, recirculation lengths between 5.4 and 7.1 are obtained. Furthermore, Ref. 3 shows that the
recirculation length can differ significantly by just changing some implementation details in the wall functions.


Figure 3: Contour plot of streamwise velocity equal to zero, coloured by $x / S$ where $S$ is the step beight.

Finally, note that the recirculation length can shift quite significantly with the mesh resolution. The current result does not shift much if the mesh is refined, but coarser meshes can yield very different recirculation lengths. This emphasizes the need to ensure that the mesh is fine enough.

## References

1. Ist NAFEMS Workbook of CFD Examples. Laminar and Turbulent Two-Dimensional Internal Flows. NAFEMS, 2000.
2. J. Kim, S.J. Kline, and J.P. Johnston, "Investigation of a Reattaching Turbulent Shear Layer: Flow Over a Backward Facing Step," Transactions of the ASME, vol. 102, p. 302, 1980.
3. D. Kuzmin, O. Mierka, and S. Turek, "On the Implementation of the $k-\varepsilon$ Turbulence Model in Incompressible Flow Solvers Based on a Finite Element

Discretization," International Journal of Computing Science and Mathematics, vol. 1, no. 2-4, pp. 193-206, 2007.

Model Library path: CFD_Module/Single-Phase_Benchmarks/ turbulent_backstep

## Modeling Instructions

From the File menu, choose New.

## MODEL WIZARD

I Go to the Model Wizard window.
2 Click the 2D button.

## 3 Click Next.

4 In the Add physics tree, select Fluid Flow $>$ Single-Phase Flow $>$ Turbulent Flow $>$ Turbulent Flow, k- $\varepsilon$ (spf).

5 Click Next.
6 Find the Studies subsection. In the tree, select Preset Studies>Stationary.

## 7 Click Finish.

## GLOBAL DEFINITIONS

## Parameters

I In the Model Builder window, right-click Global Definitions and choose Parameters.
2 In the Parameters settings window, locate the Parameters section.
3 In the table, enter the following settings:

| Name | Expression | Description |
| :--- | :--- | :--- |
| S | $0.0381[\mathrm{~m}]$ | Step height |
| hc | $0.0762[\mathrm{~m}]$ | Inlet channel height |
| H | $0.1143[\mathrm{~m}]$ | Outlet channel height |
| L1 | $0.3048[\mathrm{~m}]$ | Inlet channel length |
| L2 | $1.3335[\mathrm{~m}]$ | Outlet channel length |
| Vinl | $16.58[\mathrm{~m} / \mathrm{s}]$ | Centerline inlet velocity |
| rhof | $1.23\left[\mathrm{~kg} / \mathrm{m}^{\wedge} 3\right]$ | Density |


| Name | Expression | Description |
| :--- | :--- | :--- |
| muf | $1.79 \mathrm{e}-5[\mathrm{Pa*}]^{*}$ fact | Dynamic Viscosity |
| fact | 1.0 | Viscosity scaling factor |

## GEOMETRY I

## Rectangle I

I In the Model Builder window, under Model I right-click Geometry I and choose Rectangle.

2 In the Rectangle settings window, locate the Size section.
3 In the Width edit field, type 100*L1.
4 In the Height edit field, type hc / 2.
5 Click the Build All button.

## MATERIALS

## Material I

I In the Model Builder window, under Model I right-click Materials and choose Material.
2 In the Material settings window, locate the Material Contents section.
3 In the table, enter the following settings:

| Property | Name | Value |
| :--- | :--- | :--- |
| Density | rho | rhof |
| Dynamic viscosity | mu | muf |

Turbulent Flow, $\mathbf{k}-\varepsilon$

## Inlet |

I In the Model Builder window, under Model I right-click Turbulent Flow, k- $\varepsilon$ and choose Inlet.

2 Select Boundary 1 only.
3 In the Inlet settings window, locate the Velocity section.
4 In the $U_{0}$ edit field, type Vinl.
5 Locate the Boundary Condition section. In the $L_{\mathrm{T}}$ edit field, type 0.07 *hc.
6 In the $I_{\mathrm{T}}$ edit field, type 0.03.
Symmetry I
I In the Model Builder window, right-click Turbulent Flow, k- $\varepsilon$ and choose Symmetry.

2 Select Boundary 3 only.

## Boundary Stress I

I Right-click Turbulent Flow, k- $\varepsilon$ and choose Boundary Stress.
2 In the Boundary Stress settings window, locate the Boundary Condition section.
3 From the Boundary condition list, choose Normal stress, normal flow.
4 Select Boundary 4 only.

## MESH I

## Mapped I

In the Model Builder window, under Model I right-click Mesh I and choose Mapped.

## Distribution I

I In the Model Builder window, under Model I>Mesh I right-click Mapped I and choose Distribution.

2 Select Boundaries 1 and 4 only.
3 In the Distribution settings window, locate the Distribution section.
4 From the Distribution properties list, choose Predefined distribution type.
5 From the Distribution method list, choose Geometric sequence.
6 In the Element ratio edit field, type 10.
7 In the Number of elements edit field, type 40.
Distribution 2
I Right-click Mapped I and choose Distribution.
2 Select Boundary 2 only.
3 In the Distribution settings window, locate the Distribution section.
4 From the Distribution properties list, choose Predefined distribution type.
5 In the Number of elements edit field, type 250.
6 In the Element ratio edit field, type 2.
7 Select the Reverse direction check box.

## Distribution 3

I Right-click Mapped I and choose Distribution.
2 In the Distribution settings window, locate the Distribution section.
3 From the Distribution properties list, choose Predefined distribution type.
4 Select Boundary 3 only.

5 In the Number of elements edit field, type 250.
6 In the Element ratio edit field, type 2.
7 In the Model Builder window, right-click Mesh I and choose Build AlI.

## STUDY I

In the Model Builder window, right-click Study I and choose Compute.

## RESULTS

Check that the flow is fully developed by plotting muT along the centerline.

## ID Plot Group 4

I In the Model Builder window, right-click Results and choose ID Plot Group.
2 Right-click ID Plot Group 4 and choose Line Graph.
3 Select Boundary 3 only.
This is the top surface.
4 In the Line Graph settings window, click Replace Expression in the upper-right corner of the $\boldsymbol{y}$-Axis Data section. From the menu, choose Turbulent Flow, k- $\varepsilon>$ Turbulent dynamic viscosity (spf.muT).
5 In the Model Builder window, right-click ID Plot Group 4 and choose Rename.
6 Go to the Rename ID Plot Group dialog box and type Turbulent viscosity in the New name edit field.

7 Click OK.

## 8 Right-click ID Plot Group 4 and choose Plot.

As can be seen in the resulting plot (Figure 4), the turbulent viscosity has obtained a constant value well before the outlet


Figure 4: Turbulent viscosity along the centerline of the inlet channel.
With the initial simulation step completed, create the backstep model.

## ROOT

In the Model Builder window, right-click the root node and choose Add Model.

## MODEL WIZARD

I Go to the Model Wizard window.
2 Click the 2D button.
3 Click Next.
4 In the Add physics tree, select Recently Used>Turbulent Flow, k- $\varepsilon$ (spf).

## 5 Click Next.

6 Find the Studies subsection. In the tree, select Preset Studies for Selected Physics>Stationary.

7 Find the Selected physics subsection. In the table, enter the following settings:

| Physics | Solve for |
| :--- | :--- |
| Turbulent Flow, k- $\varepsilon$ (spf) | $\times$ |

8 Click Finish.

## GEOMETRY 2

Rectangle I
I In the Model Builder window, under Model 2 right-click Geometry 2 and choose Rectangle.

2 In the Rectangle settings window, locate the Size section.
3 In the Width edit field, type L1+L2.
4 In the Height edit field, type hc.
5 Locate the Position section. In the $\mathbf{x}$ edit field, type -L1.
6 In the $\mathbf{y}$ edit field, type S.

## Point I

I In the Model Builder window, right-click Geometry 2 and choose Point.
2 In the Point settings window, locate the Point section.
3 In the $\mathbf{x}$ edit field, type -L1.
4 In the $\mathbf{y}$ edit field, type $S+h c / 2$.
Union I
I Right-click Geometry 2 and choose Boolean Operations>Union.
2 Select the objects $\mathbf{r l}$ and $\mathbf{p t I}$ only.

## Rectangle 2

I Right-click Geometry 2 and choose Rectangle.
2 In the Rectangle settings window, locate the Size section.
3 In the Width edit field, type L2.
4 In the Height edit field, type S.
5 Click the Build Selected button.

## Union 2

I Right-click Geometry 2 and choose Boolean Operations>Union.
2 Select the objects unil and $\mathbf{r} \mathbf{2}$ only.

## Mesh Control Edges I

I Right-click Geometry 2 and choose Virtual Operations>Mesh Control Edges.
2 On the object fin, select Boundary 7 only.
3 Right-click Geometry 2 and choose Build All.
4 Click the Zoom Extents button on the Graphics toolbar.

## DEFINITIONS

## Linear Extrusion I

I In the Model Builder window, under Model I right-click Definitions and choose Model Couplings>Linear Extrusion.
2 In the Linear Extrusion settings window, locate the Source Selection section.
3 From the Geometric entity level list, choose Boundary.
4 Select Boundary 4 only.
5 In the Linear Extrusion settings window, locate the Source Vertices section.
6 Under Source vertex I, click Activate Selection.
7 Select Point 3 only.
8 In the Linear Extrusion settings window, locate the Source Vertices section.
9 Under Source vertex 2, click Activate Selection.
10 Select Point 4 only.
II In the Linear Extrusion settings window, click to expand the Destination section.
$\mathbf{1 2}$ From the Destination geometry list, choose Geometry 2.
I3 Locate the Destination Vertices section. Under Destination vertex I, click Activate Selection.

14 Select Point 1 only.
15 In the Linear Extrusion settings window, locate the Destination Vertices section.
16 Under Destination vertex 2, click Activate Selection.
17 Select Point 2 only.

## Linear Extrusion 2

I In the Model Builder window, right-click Definitions and choose Model Couplings>Linear Extrusion.
2 In the Linear Extrusion settings window, locate the Source Selection section.
3 From the Geometric entity level list, choose Boundary.

4 Select Boundary 4 only.
5 In the Linear Extrusion settings window, locate the Source Vertices section.
6 Under Source vertex I, click Activate Selection.
7 Select Point 3 only.
8 In the Linear Extrusion settings window, locate the Source Vertices section.
9 Under Source vertex 2, click Activate Selection.
10 Select Point 4 only.
II In the Linear Extrusion settings window, locate the Destination section.
12 From the Destination geometry list, choose Geometry 2.
I3 Locate the Destination Vertices section. Under Destination vertex I, click Activate Selection.

14 Select Point 3 only.
15 In the Linear Extrusion settings window, locate the Destination Vertices section.
16 Under Destination vertex 2, click Activate Selection.
17 Select Point 2 only.

## MATERIALS

## Material 2

I In the Model Builder window, under Model 2 right-click Materials and choose Material.
2 In the Material settings window, locate the Material Contents section.
3 In the table, enter the following settings:

| Property | Name | Value |
| :--- | :--- | :--- |
| Density | rho | rhof |
| Dynamic viscosity | mu | muf |

Turbulent Flow, k- $\mathbf{2}$

## Inlet I

I In the Model Builder window, under Model 2 right-click Turbulent Flow, k- $\mathcal{2}$ and choose Inlet.

2 Select Boundary 1 only.
3 In the Inlet settings window, locate the Velocity section.
4 Click the Velocity field button.

5 In the $\mathbf{u}_{0}$ table, enter the following settings:

| mod1.linext1(mod1.u) | $x$ |
| :--- | :--- |
| 0 | $y$ |

6 Locate the Boundary Condition section. Click the Specify turbulence variables button.
7 In the $k_{0}$ edit field, type mod1. linext1 (mod1.k).
8 In the $\varepsilon_{0}$ edit field, type mod1.linext1(mod1.ep).

## Inlet 2

I In the Model Builder window, right-click Turbulent Flow, k- $\mathbf{2}$ and choose Inlet.
2 Select Boundary 3 only.
3 In the Inlet settings window, locate the Velocity section.
4 Click the Velocity field button.
5 In the $\mathbf{u}_{0}$ table, enter the following settings:

| mod1.linext2(mod1.u) | $x$ |
| :--- | :--- |
| 0 | $y$ |

6 Locate the Boundary Condition section. Click the Specify turbulence variables button.
7 In the $k_{0}$ edit field, type mod1. linext2(mod1.k).
8 In the $\varepsilon_{0}$ edit field, type mod1.linext2(mod1.ep).

## Outlet I

I Right-click Turbulent Flow, k- $\mathbf{2}$ and choose Outlet.
2 In the Outlet settings window, locate the Boundary Condition section.
3 From the Boundary condition list, choose Pressure.
4 Select Boundary 7 only.

MESH 2
I In the Model Builder window, under Model 2 click Mesh 2.
2 In the Mesh settings window, locate the Mesh Settings section.
3 From the Element size list, choose Coarse.
Size 1
I Right-click Model 2>Mesh 2 and choose Edit Physics-Induced Sequence.

2 In the Model Builder window, under Model 2>Mesh 2 right-click Size I and choose Build Selected.

Size 2
I Right-click Mesh 2 and choose Size.
2 In the Size settings window, locate the Geometric Entity Selection section.
3 From the Geometric entity level list, choose Boundary.
4 Select Boundary 9 only.
5 Locate the Element Size section. Click the Custom button.
6 Locate the Element Size Parameters section. Select the Maximum element growth rate check box.

7 In the associated edit field, type 1.03.

## Size 3

I Right-click Mesh 2 and choose Size.
2 In the Size settings window, locate the Geometric Entity Selection section.
3 From the Geometric entity level list, choose Point.
4 Locate the Element Size section. Click the Custom button.
5 Locate the Element Size Parameters section. Select the Maximum element size check box.

6 Select Point 5 only.
7 In the associated edit field, type 5e-4.

## Boundary Layer Properties I

I In the Model Builder window, expand the Model $\mathbf{2} \boldsymbol{>}$ Mesh $\mathbf{2 > B o u n d a r y}$ Layers I node, then click Boundary Layer Properties I.

2 In the Boundary Layer Properties settings window, locate the Boundary Layer Properties section.

3 In the Thickness adjustment factor edit field, type 2.
4 In the Number of boundary layers edit field, type 6.
5 Click the Build All button.

STUDY 2

Step I: Stationary
I In the Model Builder window, expand the Study $\mathbf{2}$ node, then click Step I: Stationary.

2 In the Stationary settings window, click to expand the Study Extensions section.
3 Select the Continuation check box.
4 Click Add.
5 In the table, enter the following settings:

| Continuation parameter | Parameter value list |
| :--- | :--- |
| fact | 51 |

6 Click to expand the Values of Dependent Variables section. Select the Values of variables not solved for check box.

7 From the Method list, choose Solution.
8 From the Study list, choose Study I, Stationary.
9 In the Model Builder window, right-click Study 2 and choose Compute.

## RESULTS

Check that the wall lift-off is 11.06 almost everywhere by selecting the Wall Resultion (spf2) plot group.

Wall Resolution (spf2)
Next, reproduce the flow-field plot with the following steps:

## Velocity (spf2)

I In the Model Builder window, under Results right-click Velocity (spf2) and choose Streamline.

2 In the Streamline settings window, locate the Streamline Positioning section.
3 From the Positioning list, choose Uniform density.
4 In the Separating distance edit field, type 0.007.
5 Locate the Coloring and Style section. From the Color list, choose White.
6 Click the Plot button.
Finally, visualize the recirculation length.
2D Plot Group 8
I In the Model Builder window, right-click Results and choose 2D Plot Group.
2 In the 2D Plot Group settings window, locate the Data section.
3 From the Data set list, choose Solution 3.
4 Right-click Results>2D Plot Group 8 and choose Contour.

5 In the Contour settings window, click Replace Expression in the upper-right corner of the Expression section. From the menu, choose Turbulent Flow, k- $\mathbf{2} \mathbf{2}$ Velocity field $>$ Velocity field, $x$ component ( $\mathbf{u} 2$ ).

6 Locate the Levels section. From the Entry method list, choose Levels.
7 Right-click Results $\mathbf{> 2 D}$ Plot Group 8>Contour I and choose Color Expression.
8 In the Color Expression settings window, locate the Expression section.
9 In the Expression edit field, type x/S.
IO In the Model Builder window, right-click 2D Plot Group 8 and choose Rename.
II Go to the Rename 2D Plot Group dialog box and type Recirculation length in the New name edit field.

12 Click OK.
I3 In the 2D Plot Group settings window, locate the Data section.
14 From the Data set list, choose Solution 3.
15 Click the Plot button.

