數值分析於大地工程上的應用
Application of numerical simulation in geotechnical engineering

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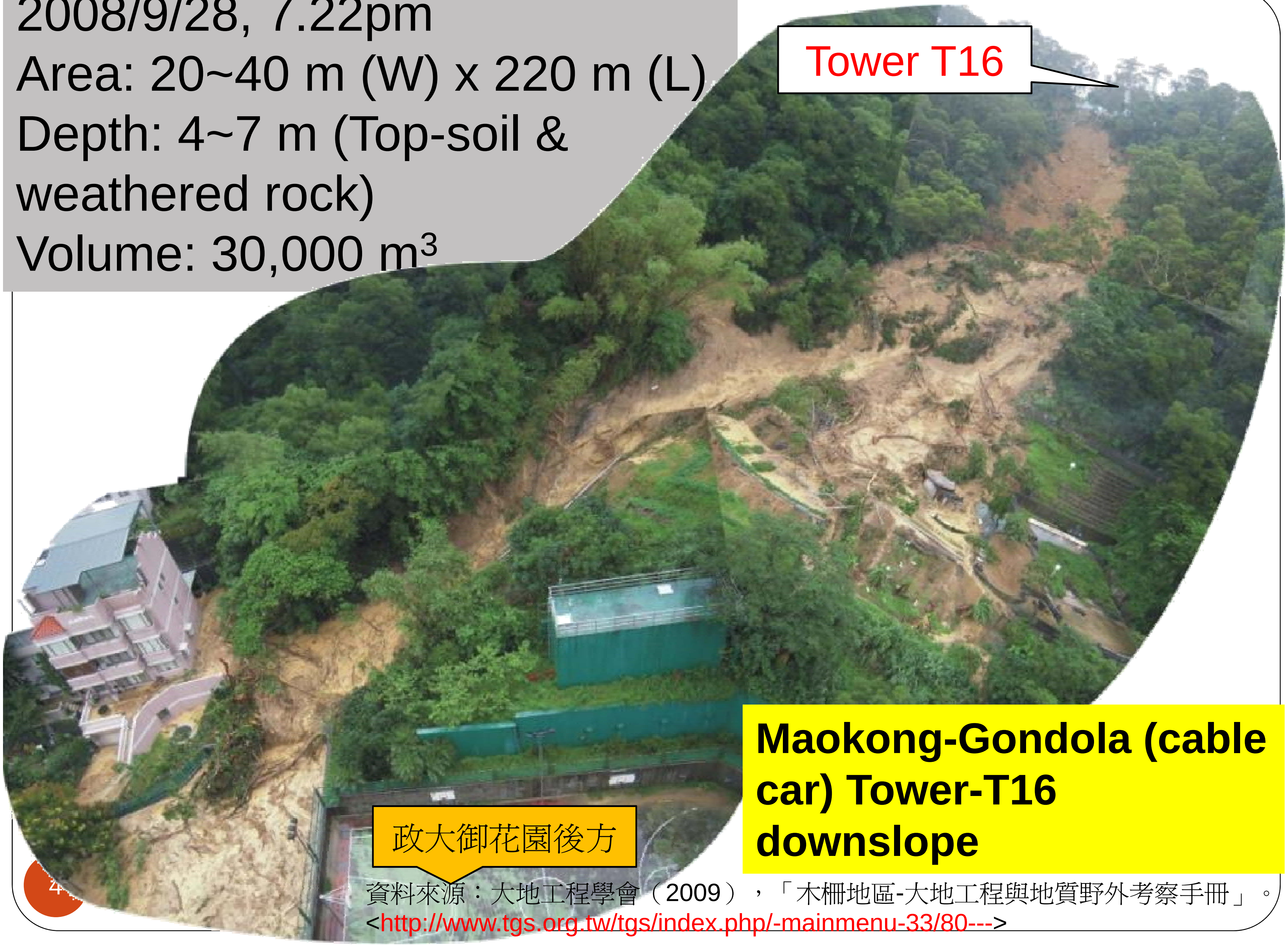
Outlines:

1. Introduction
2. Theoretical background
3. Verification of formulation
4. Real case study
5. Conclusions

Introduction

2008/9/28, 7.22pm
Area: 20~40 m (W) x 220 m (L)
Depth: 4~7 m (Top-soil & weathered rock)
Volume: 30,000 m^3

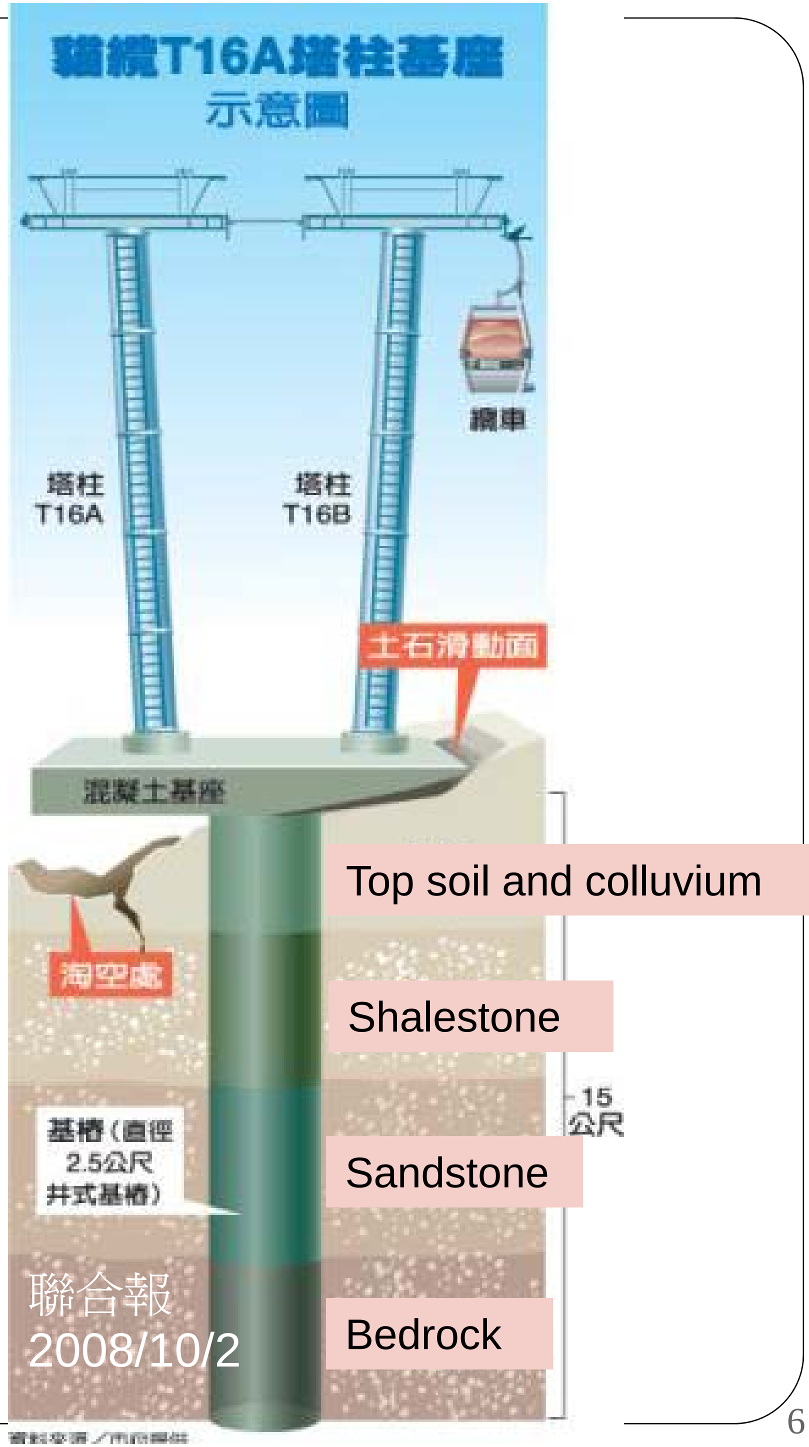


Maokong-Gondola (cable car) Tower-T16 downslope

資料來源：大地工程學會（2009），「木柵地區-大地工程與地質野外考察手冊」。
<http://www.tgs.org.tw/tgs/index.php/-mainmenu-33/80--->


齊柏林

聯合報
2008/12/1



貓纜T16A塔柱基座
示意圖
塔柱
T16A
塔柱
T16B
纜車
土石滑動面
混凝土基座
淘空處
基樁（直徑
2.5公尺
井式基樁）
15
公尺
Top soil and colluvium
Shalestone
Sandstone
Bedrock
聯合報
2008/10/2

Theoretical Background

Equilibrium eqn for saturated soil:

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} = 0$$
$$\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{zy}}{\partial z} = 0$$
$$\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \sigma_z}{\partial z} = -\gamma$$

Effective stress for unsaturated soil:

$$\sigma' = (\sigma - u_a) + \beta(u_a - u_w)$$

Compatibility equation:

$$\begin{cases} \varepsilon_x = \frac{\partial u}{\partial x} \; ; \; \varepsilon_y = \frac{\partial v}{\partial y} \; ; \; \varepsilon_x = \frac{\partial w}{\partial z} \\ \gamma_{yz} = \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial z}\right) \; ; \; \gamma_{xz} = \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right) \\ \gamma_{xy} = \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right) \; ; \; \tau = G\gamma \end{cases}$$

Soil structure constitutive relations for unsaturated soil:

$$\sigma_x = \frac{E}{(1+\mu)}\varepsilon_x + \frac{E\mu\varepsilon_v}{(1+\mu)(1-2\mu)} + u_a - \beta(u_a - u_w)$$
$$\sigma_y = \frac{E}{(1+\mu)}\varepsilon_y + \frac{E\mu\varepsilon_v}{(1+\mu)(1-2\mu)} + u_a - \beta(u_a - u_w)$$
$$\sigma_z = \frac{E}{(1+\mu)}\varepsilon_z + \frac{E\mu\varepsilon_v}{(1+\mu)(1-2\mu)} + u_a - \beta(u_a - u_w)$$

$$G = \frac{E}{2(1+\mu)}; \; \beta = \frac{E}{H(1-2\mu)}$$

Equilibrium DE in terms of $(u_a - u_w)$ and ε_v for unsaturated soil:

$$G\nabla^2 u + \frac{G}{1-2\mu}\frac{\partial \varepsilon_v}{\partial x} - \beta\frac{\partial (u_a - u_w)}{\partial x} + \frac{\partial u_a}{\partial x} = 0$$

$$G\nabla^2 v + \frac{G}{1-2\mu}\frac{\partial \varepsilon_v}{\partial y} - \beta\frac{\partial (u_a - u_w)}{\partial y} + \frac{\partial u_a}{\partial y} = 0$$

$$G\nabla^2 w + \frac{G}{1-2\mu}\frac{\partial \varepsilon_v}{\partial z} - \beta\frac{\partial (u_a - u_w)}{\partial z} + \frac{\partial u_a}{\partial z} = -\gamma$$

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

u, v, w, (u_a-u_w) = four unknowns to solve for but with only three equations

Continuity equation eqn for water flow through an unsaturated soil:

$$\nabla \cdot \left[k_w^x \frac{\partial h}{\partial x} + k_w^y \frac{\partial h}{\partial y} + k_w^z \frac{\partial h}{\partial z}\right] = \left(\frac{m_1^w}{m_2^s}\right)\frac{\partial \varepsilon_v}{\partial t} + \gamma_w \left(m_2^w - \frac{m_1^w m_2^s}{m_1^s}\right)\frac{\partial h}{\partial t}$$

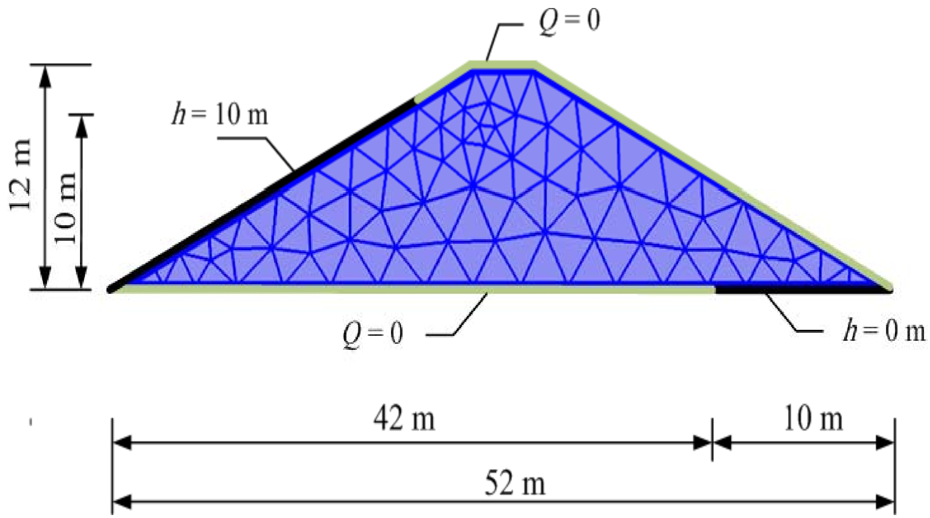
$$m_1^s = \frac{d\varepsilon_v}{d(\sigma_{mean} - u_a)} = \frac{1}{(1+e_0)}\frac{de}{d(\sigma_{mean} - u_a)}$$

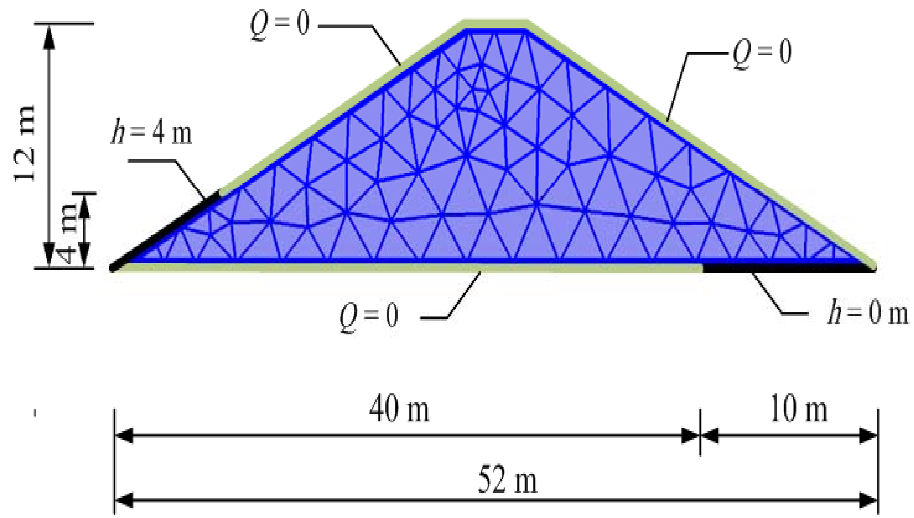
$$m_2^s = \frac{d\varepsilon_v}{d(u_a - u_w)} = \frac{1}{(1+e_0)}\frac{de}{d(u_a - u_w)}$$

$$m_1^w = \frac{\partial \theta_w}{\partial(\sigma_{mean} - u_a)}\frac{S}{(1+e_0)}\frac{de}{d(\sigma_{mean} - u_a)}\frac{e}{(1+e_0)}\frac{dS}{d(\sigma_{mean} - u_a)}$$

$$m_2^w = \frac{\partial \theta_w}{\partial(u_a - u_w)} = \frac{S}{(1+e_0)}\frac{de}{d(u_a - u_w)} + \frac{e}{(1+e_0)}\frac{dS}{d(u_a - u_w)}$$

Verification



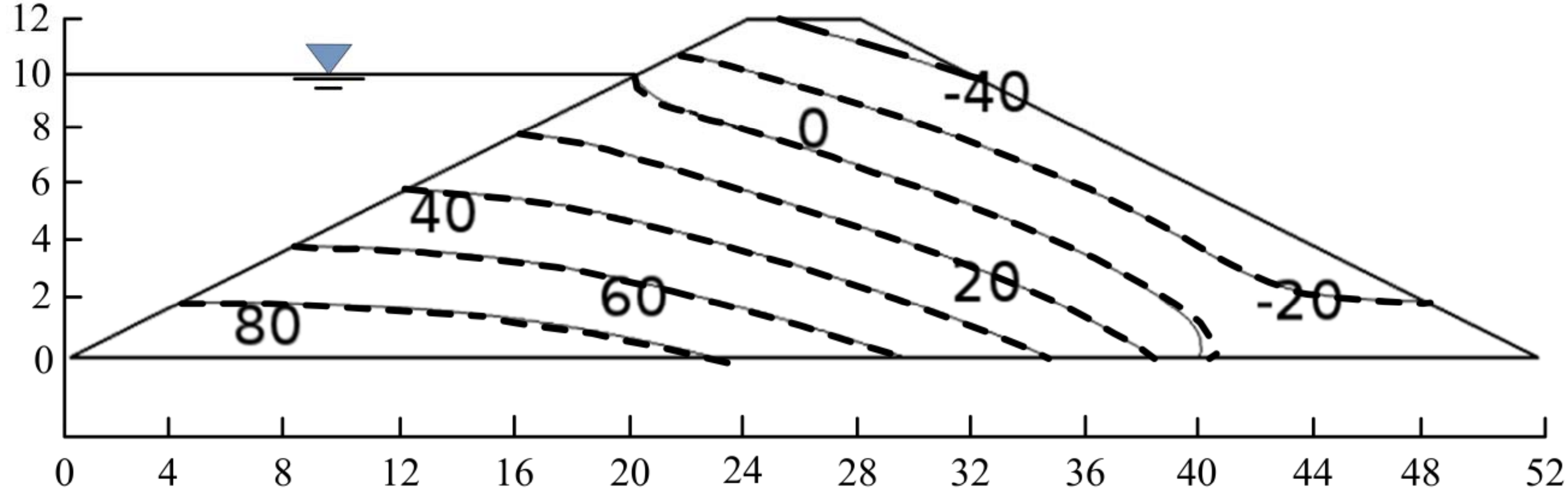


$$\frac{\partial}{\partial x}\left(k_x \frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(k_y \frac{\partial h}{\partial y}\right) = m_2^w \rho_w g \frac{\partial h}{\partial t}$$

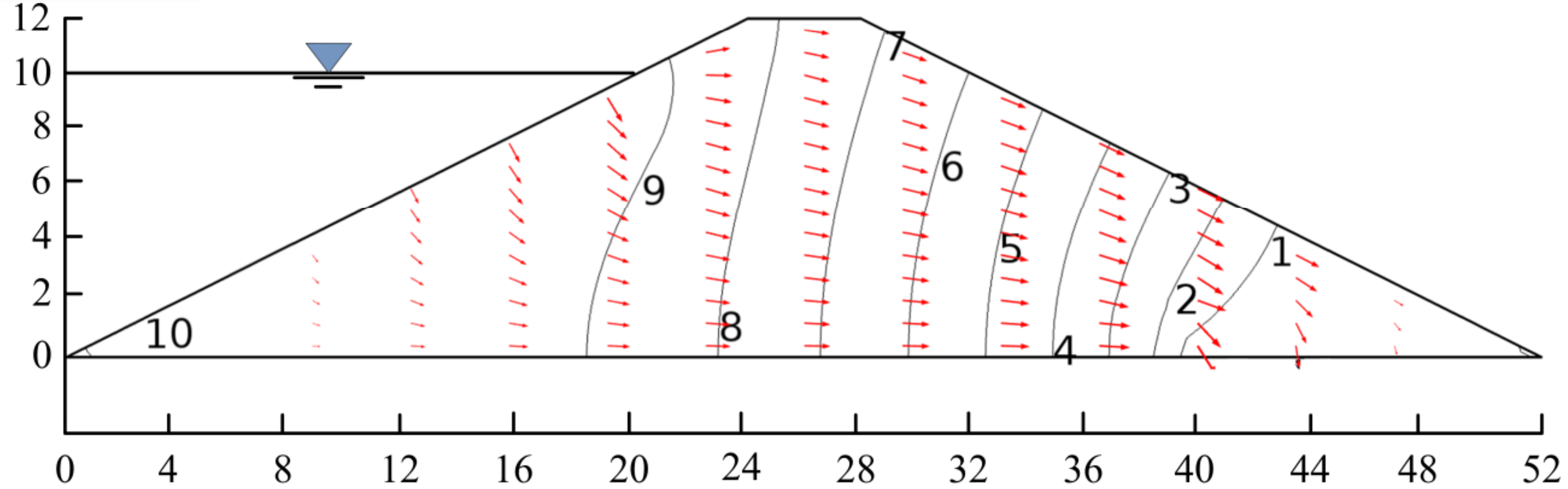
van Genuchten (1980)	回歸係數		
	a	**n**	**m**
儲水方程式： $m_2^w = -\frac{\theta_s}{(1+(a\psi)^n)^m}(a\psi)^n \frac{mn}{\psi(1+(a\psi)^n)}$ $\theta_s = 0.381$	4.391×10[-2]	46.754	0.036
非飽和滲透係數方程式： $k_w = k_{sat}\left[\frac{[1-(a\psi)^{n-1}[1+(a\psi)^n]^{-m}]^2}{[1+(a\psi)^n]^{\frac{m}{2}}}\right]$ $k_{sat} = 2.5\times10^{-7}$ (m/sec)	4.049×10[-2]	8.852	0.887

穩態-蓄水面 10 m
（孔隙水壓力）

Thieu (2001) 虛線
Simulation 實線

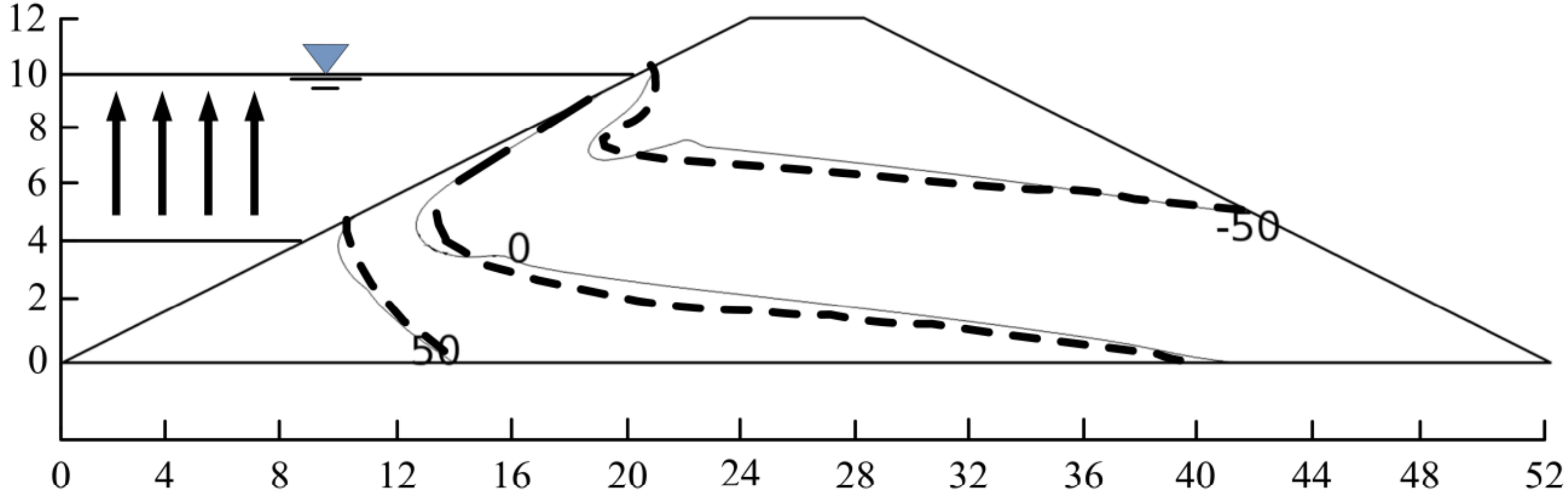


等勢能線(實線)
流線(箭頭)

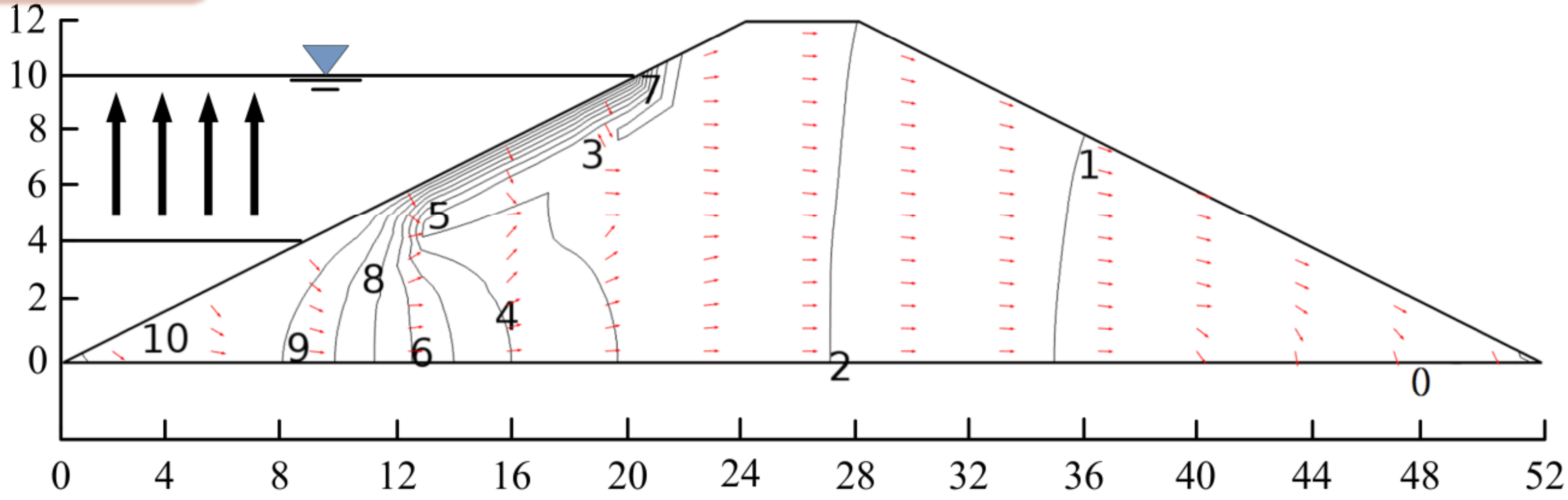


暫態-蓄水面 4 m 上升至 10 m
（孔隙水壓力）

Thieu (2001) 虛線
Simulation 實線

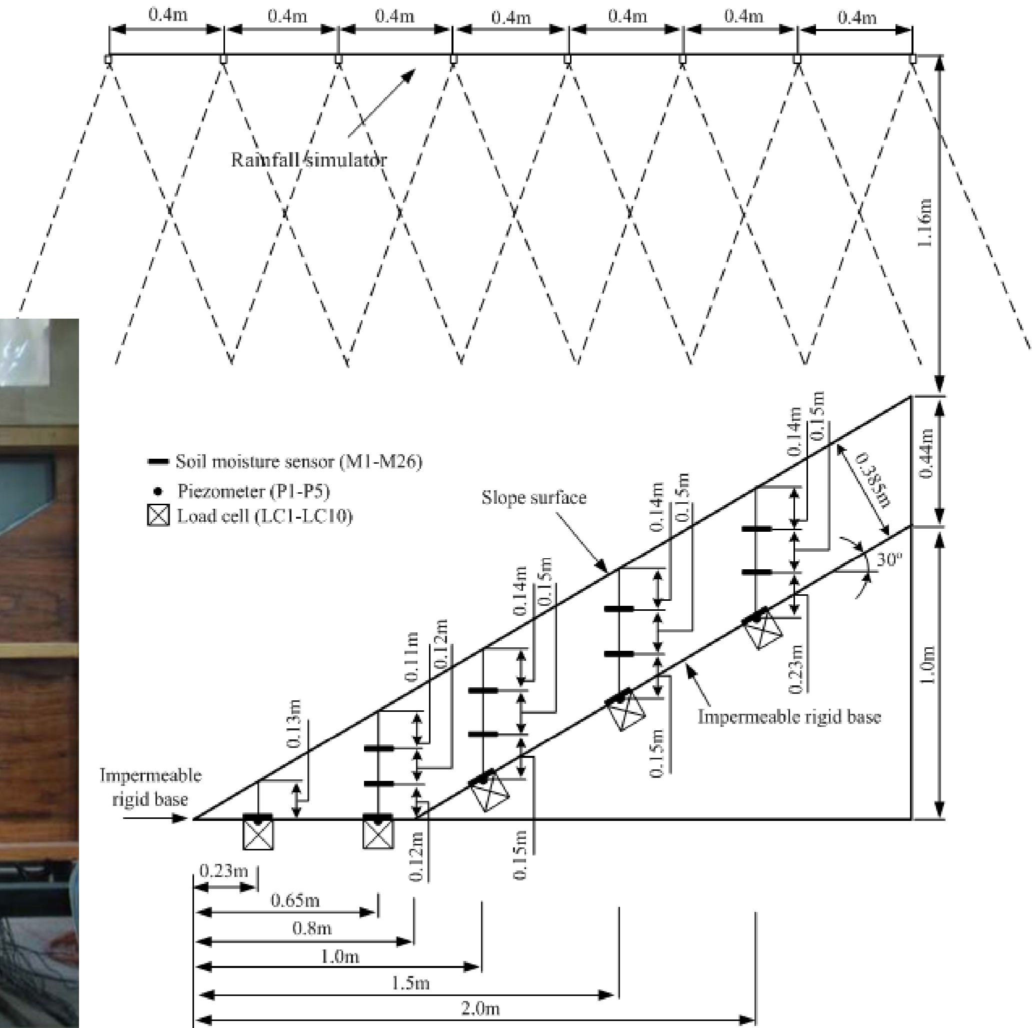


等勢能線(實線)
流線(箭頭)

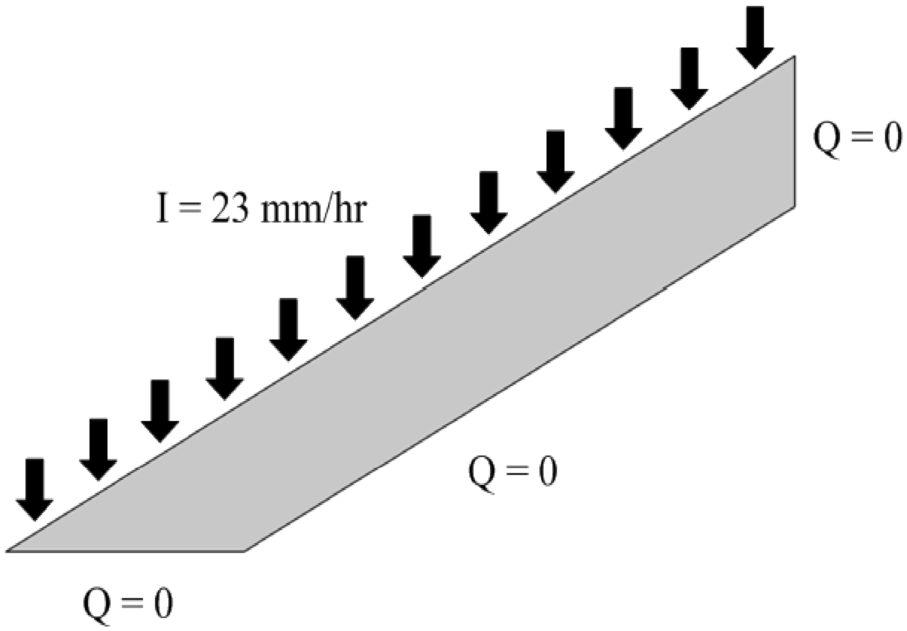


Simulation of
Laboratory Model Test

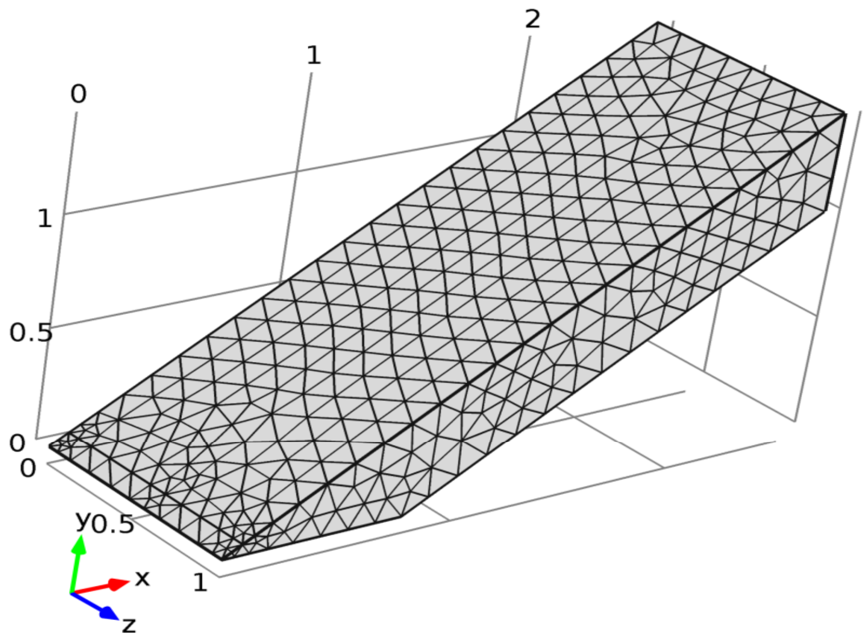
Simulate Lan's (2008) man-made rainfall slope model test and compared the simulated and experimental distribution of degree of saturation (water content).



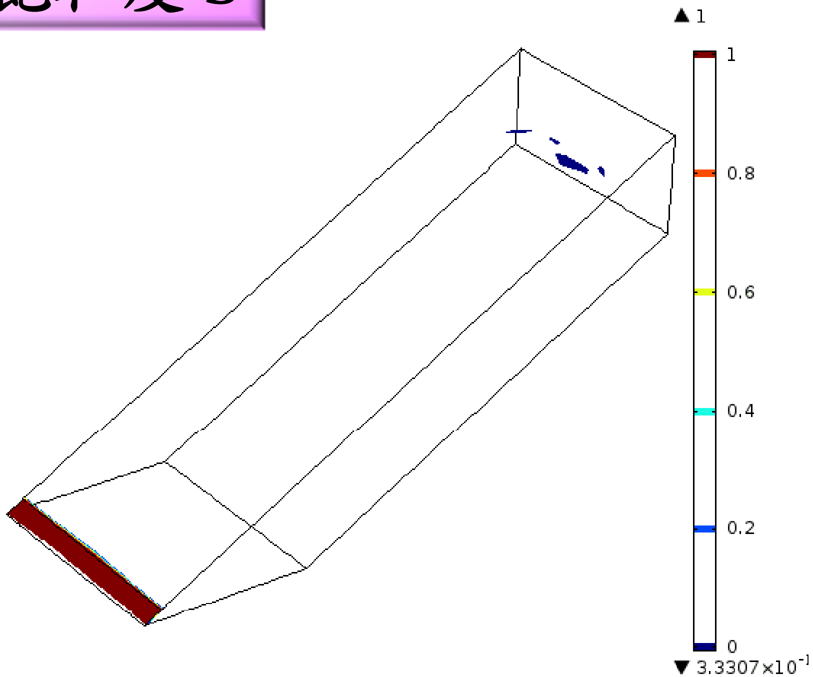
邊坡模型邊界狀態



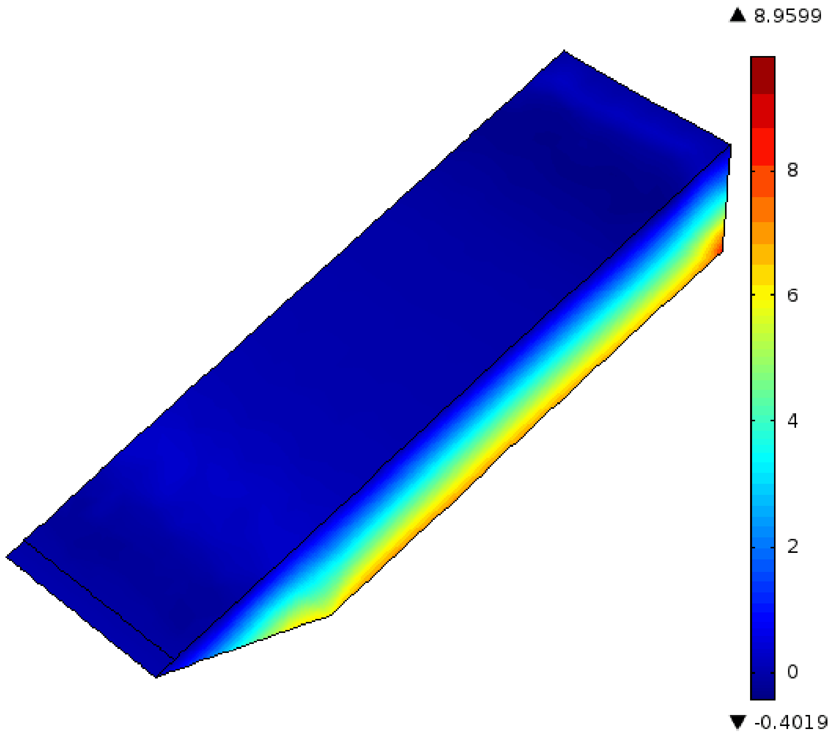
網格



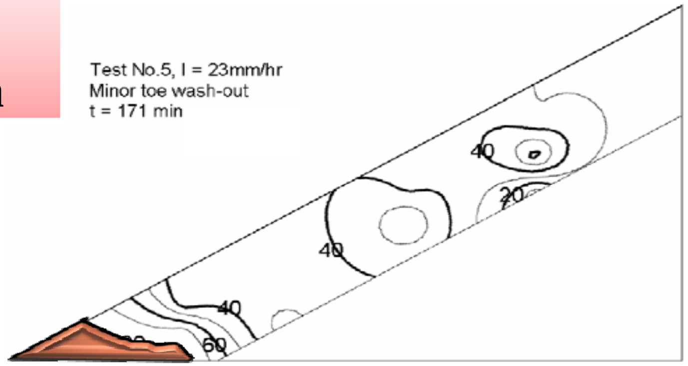
飽和度 S



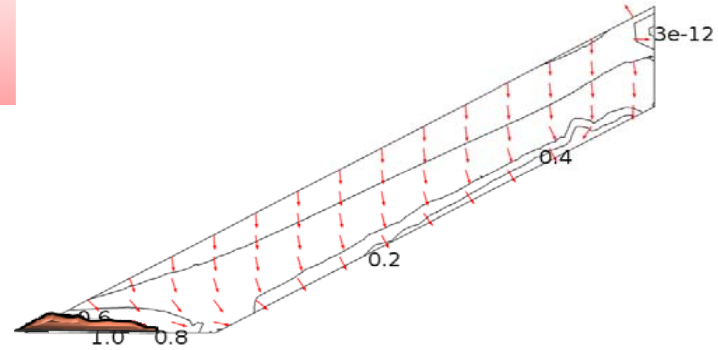
正向應力 kPa ＋ 總位移



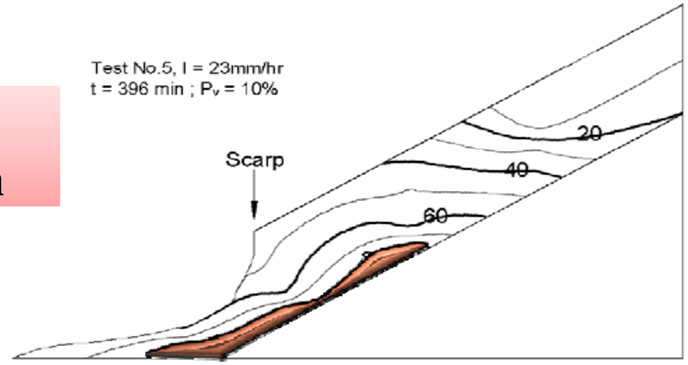
藍少村
171 min



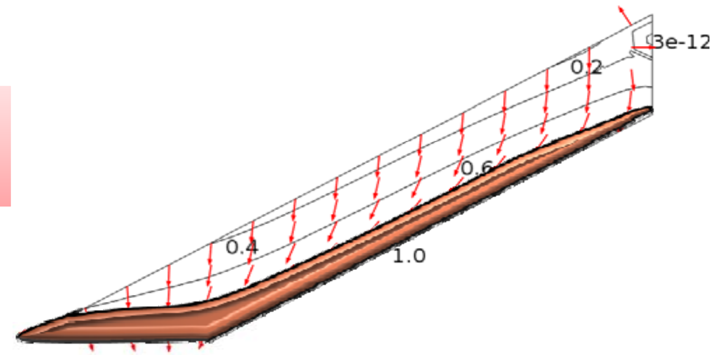
Simulation
171 min



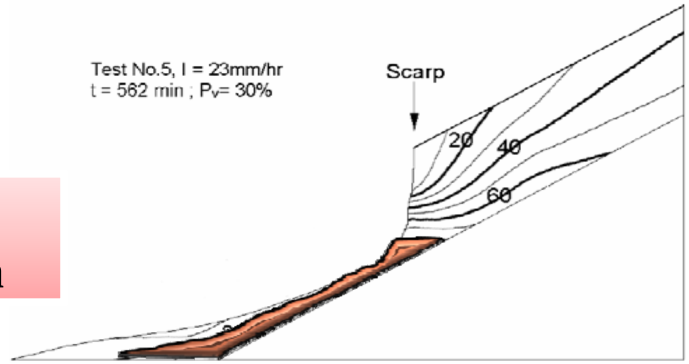
藍少村
396 min



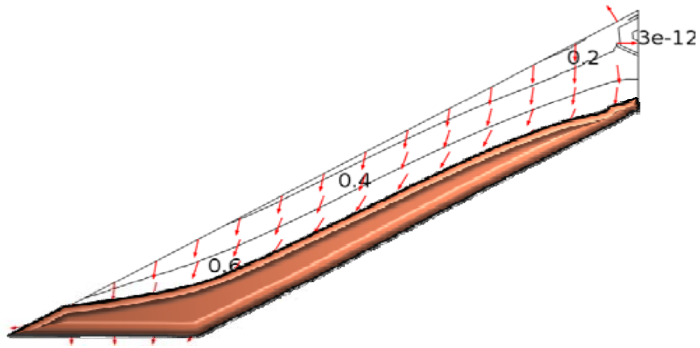
Simulation
396 min



藍少村
562 min

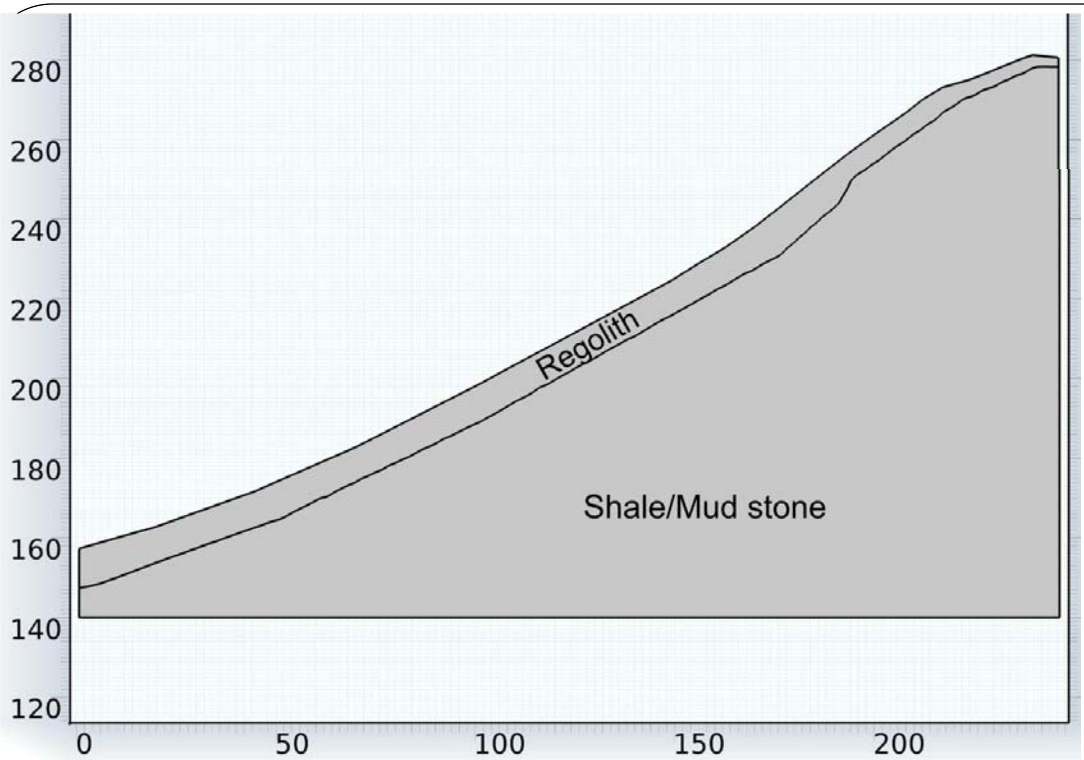


Simulation
562 min

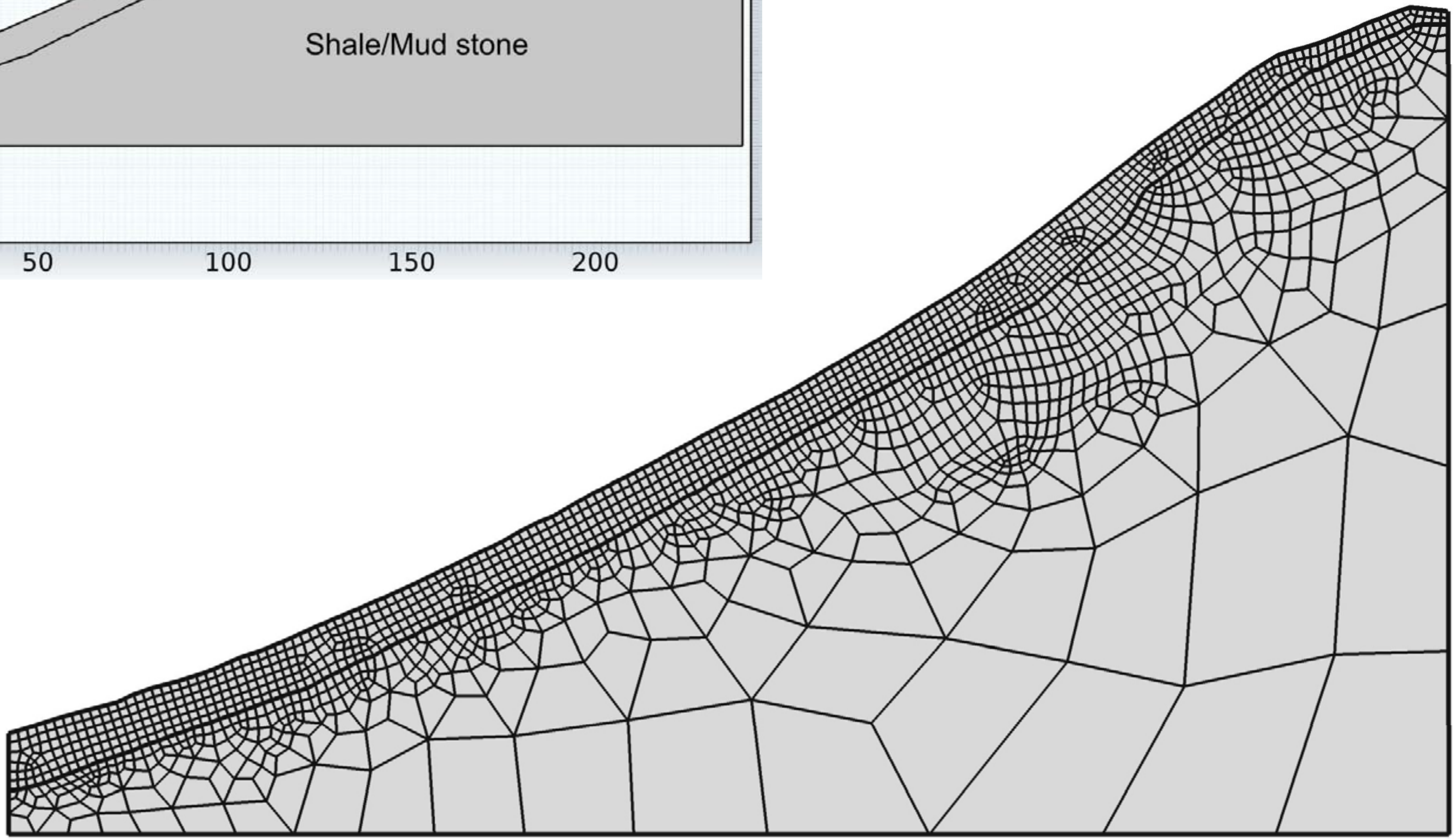


Real Case Study

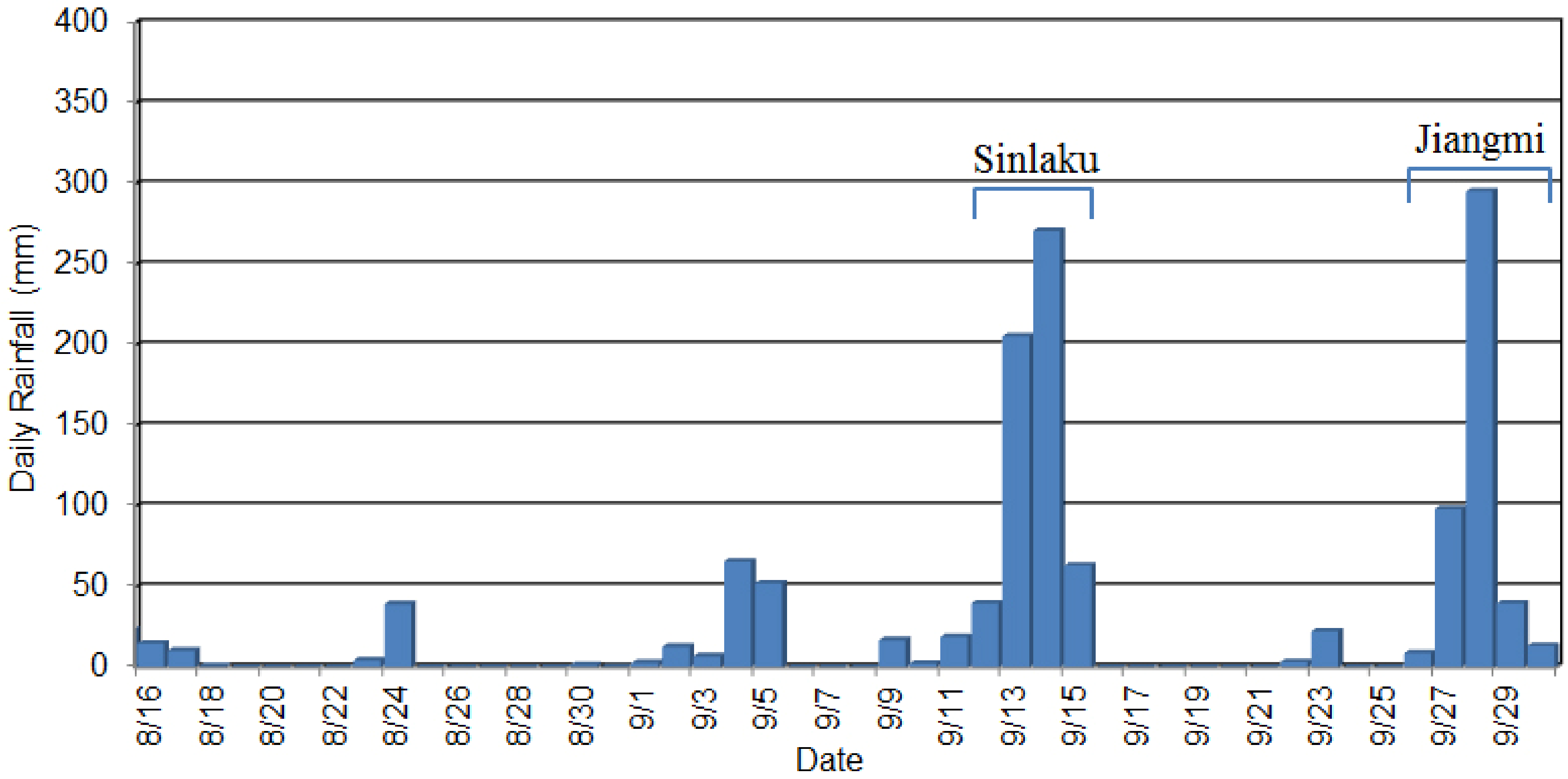
Simulation of Maokong Landslide



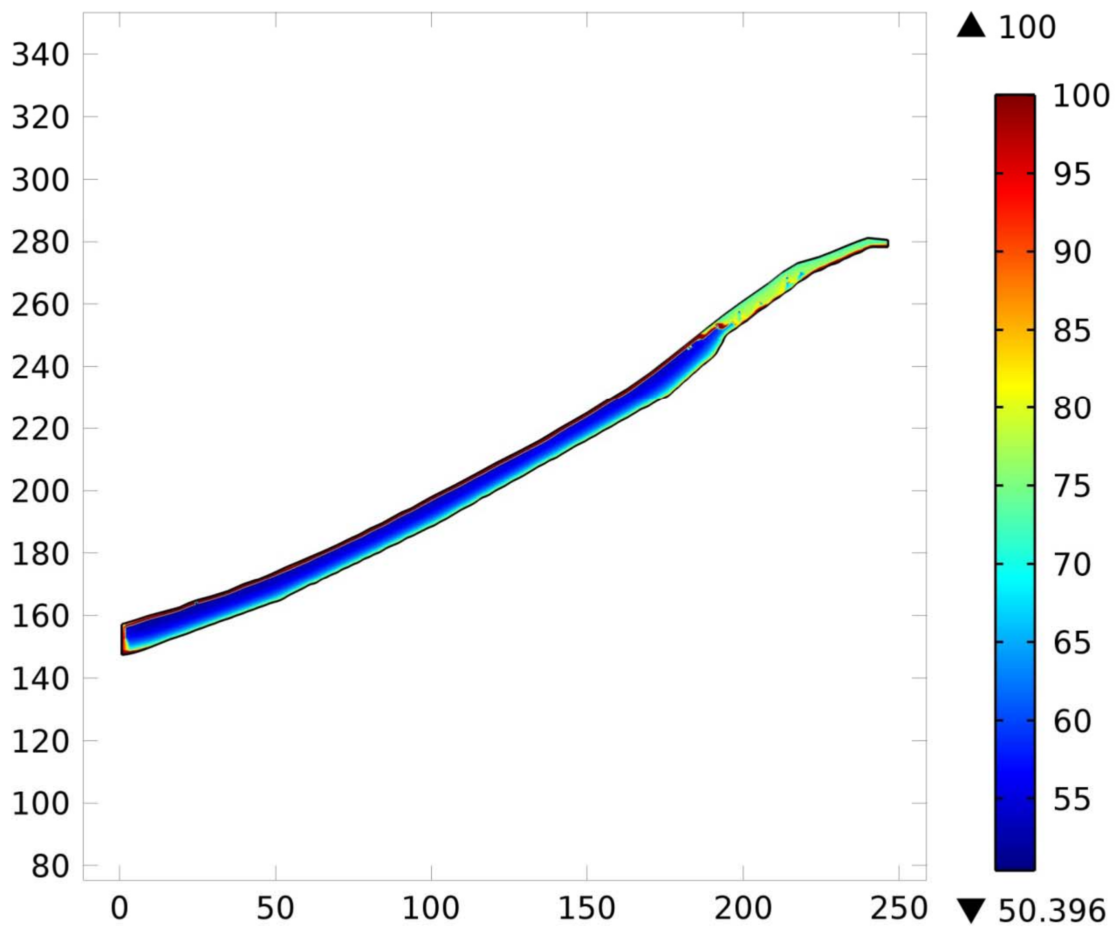
280
260
240
220
200
180
160
140
120
0
50
100
150
200
Regolith
Shale/Mud stone



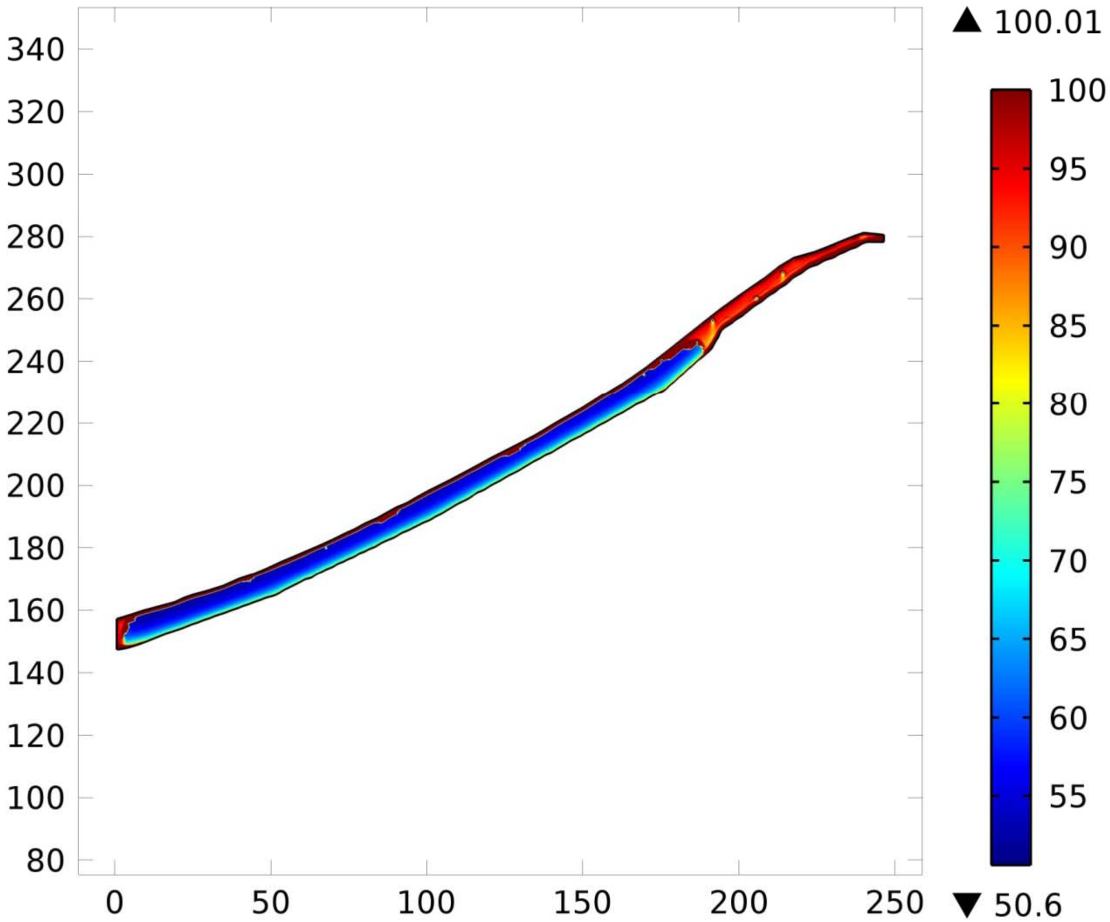
Daily rainfall recorded between 8/1-9/30



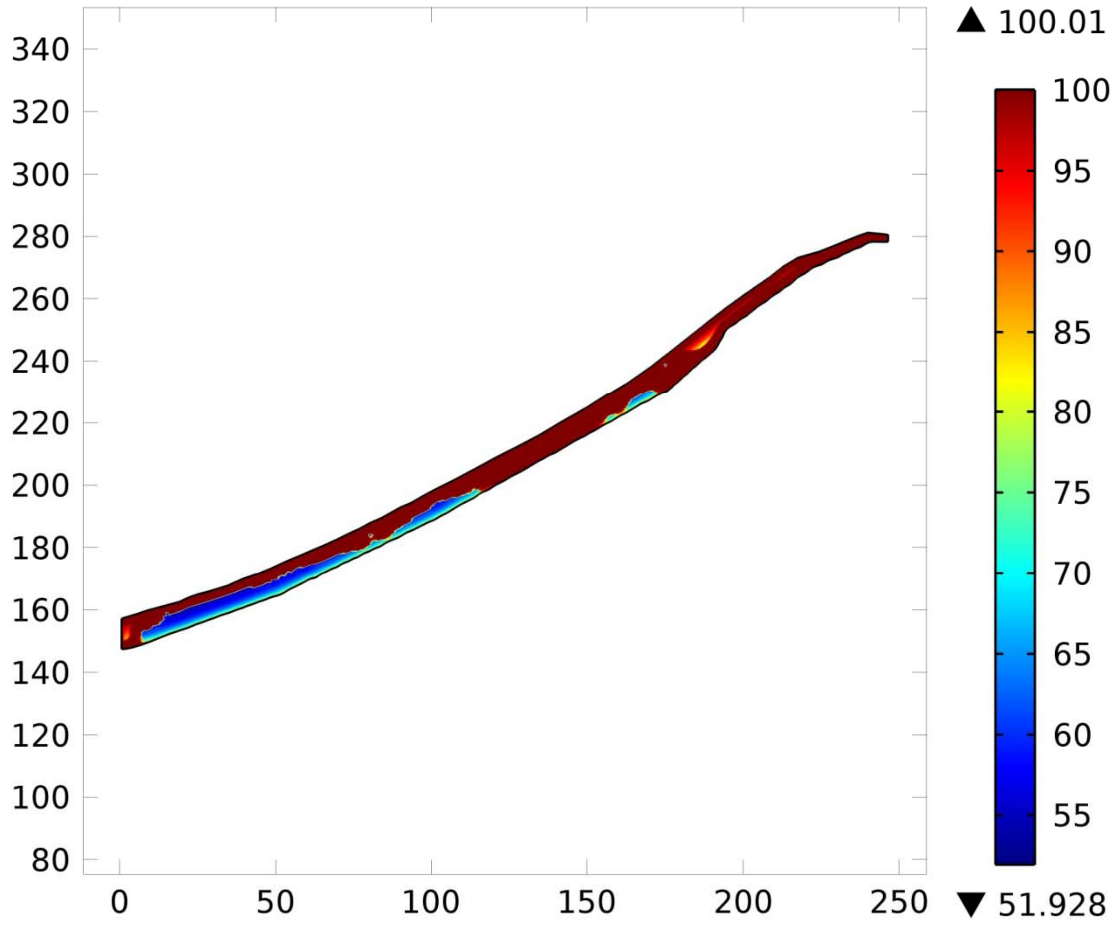
Degree of saturation on 8/30



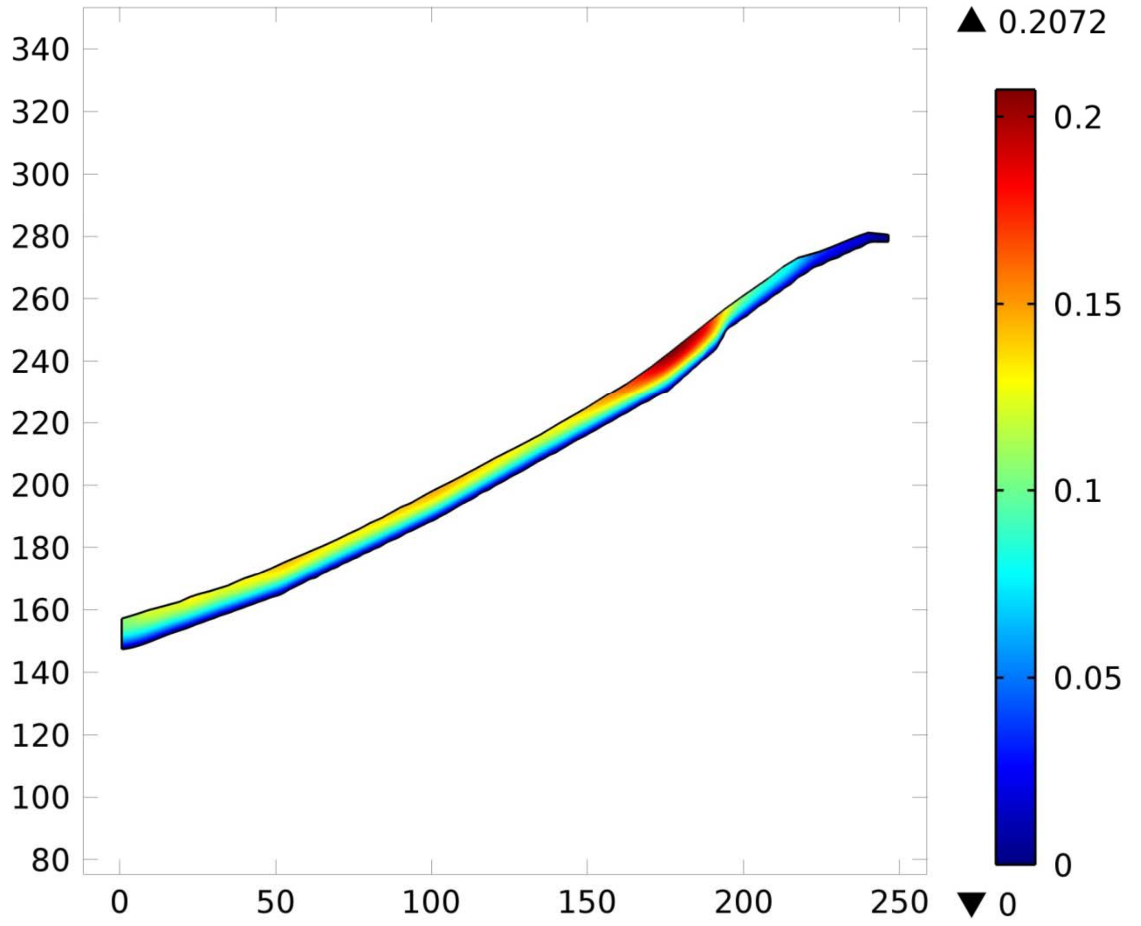
Degree of saturation on 9/14



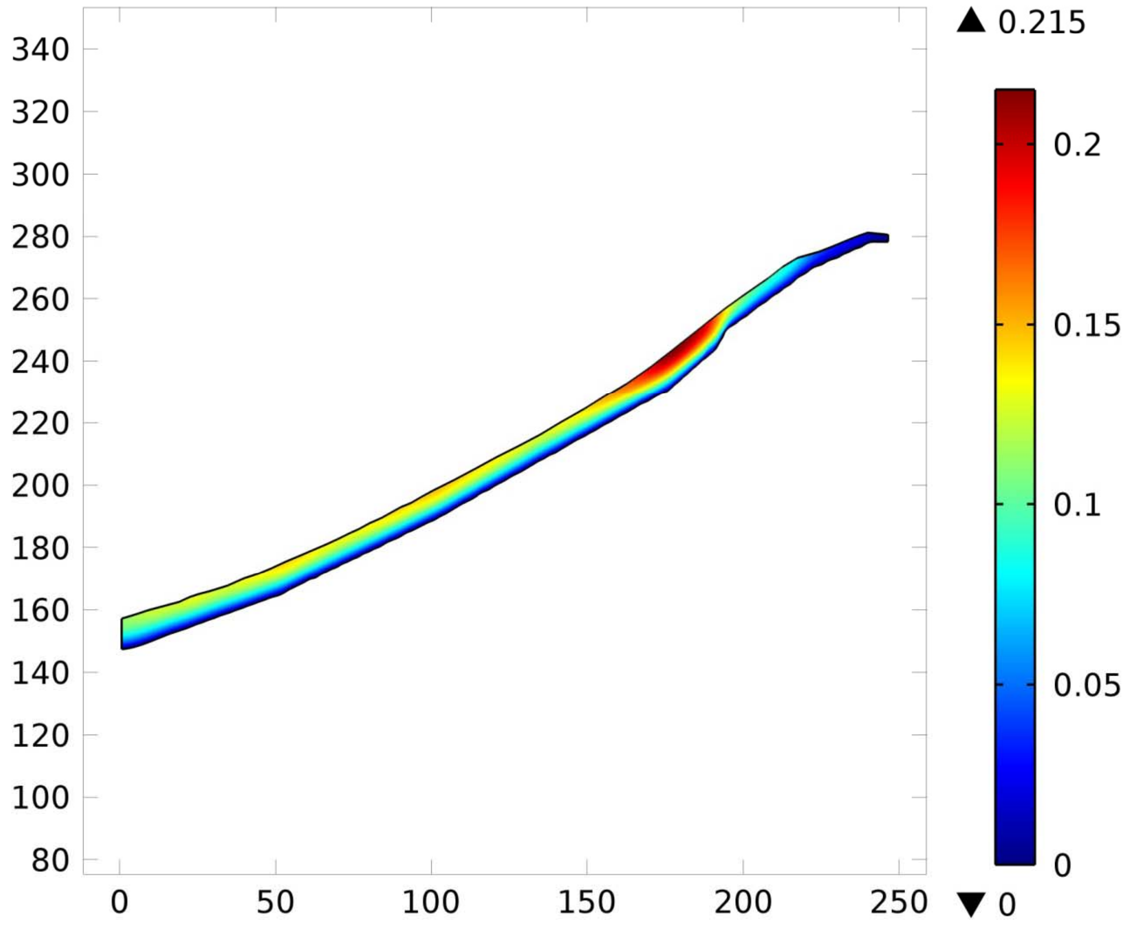
Degree of saturation on 9/28



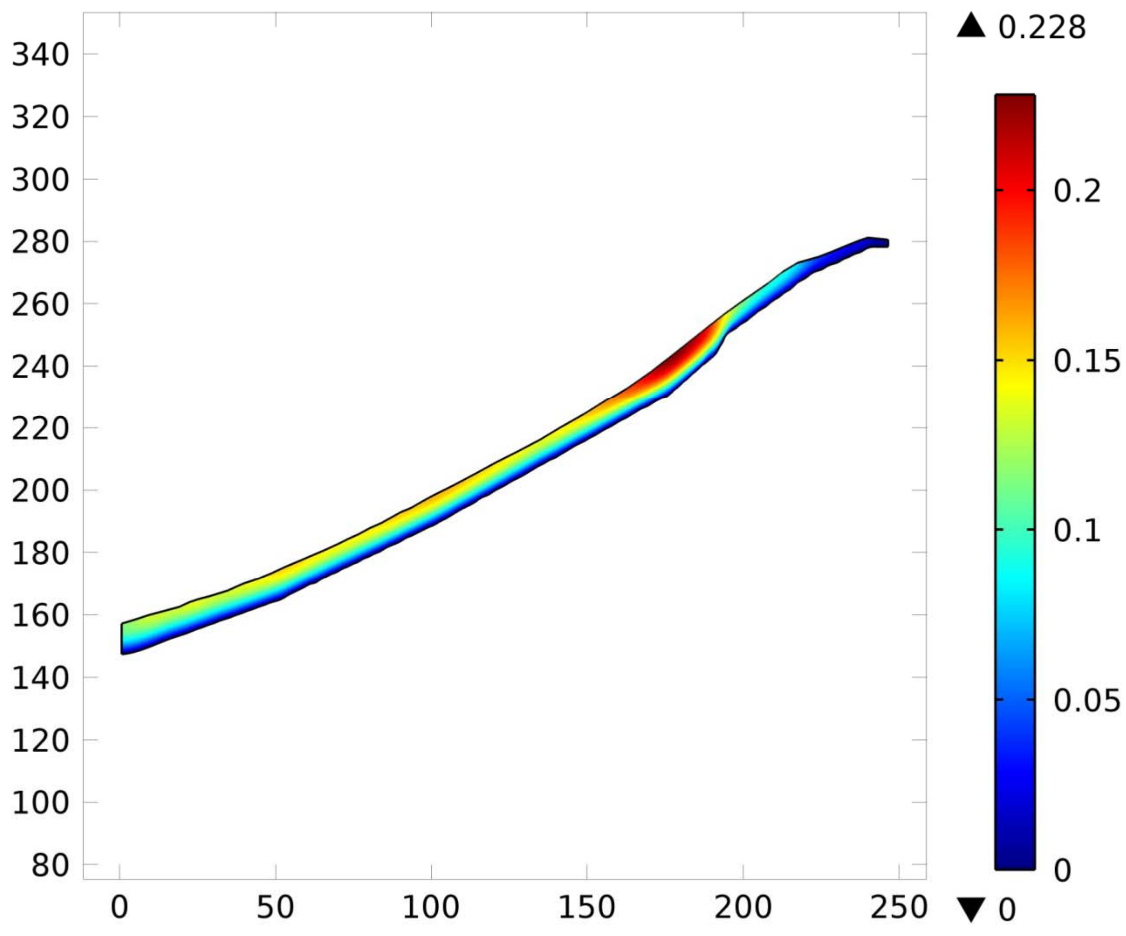
Displacement on 8/30



Displacement on 9/14



Displacement on 9/28



CONCLUSIONS

- A series of non linear PDESs has been used to asses the initiation of the mechanism of the Maokong landslide.
- Geotechincal problems related to contamination, groundwater flow, climate change, etc. can all be represented by non-linear PDEs and be solved using the increasingly powerful COMSOL.

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- 楊棟焜經理—皮托科技股份有限公司

THE END