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A Theoretical Model for the Control of Color Degradation and Microbial Spoilage Occurring in Food Convective Drying

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Outline of the talk

Formulation of a transport model describing the simultaneous transfer of momentum, heat and mass occurring in a convective drier where hot dry air flows, in turbulent conditions, around a cylindrical potato sample

Description of microbial inactivation kinetics of *Listeria monocytogenes*
(Valdramidis et al., Journal of Food Engineering, 2006, 76, 79)

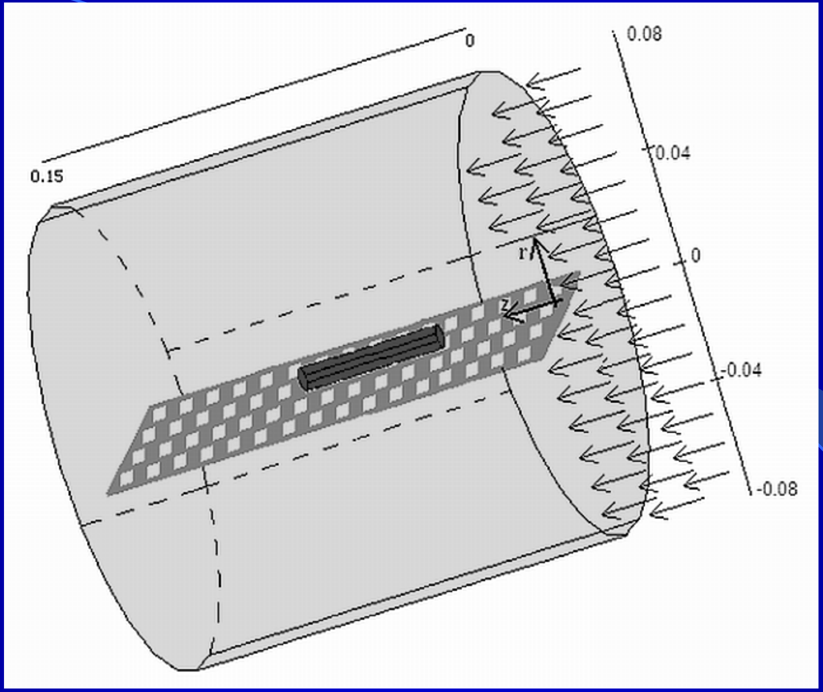
Prediction of the time evolution of dried potato quality expressed in terms of the color degradation (Krokida et al., DRYING TECHNOLOGY, 1998, 16(3-5), 667)

Formulation of a general model given as the combination of the transport model, of the product decontamination model and of the model aimed at predicting the kinetics of color changes occurring during drying.

Aim of the work

Identification, on the basis of a dynamic optimization algorithm, of a set of operating conditions that are to be chosen so as to achieve, at the same time, high-quality and safe dried foods.

Formulation of the transport model



The unsteady-state momentum transfer referred to the drying air was expressed in terms of the k-ω model

Besides liquid water and energy conservation, also the transport of vapor was accounted for (multiphase approach)

The proposed model did not rely on the specification of the interfacial heat and mass transfer coefficients (continuity of both heat and mass fluxes at the food/air interfaces)

Food sample

Conservation equations for liquid water and vapor

$$\partial C_w \big/ \partial t + \underline{\nabla} \cdot \left(- D_w \underline{\nabla} C_w \right) + \dot{I} = 0$$

$$\partial C_v \big/ \partial t + \underline{\nabla} \cdot \left(- D_v \underline{\nabla} C_v \right) - \dot{I} = 0$$

Energy conservation

$$\rho_s C_{p_s} \partial T \big/ \partial t - \underline{\nabla} \cdot \left(k_{eff} \underline{\nabla} T \right) + \lambda \cdot \dot{I} = 0$$

$\dot{I}$ Volumetric rate of evaporation

Main hypotheses:

- vapor and liquid water in phase equilibrium at any time
- vapor pressure function of the local values of temperature and moisture content
- evaporation occurred over the entire food domain and also at food outer surfaces
- convective transport negligible
- the conservation equation referred to air transport negligible

The above equations were coupled, by a set of boundary conditions, expressing the continuity at food/air interfaces, to the conservation equations in the air

No heat/mass transfer coefficient is, therefore, needed; the proposed approach is useful when food shape changes irregularly with time (shrinkage)

Drying air

Momentum balance and continuity equation k-ω model (Wilcox)

$$\partial\rho_a/\partial t + \underline{\nabla}\cdot\rho_a\underline{u} = 0$$

$$\rho_a\,\partial\underline{u}/\partial t + \rho_a\underline{u}\cdot\underline{\nabla u} + \underline{\nabla}\cdot\left(\overline{\rho_a\underline{u}'\otimes\underline{u}'}\right) = -\underline{\nabla}p + \underline{\nabla}\cdot\left[\eta_a\left(\underline{\nabla u} + (\underline{\nabla u})^T\right)\right]$$

Turbulent kinetic energy

$$\rho_a\,\partial k/\partial t + \rho_a\underline{u}\cdot\underline{\nabla}k = \underline{\nabla}\cdot\left[(\eta_a + \sigma_k\eta_t)(\underline{\nabla}k)\right] + \frac{\eta_t}{2}\left(\underline{\nabla u} + (\underline{\nabla u})^T\right)^2 - \beta_k\rho_a k\omega$$

Dissipation per unit turbulent kinetic energy

$$\rho_a\,\partial\omega/\partial t + \rho_a\underline{u}\cdot\underline{\nabla}\omega = \underline{\nabla}\cdot\left[(\eta_a + \sigma_\omega\eta_t)(\underline{\nabla}\omega)\right] + (\alpha\omega/2k)\eta_t\left(\underline{\nabla u} + (\underline{\nabla u})^T\right)^2 - \beta\rho_a\omega^2$$

Where the eddy viscosity was

$$\eta_t = \rho_a k/\omega$$

Mass balance referred to vapor and Energy conservation

$$\partial C_2/\partial t + \underline{\nabla}\cdot\left(-D_a\underline{\nabla}C_2\right) + \underline{u}\cdot\underline{\nabla}C_2 = 0$$

$$\rho_a C_{pa}\,\partial T_2/\partial t - \underline{\nabla}\cdot\left(k_a\underline{\nabla}T_2\right) + \rho_a C_{pa}\underline{u}\cdot\underline{\nabla}T_2 = 0$$

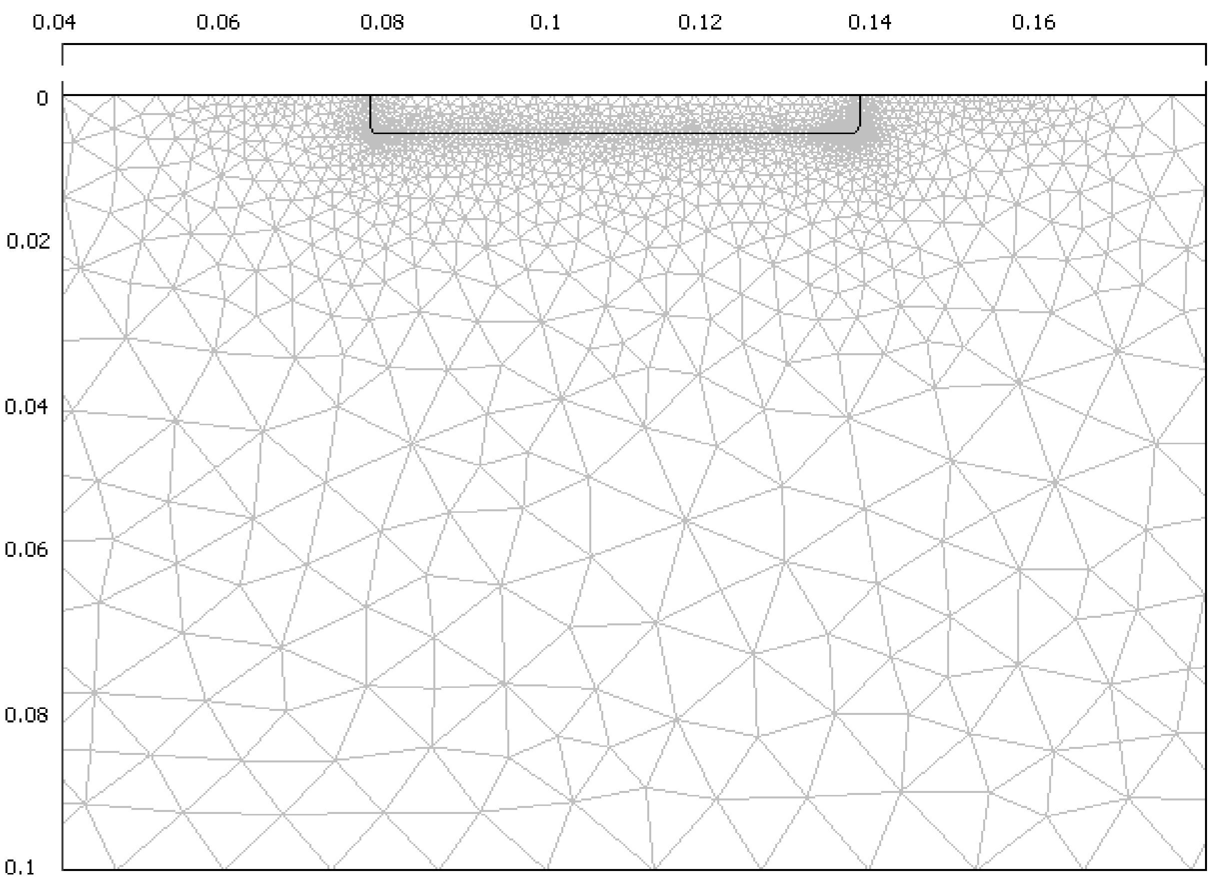
System of non-linear PDEs solved by FEM (Comsol Multiphysics)

Total number of 16842 triangular finite elements leading to about 142000 DOFs.

The mesh consisted of 8505 and 7977 elements for food and air domains, respectively.

The considered mesh provided a good spatial resolution and the solution was independent on the grid size even with further refinements.

Lagrange finite elements of order two were chosen for all the variables.



The product decontamination was described considering the microbial inactivation kinetics of Listeria monocytogenes (Valdramidis et al., Journal of Food Engineering, 2006, 76, 79)

The kinetics of color changes was described in terms of the so called Hunder parameters: C was each of the color parameter (a, b, L), ((Krokida et al., DRYING TECHNOLOGY, 1998, 16(3-5), 667)

$$\frac{dN}{dt} = -k_{\max} \cdot \left(\frac{1}{1+Cc} \right) \cdot N$$

$$\frac{dCc}{dt} = -k_{\max} \cdot Cc$$

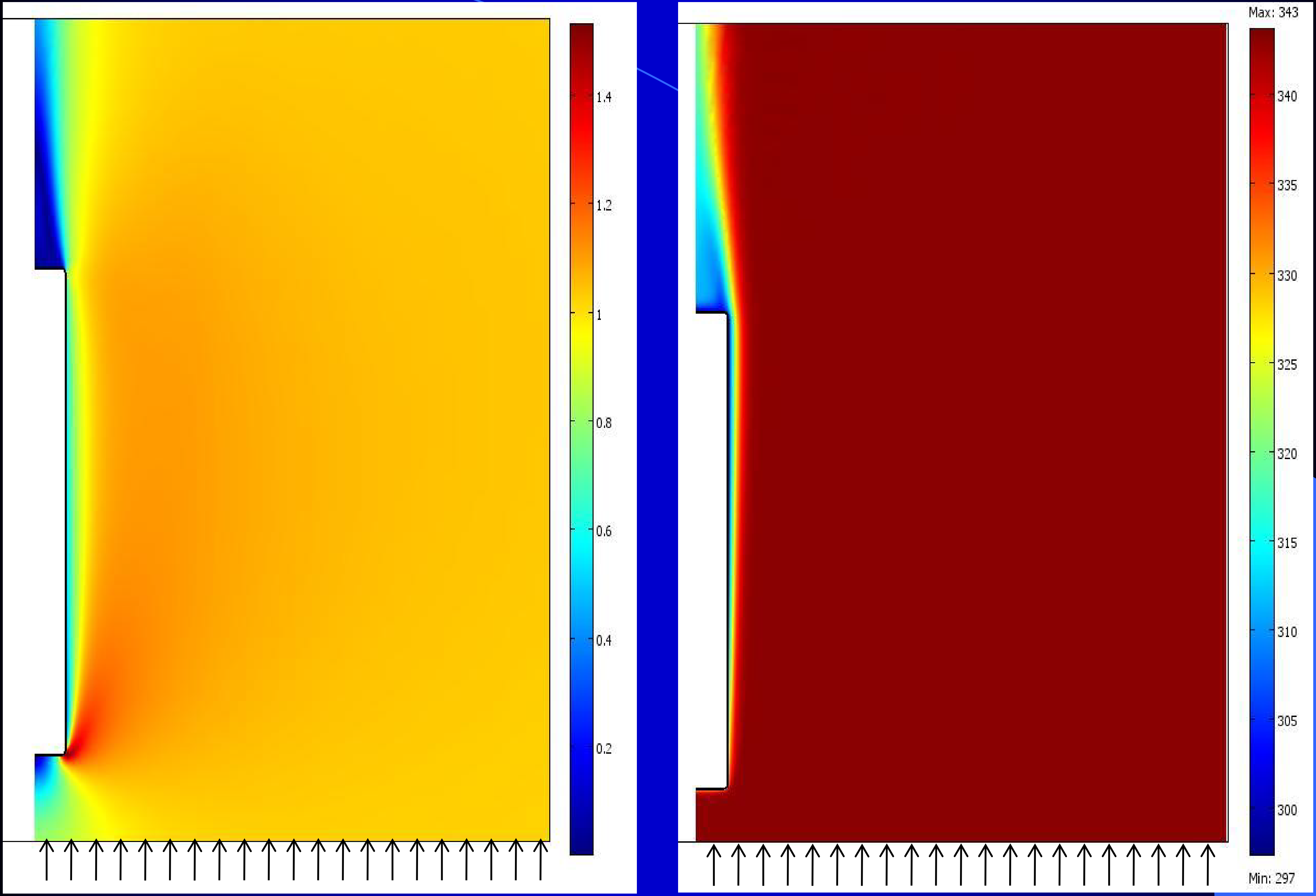
$$k_{\max}(T, a_w) = \frac{\ln 10}{1.8} \exp\left(\frac{\ln 10}{7.11}(T-60) \right) \cdot \exp\left(\frac{\ln 10}{0.231}(a_w - 1) \right)$$

$$\frac{C - C_e}{C_0 - C_e} = \exp(-k_c t)$$

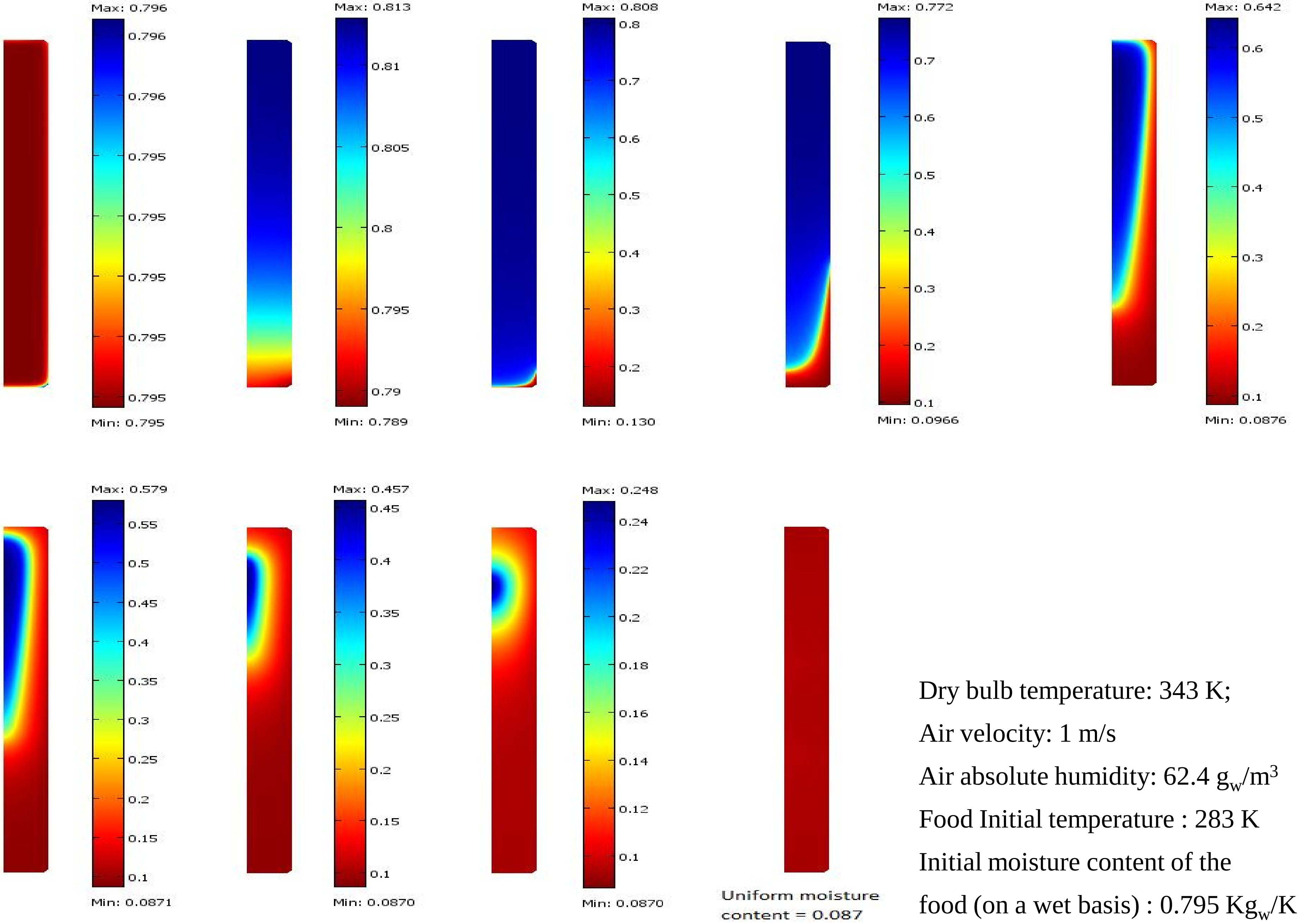
$$Ce = C_{e0}(T_a / 70)^{a_T} (H / 30)^{a_H}$$

$$k_c = K_{c0}(T_a / 70)^{m_T} (H / 30)^{m_H}$$

Velocity field and temperature profile close to food sample



Time evolution of potato moisture content (on a wet basis)



Dry bulb temperature: 343 K;

Air velocity: 1 m/s

Air absolute humidity: 62.4 g_w/m^3

Food Initial temperature : 283 K

Initial moisture content of the food (on a wet basis) : 0.795 Kg_w/Kg

Model exploitation

The proposed transport model predicts the time evolutions of both moisture and temperature distributions. This allows characterizing drying process behavior as a function of the operating conditions and detecting, for instance, the regions where high values of moisture content can promote microbial spoilage.

In general, the exploitation of drastic operating conditions, namely high dry bulb temperatures and low values of relative humidity, has the following effects:

- the drying rate is high and the process goes to completion faster;

- the reduction of microbial population is more efficient and food safety is improved;

the organoleptic characteristics of food tend to deteriorate, thus determining a significant worsening of dried products quality.

On the contrary, the utilization of mild or very mild operating conditions determines an opposite behavior:

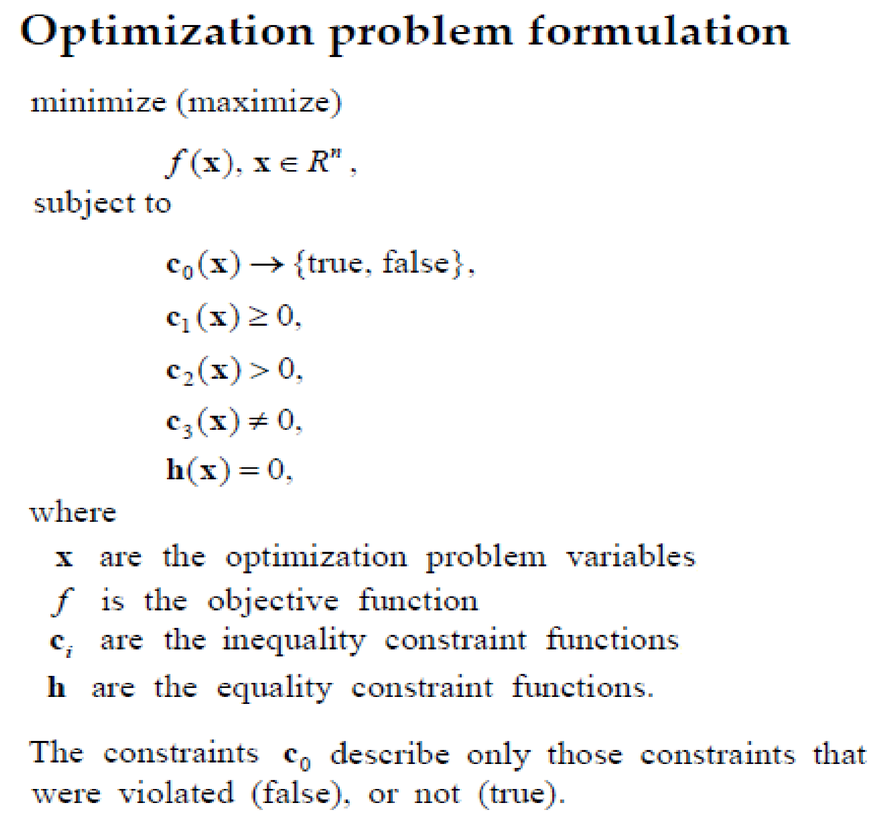
the organoleptic properties of dried foods do certainly improve, but a proper decontamination of the final product could not be assured.

It is, therefore, necessary to identify a definite set of operating conditions, possibly changing during drying process, that has to be chosen so as to achieve, at the same time, high quality and safe dried foods.

Optimization Problem

Five elements are common to all optimization problems:

- **the identification of the design variables that are be controlled;**
- **the mathematical model describing the process behavior;**
- **the requirements that must be met (constraints or restrictions);**
- **the definition of the objective function (the mathematical expression of what is to be optimized);**
- **a proper optimization technique .**



Optimization problem formulation

minimize (maximize)

$$f(\mathbf{x}),\ \mathbf{x} \in R^n,$$

subject to

$$\mathbf{c}_0(\mathbf{x}) \to \{\text{true, false}\},$$
$$\mathbf{c}_1(\mathbf{x}) \geq 0,$$
$$\mathbf{c}_2(\mathbf{x}) > 0,$$
$$\mathbf{c}_3(\mathbf{x}) \neq 0,$$
$$\mathbf{h}(\mathbf{x}) = 0,$$

where

$\mathbf{x}$ are the optimization problem variables
f is the objective function
$\mathbf{c}_i$ are the inequality constraint functions
$\mathbf{h}$ are the equality constraint functions.

The constraints $\mathbf{c}_0$ describe only those constraints that were violated (false), or not (true).

Formulation of a general optimization model

Identification of the design variables that are to be controlled:

Average moisture content of the food; microbial population, food quality (color degradation). Also the input variables are known or can be fixed

Mathematical model describing the process behavior:

The already-described combination of the transport phenomena model, of the product decontamination model predicting the microbial inactivation kinetics of *Listeria monocytogenes* and of the model describing the kinetics of color changes

Requirements that must be met:

Proper decontamination of the final product (attention to the critical points of each exposed surface, in particular the rear one); high organoleptic properties of dried potatoes; attainment of a limit value of food moisture content corresponding to water activity values smaller than a definite threshold

Definition of the objective function:

Analysis of different scenarios

The optimization technique:

A derivative-free method CDOS (Conjugate Direction with Orthogonal Shift) available in Maple → Proper integration between Comsol and Maple

Analysis of different scenarios

Minimize the color difference Δb (yellowness);
t ∈ [0, 180 min], H ∈ [20, 50], Ta ∈ [323 K, 363 K];
Subject to : $^{N}/_{N0} \le 10^{-6}$; $\overline{X_b} \le 0.75$; Δa≤3

t = 174 min, H = 28.3%,
Ta = 350 K; Δb=4.42
$^{N}/_{N0} = 10^{-6}$; $\overline{X_b} = 0.75$;
Δa=3

Minimize the color difference Δa (redness);
t ∈ [0, 180 min], H ∈ [20, 50], Ta ∈ [323 K, 363 K];
Subject to : $^{N}/_{N0} \le 10^{-6}$; $\overline{X_b} \le 0.75$; Δb≤ 3.5

t = 162 min, H = 25%,
Ta = 360.15 K; Δa=4.13
$^{N}/_{N0} = 2.21\ 10^{-33}$;
$\overline{X_b} = 0.75$; Δb=3.5

Minimize $\overline{X_b}$;
t ∈ [0, 180 min], H ∈ [20, 50], Ta ∈ [323 K, 363 K];
Subject to : $^{N}/_{N0} \le 10^{-6}$; Δa ≤ 3; Δb≤ 3.5

t = 179 min, H = 25.6%,
Ta = 356.27 K; $\overline{X_b} = 0.66$
$^{N}/_{N0} = 1.42\ 10^{-16}$;
Δa = 4.0; Δb=4.0

Step-by-step Optimization

The complete time horizon was subdivided into N subintervals:
$I_1=[0, t_1]$, ….. , $I_i = [t_{i-1}, t_i]$, $I_N = [t_{N-1}, 180]$
Minimize $\overline{X_{b,i}}$ in each subinterval and determine $t_i \in I_i$, $H_i \in [20, 50]$,
$Ta_i \in [323\text{ K}, 363\text{ K}]$;
Subject to : $^{N_i}/_{N0} \le \alpha_i$; $\Delta a_i \le \beta_i$; $\Delta b_i \le \gamma_i$
where $\alpha_i, \beta_i, \gamma_i$ defined according to a specific pattern

Conclusions and Future Developments

A general predictive tool given as a combination of a transport phenomena model, of a product decontamination model and of a model describing the kinetics of color changes (quality parameter) was developed

An optimization model was also formulated; a set of operating conditions that allows attaining specific control objectives represented by the determination of a trade-off condition between quality and safety was determined. It is therefore possible to minimize expensive and time-consuming pilot test-runs.

Experimental validation of the predictions provided by the developed optimization model

Formulation of a complete model accounting also for other quality parameters (e.g. shrinkage). The work is actually in progress (Stress-strain analysis coupled to momentum, heat and mass transfer in a time-dependent deformed mesh – ALE procedure)

Formulation of a more general optimization model accounting also for process economics

Thank you
for your kind
attention