

# **Finite Element Solution of Nonlinear Transient Rock Damage with Application in Geomechanics of Oil and Gas Reservoirs**

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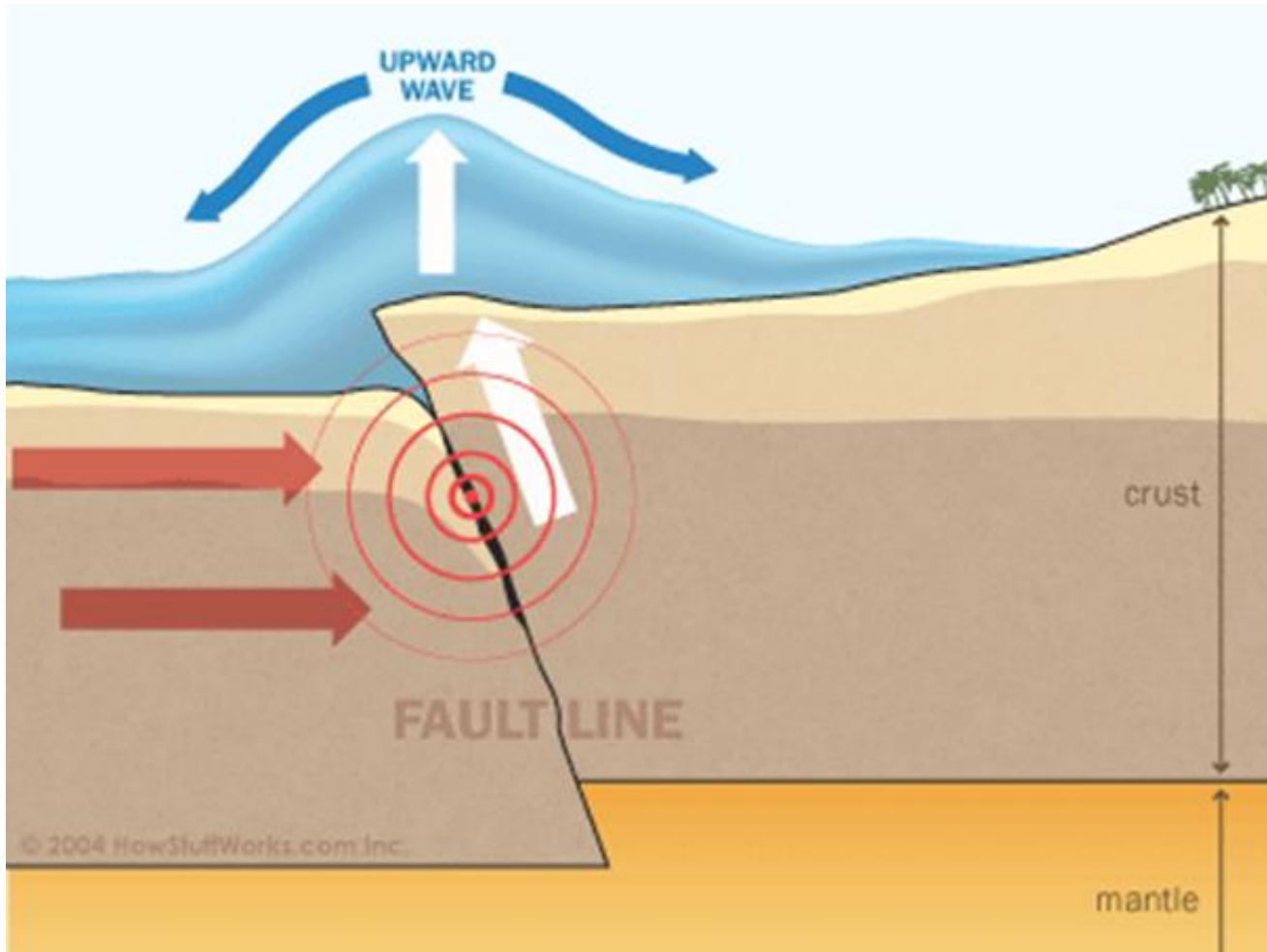
Department of Petroleum and Geosystems Engineering

Excerpt from the Proceedings of the 2012 COMSOL Conference in Boston

COMSOL  
CONFERENCE  
BOSTON  
2012

# Fractures everywhere

Faults, **Km** scale



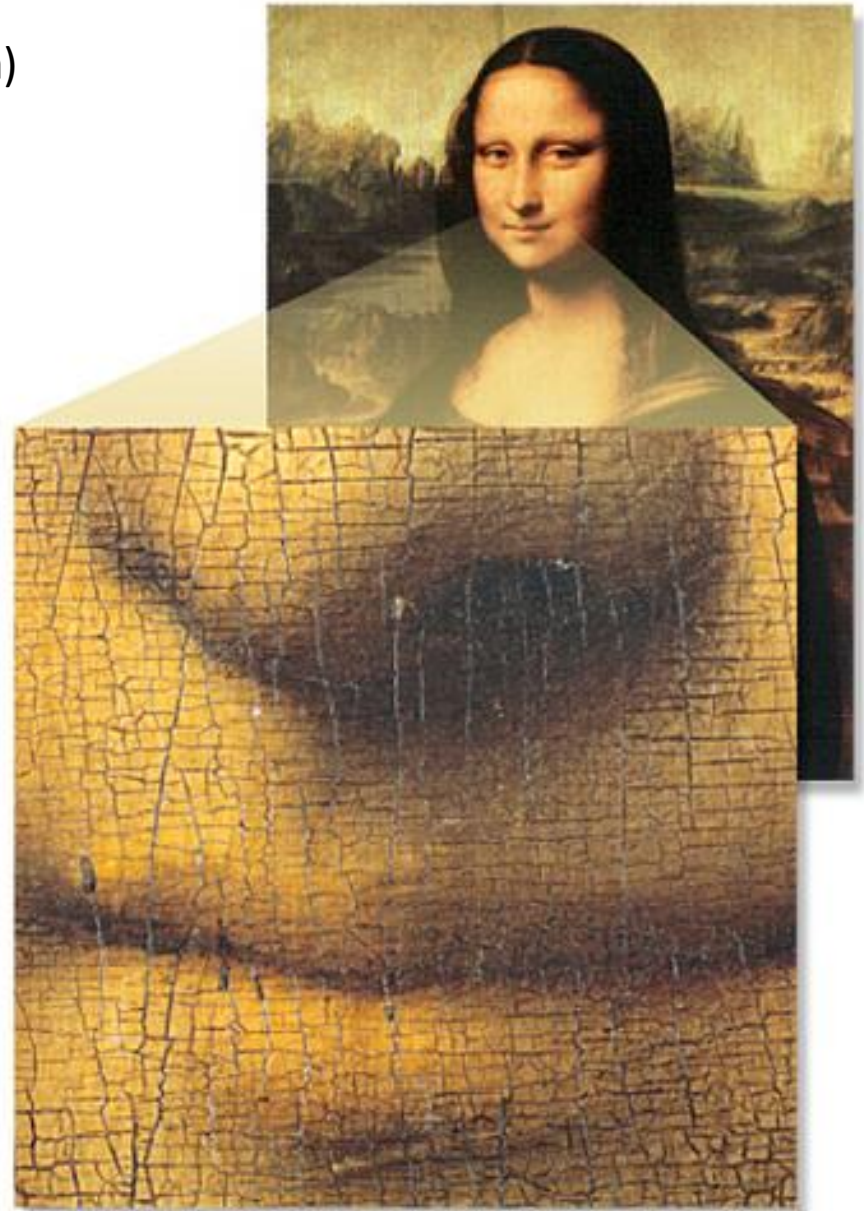
# Fractures everywhere

Vertical Stabilizer After Recovery From Jamaica Bay (Scale:m)



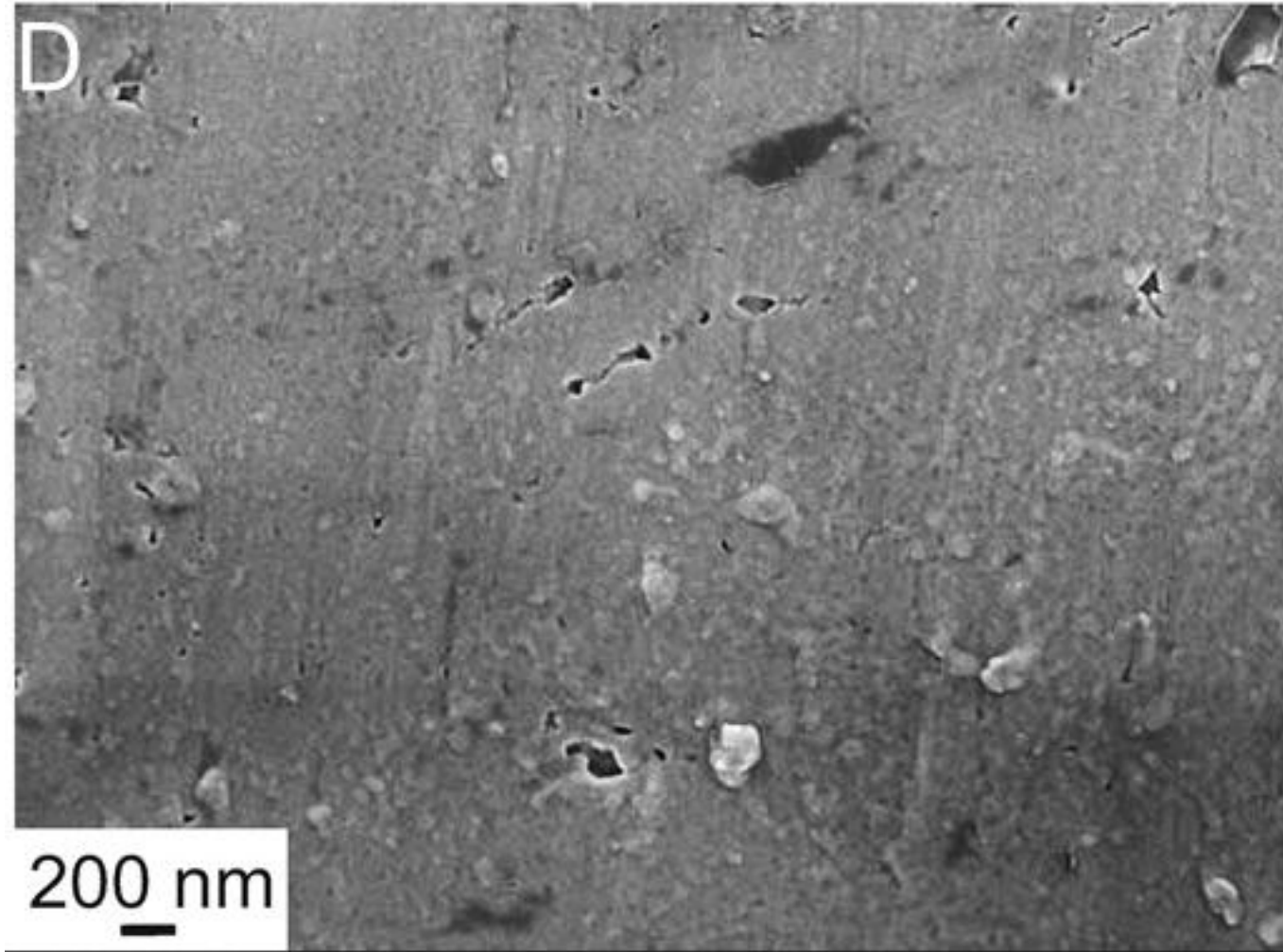
# Fractures everywhere

Age crackling in the Mona Lisa (Scale:mm-cm)

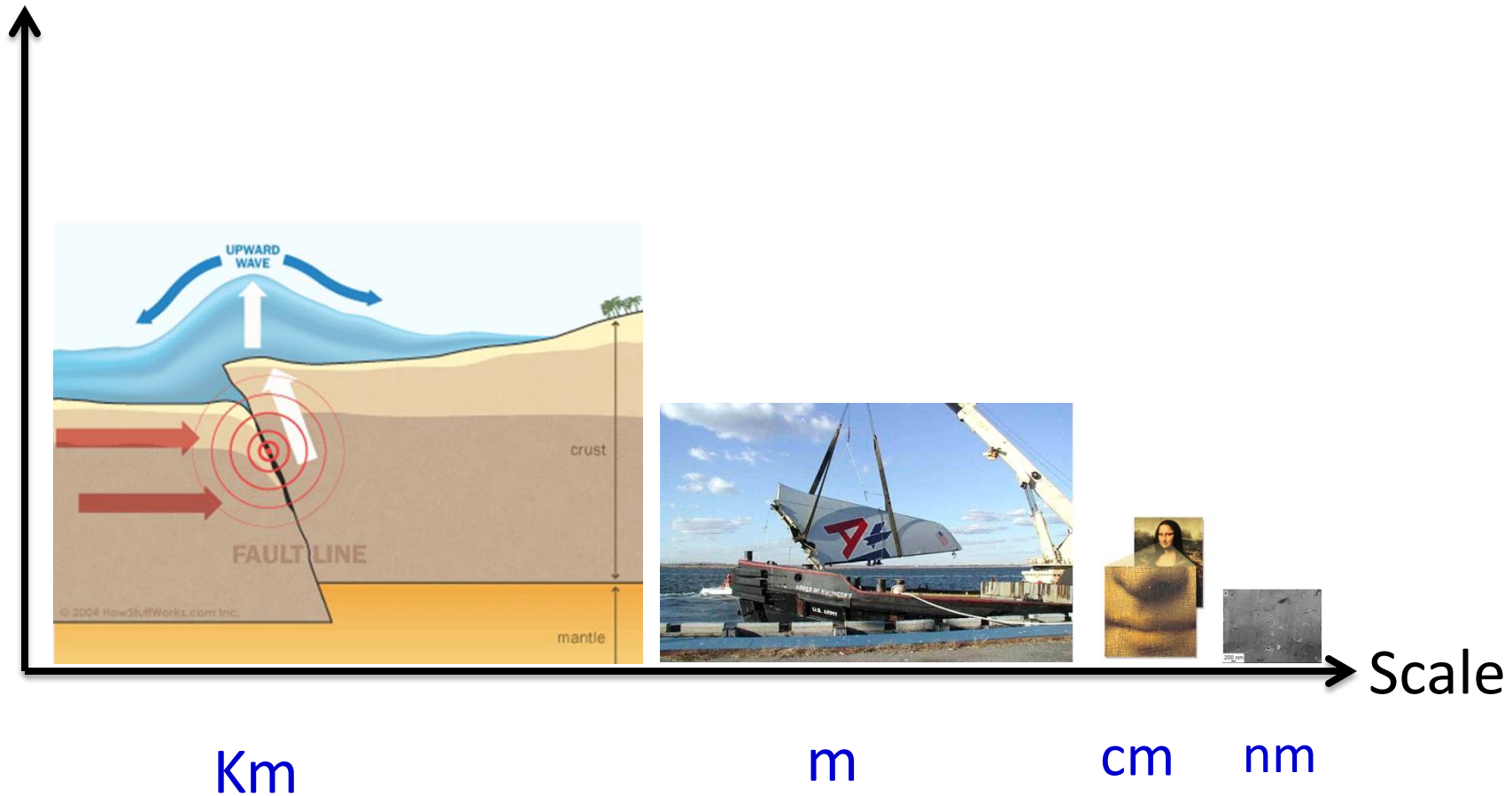


# Fractures everywhere

Fractures in rock (Scale:nm- $\mu$ m)

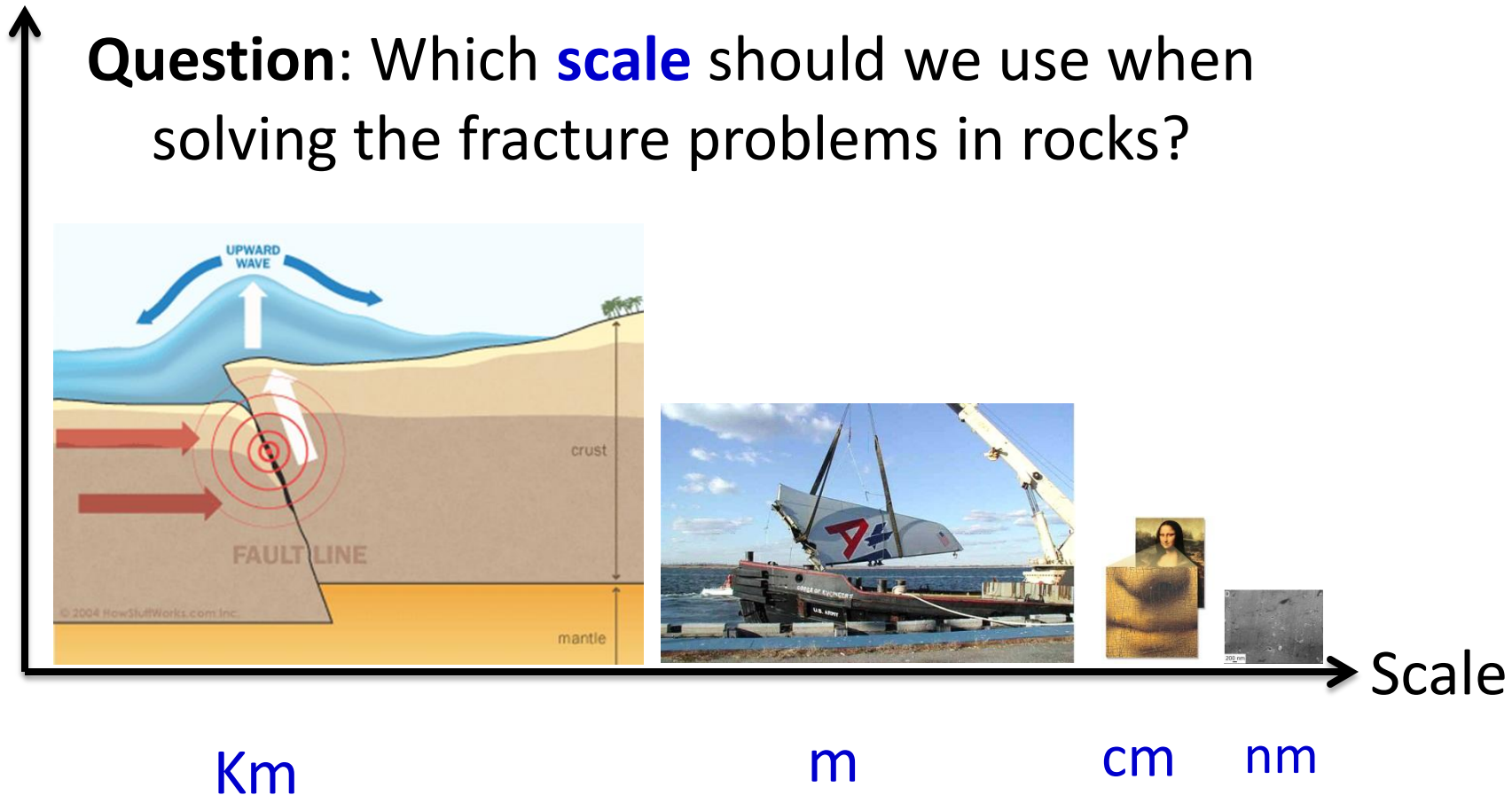


# Fracture: A multiscale problem



# Fracture: A multiscale problem

Question: Which **scale** should we use when solving the fracture problems in rocks?

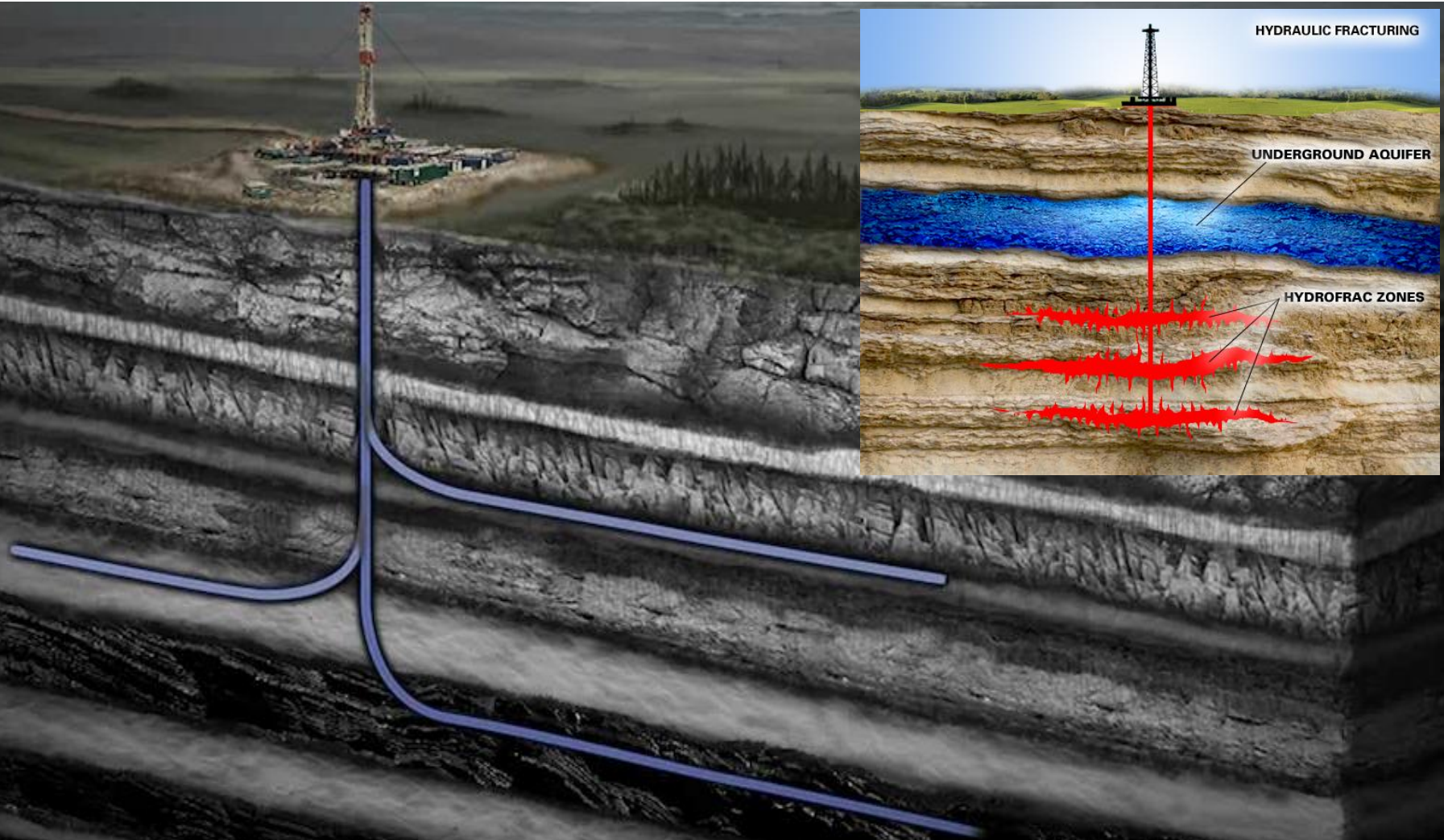


# Application: Rock (Hydraulic) Fracturing





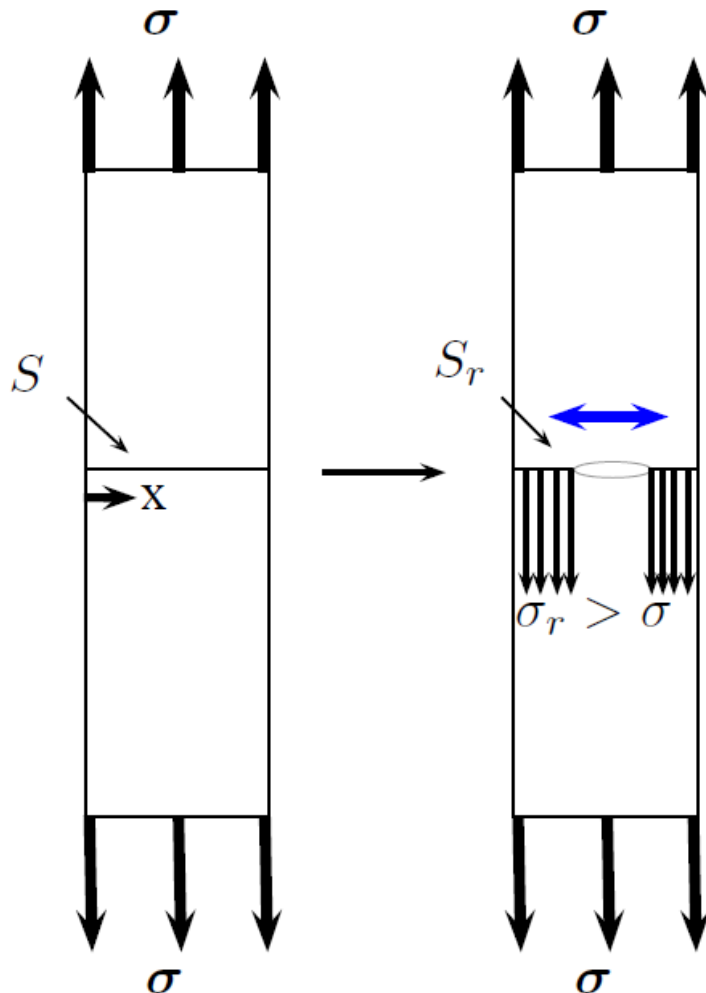
# Application: Rock (Hydraulic) Fracturing



# Description of Problem

- How does fracture propagate in rock?
- What is the governing equation of **rock damage**?
- How can we solve the equation using COMSOL?

# Description of Problem: **rock damage**



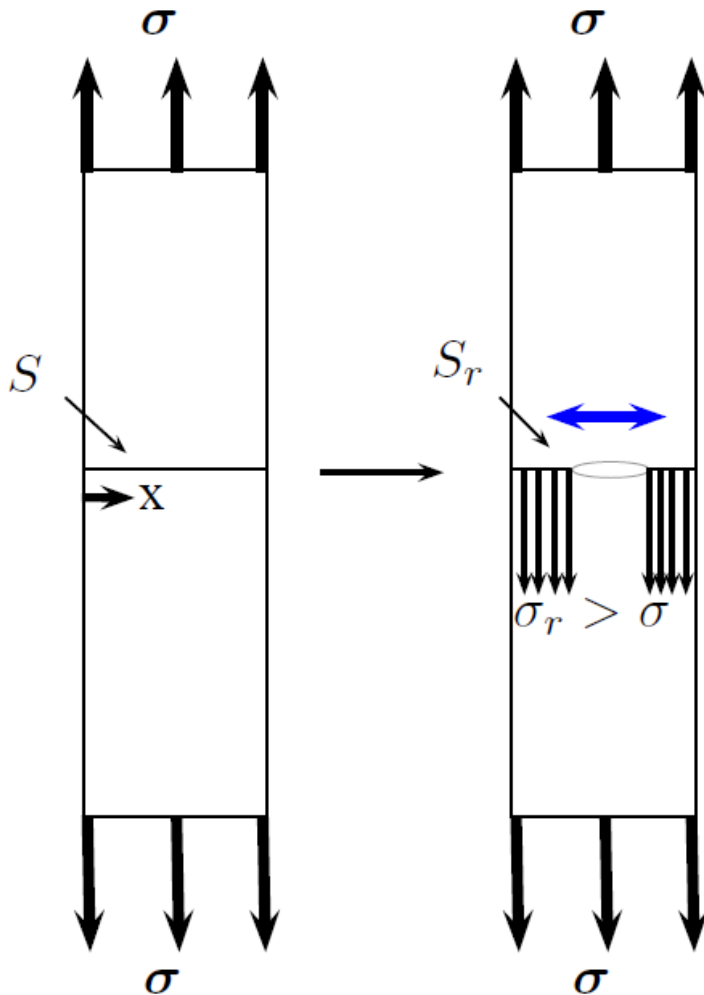
$$\omega = \frac{S - S_r}{S}$$

Damage parameter  $\omega$  varies between 0 and 1.

**0**: intact rock

**1**: fully pulverized rock

# Description of Problem: **rock damage**



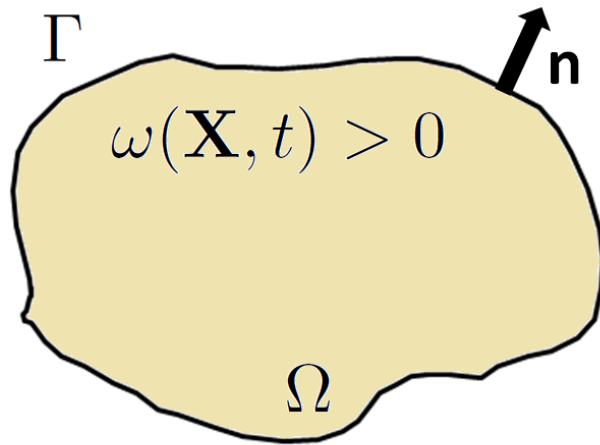
$$\omega = \frac{\begin{array}{c} \text{Total} \\ \downarrow \\ S \end{array} - \begin{array}{c} \text{Remaining} \\ \downarrow \\ S_r \end{array}}{S}$$

Damage parameter  $\omega$  varies between 0 and 1.

**0**: intact rock

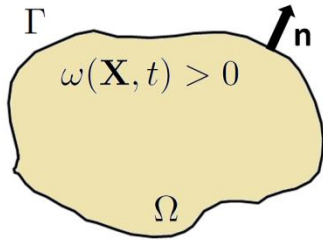
**1**: fully pulverized rock

# Description of Problem: **rock damage**



$$\left\{ \begin{array}{l} \omega : \Omega \rightarrow R \\ \omega = 0, \quad D \frac{\partial \omega}{\partial n} = 0 \quad \text{on } \Gamma \end{array} \right.$$

# Description of Problem: **rock damage**



$$\begin{cases} \omega : \Omega \rightarrow R \\ \omega = 0, D \frac{\partial \omega}{\partial n} = 0 \quad \text{on } \Gamma \end{cases}$$

$$\frac{\partial \omega(X, t)}{\partial t} = [\nabla \cdot [\kappa \nabla (f(\omega))] + f(\omega)]_+$$

# Coefficient form of PDE in COMSOL

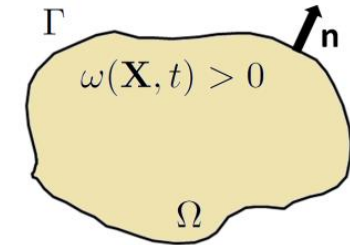
▲  $\Delta u$  Coefficient Form PDE (c)

Coefficient Form PDE 1

Zero Flux 1

Initial Values 1

Mesh 1



$$\begin{cases} \omega : \Omega \rightarrow R \\ \omega = 0, D \frac{\partial \omega}{\partial n} = 0 \quad \text{on } \Gamma \end{cases}$$

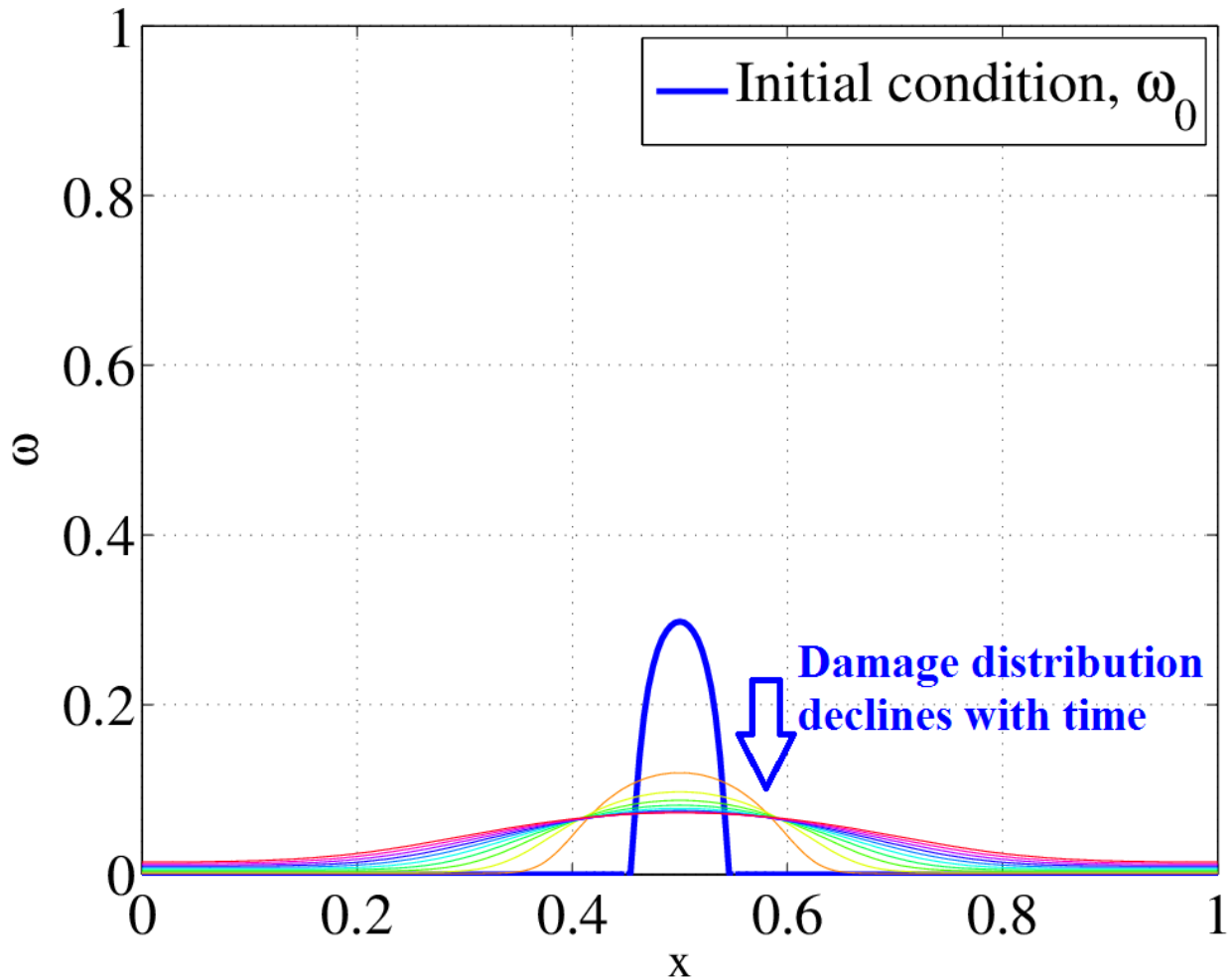
$$e_a \frac{\partial^2 u}{\partial t^2} + d_a \frac{\partial u}{\partial t} + \nabla \cdot (-c \nabla u - \alpha u + \gamma) + \beta \cdot \nabla u + a u = f$$

$$f = (1 - u) \exp\left(\frac{\mu_0}{1 - u}\right)$$

$$c = \kappa \left( \frac{\mu_0 + u - 1}{1 - u} \right) \exp\left(\frac{\mu_0}{1 - u}\right)$$

Use zero for  $e_a, \alpha, \gamma, \beta$

# Results



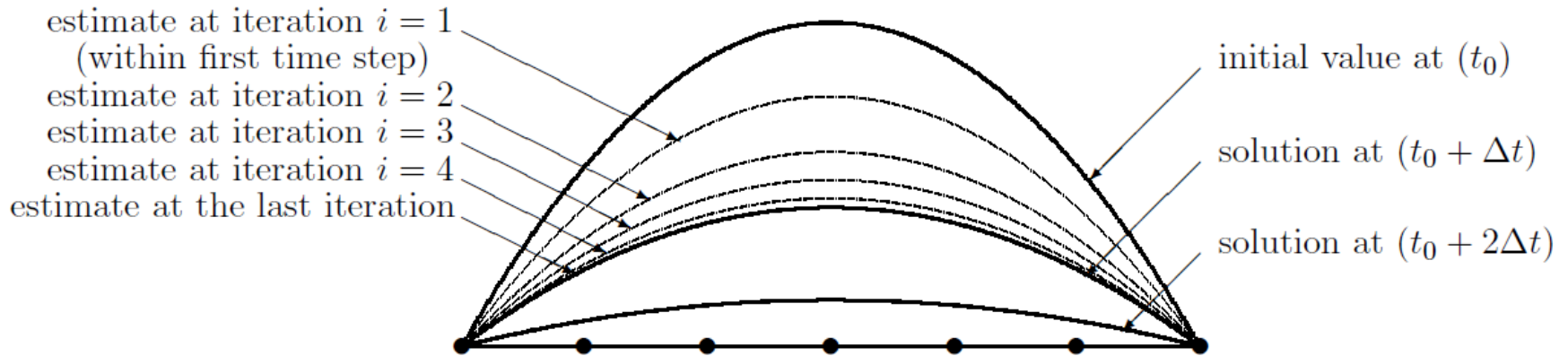
Distribution of damage in a bar under tension



# Use MATLAB to fix the problem

ESTIMATE OF SOLUTIONS AT N.R. ITERATIONS

SOLUTIONS AT TIME STEPS



# Use MATLAB to fix the problem

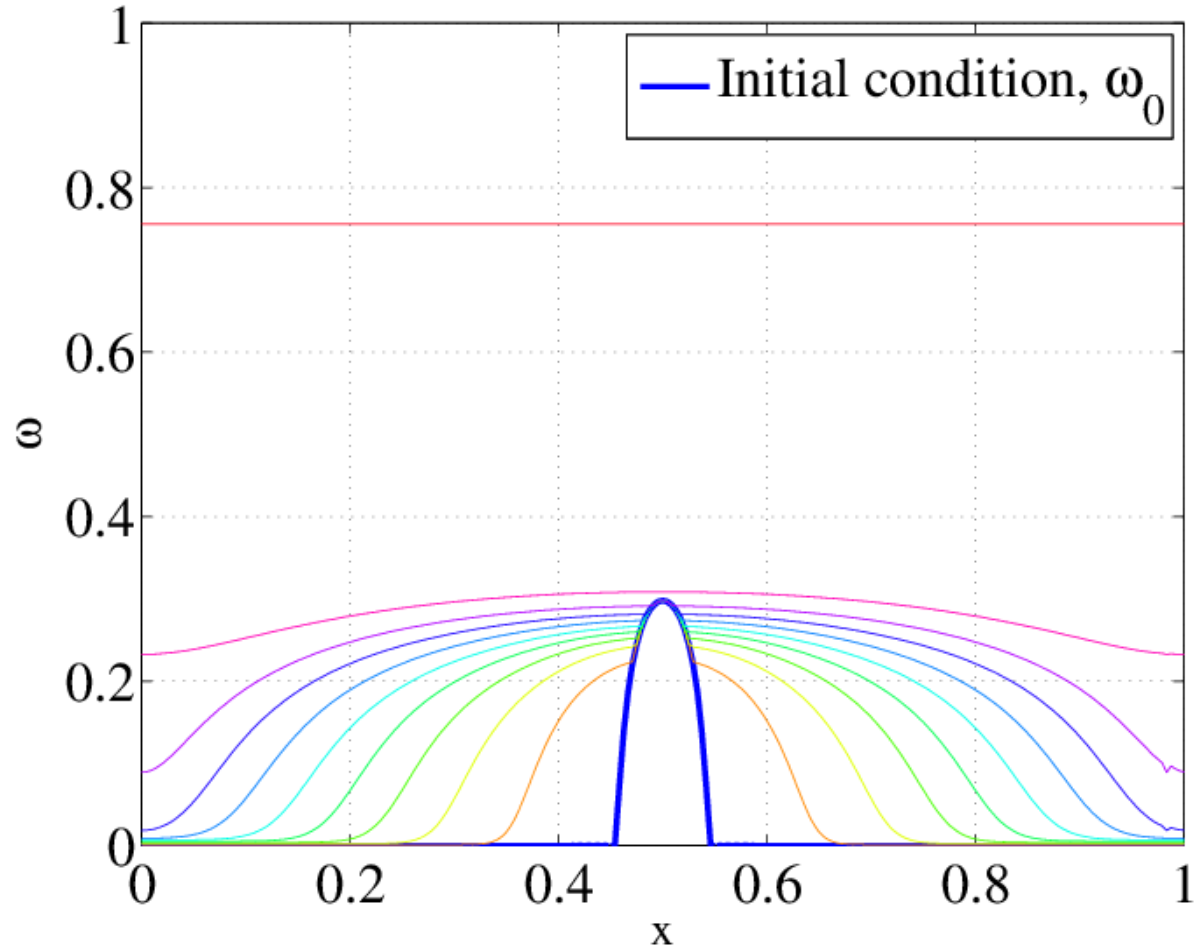
## STEP 1: Obtain solution data

```
nodes = xmeshinfo(fem, 'out', 'nodes');  
dofs=nodes.dofs;      %Obtain degrees of freedom  
coords=nodes.coords; %Obtain coordinates for the DOFs  
X=fem.sol.u;         %Store solution vector
```

## STEP 2: Remove negative results

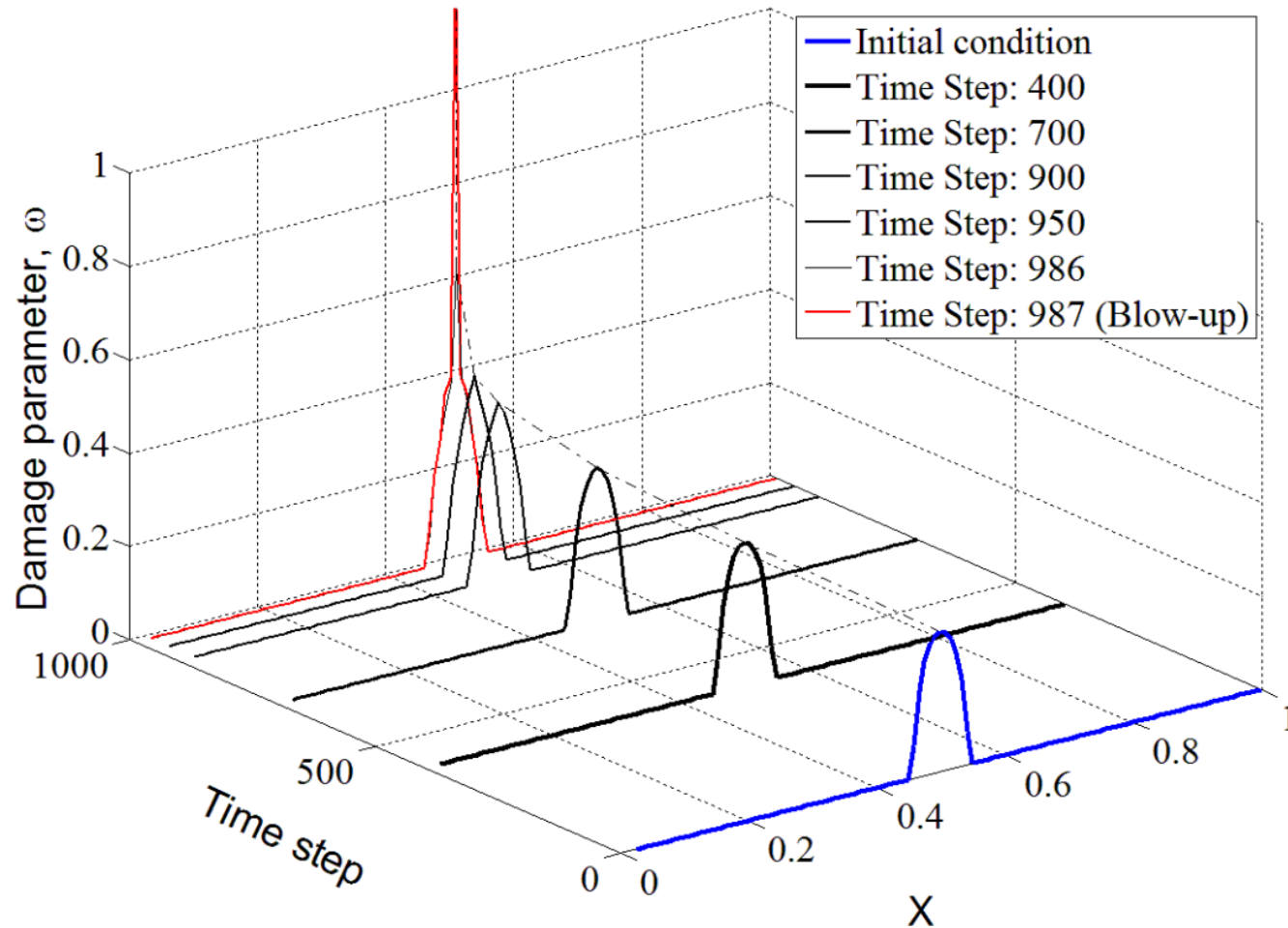
```
X=fem.sol.u; %Assigns the entire solution vector to X  
for il=1:length(dofs)  
    if (X(il,2))<(X(il,1))  
        X(il,2)=X(il,1);  
    end  
end
```

# Results



Distribution of damage in a bar under tension

# Results



Distribution of damage : Blow-up of solution (**Fracture initiation**)

# Conclusions

- In this paper we have used COMSOL Multiphysics and COMSOL Script to solve the transient rock damage problem. **We have analyzed the non-healing process and incorporated the positive rate of damage** in the finite element solution we obtained from COMSOL.
- Due to the nonlinearity of damage problem, to obtain an accurate converged solution, time steps have to be very small. Our numerical results indicate that for the number of **time steps beyond 10,000**, the solution converges.
- Inducing rock damage, enhances the permeability of rock and **increases productivity of oil and gas wells.**

**Thank You**