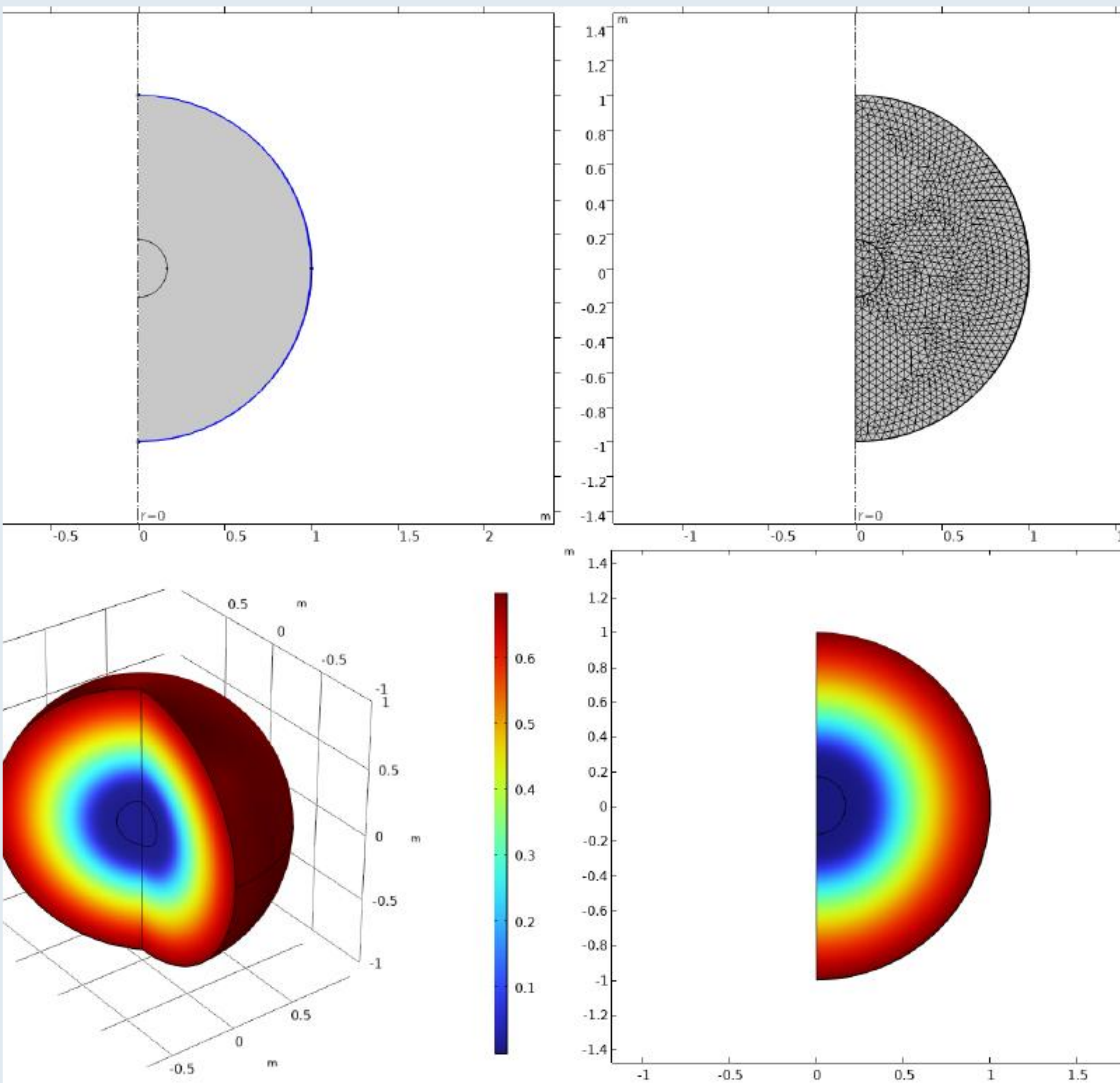




A Multiphysics Framework for Quantitative Assessment of Operational Resource Depletion

This work explores theoretically and numerically the resource accessibility under finite transport capacity, taking also into account the possible effects of the improvements in efficiency due to new technological advancements.

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Introduction

The existence of a large external resource reservoir does not imply that a resource-dependent system can sustain an arbitrarily large rate of consumption. A distinction must be made between the *amount of resource that exists* and the *rate at which that resource can be accessed, transported, and delivered* to the region where it is required. The present work develops a spatially resolved framework for investigating this mechanism in a generic way: the transported quantity may represent matter, an energy carrier, nutrients, raw materials, or any other resource whose use requires transfer from an external reservoir toward a localized consumer using a standard transport equation with a diffusion term together with a consumption sink region situated in a particular place inside the simulated domain.

The system combines three physical processes: internal redistribution of the resource, finite-rate access from an external reservoir, and localized consumption driven by a time-dependent demand.

Technological efficiency is subsequently incorporated to determine whether improved resource utilization can eliminate the accessibility constraint or only postpone its onset. Finally, uncertainty is introduced into the external access rate in order to study how fluctuations in supply capacity propagate into the operational lifetime of the system.

Let $C(x, t)$ denote the local resource availability or concentration. Conservation of the resource inside the computational domain is described by

where D is the effective transport coefficient

$$Q_{\text{cons}}(\mathbf{x}, t) = \chi(\mathbf{x}) q_{\text{eff}}(t) \frac{C}{K_m + C},$$

For the supposed efficiency, we adopt the bounded law

$$\frac{\partial C}{\partial t} = \nabla \cdot (D \nabla C) - \chi(r, z) \frac{q_0 e^{g t}}{\eta(t)} \frac{C}{K_m + C},$$

$$\eta(t) = \eta_0 + (\eta_{\max} - \eta_0)(1 - e^{-\alpha t}),$$

$$J_{\text{in}} = h_{\text{acc}}(C_{\text{ext}} - C),$$

$$C(r, z, 0) = C_{\text{init}}.$$

$$\eta(t) = \eta_0 + (\eta_{\max} - \eta_0)(1 - e^{-\alpha t}),$$

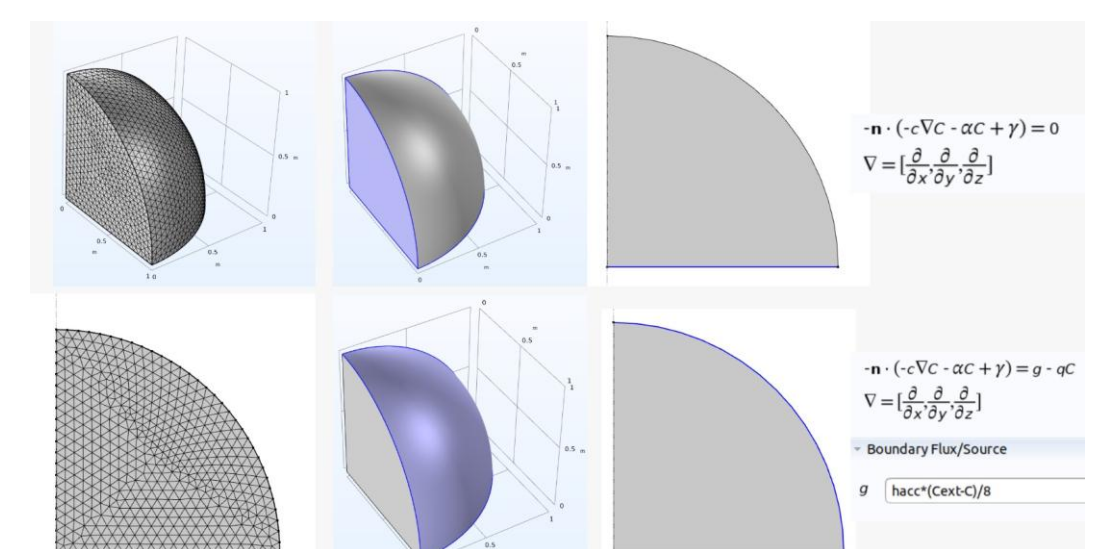
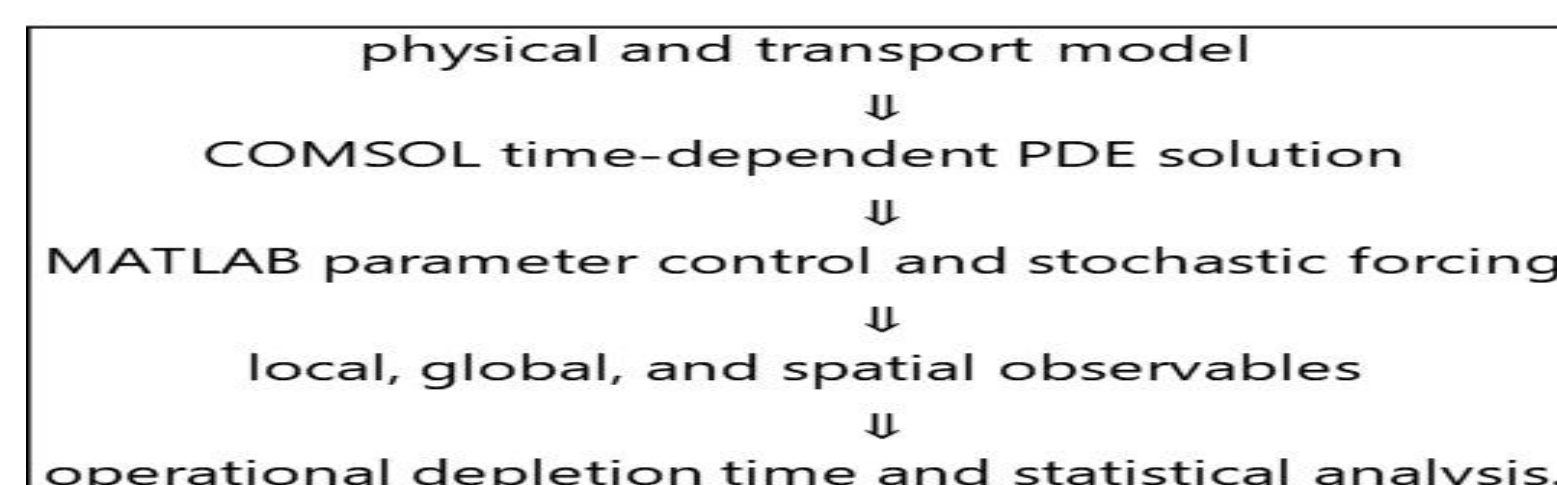
where η_0 and $\eta_{\max}$ are the initial efficiency and maximum possible attainable, and α determines the rate of technological improvement.

Resource transfer across the external boundary is described by the Robin condition on the inward resource flux $J_{\text{in}} : J_{\text{in}} = h_{\text{acc}}(C_{\text{ext}} - C)$, and h_{acc} is an effective access or transfer coefficient.

The diffusion term redistributes the resource through the domain, whereas the sink term removes resource in the consumption region. The evolution is therefore controlled by the competition between spatial replenishment and local extraction.

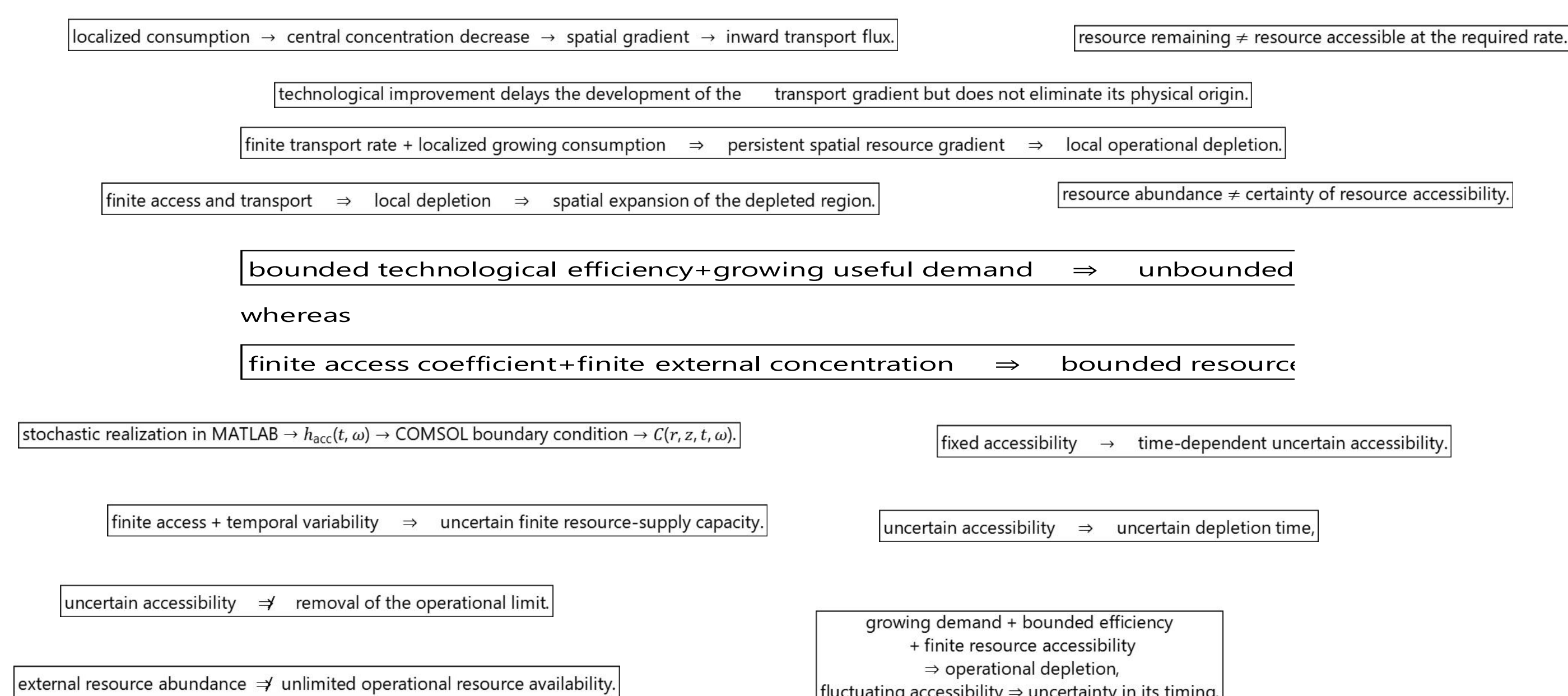
Objective and Methodology

The central objective of the simulations is to determine whether a system connected to a permanently available external resource reservoir can nevertheless undergo operational depletion when the rate at which the resource can enter and propagate through the system is finite. The external resource is therefore not assumed to disappear during the simulation. Instead, the limitation arises from its finite accessibility. Within this framework, technological progress modifies the amount of resource required to sustain a prescribed useful demand, while the physical transport mechanism determines whether this requirement can actually be supplied to the consuming region. The complete computational workflow can therefore be represented schematically as



FIGURES 1 to 4 (from above-left to below-right): 1.-Temporal evolution of the central resource availability for different technological-improvement rates. 2.- Global quantities calculated from the numerical solutions. 3.- Total instantaneous resource consumption. 4.- Operational depletion time as a function of the technological-improvement rate.

Results and conclusions



The results provides a direct numerical confirmation that technological improvement can delay operational depletion, whereas faster improvement reduces cumulative resource extraction during the technological transient, preserving both the internal inventory and the concentration in the consuming region.

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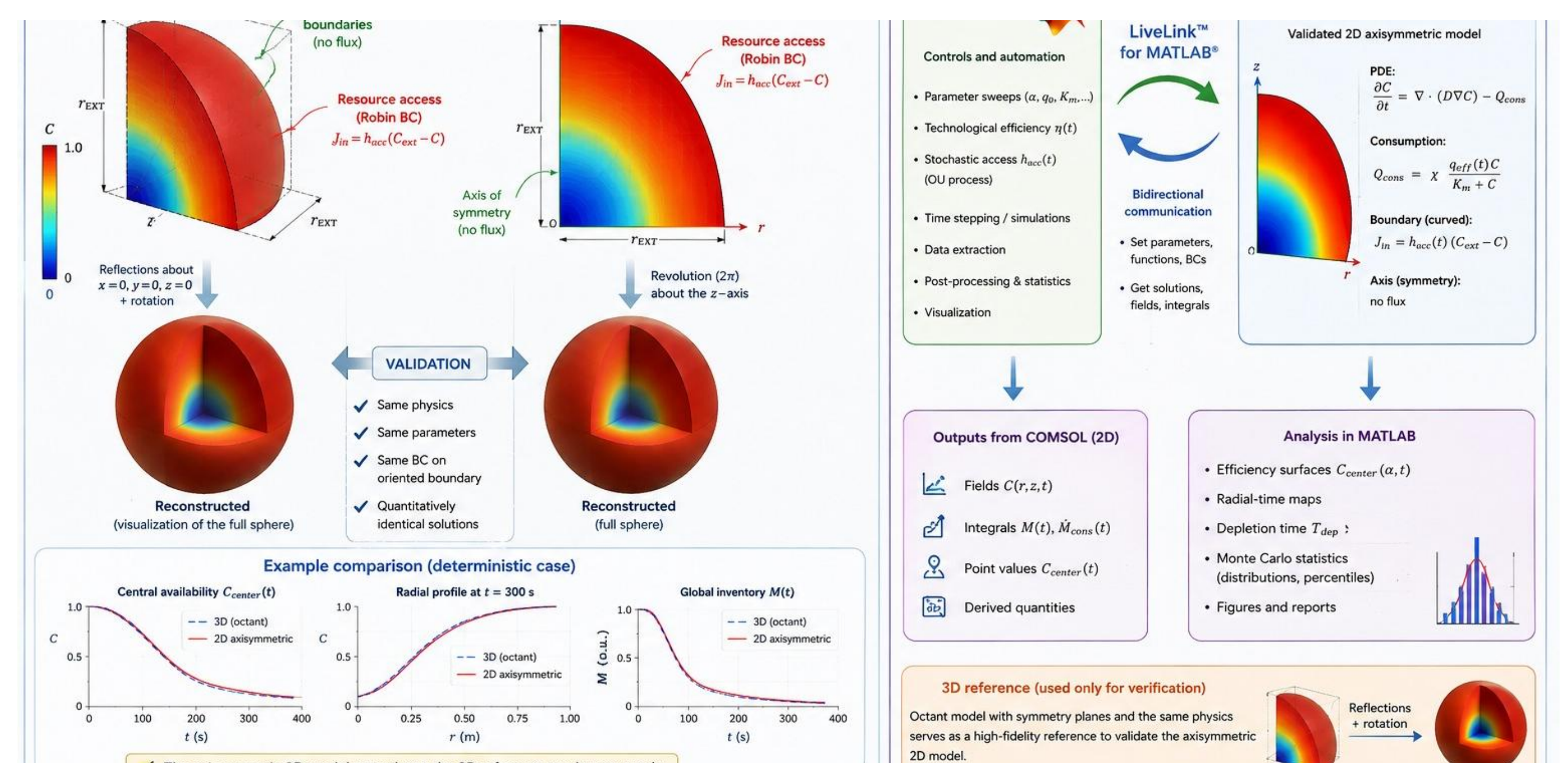


FIGURE 5.- Validation of the 3D vs. 2D representation of the central resource availability and global inventory as a function of time and technological-improvement rate, together with the combination of the COMSOL/MATLAB® analysis procedure.

