Solving the Inverse Problem of Resonant Ultrasound Spectroscopy on Dumbbell-shaped Compression Samples Using COMSOL Multiphysics®

M. C. Golt[1]

[1]U.S. Army Research Laboratory*, Aberdeen Proving Ground, MD. *Contracted through Dynamic Science, Inc.

Abstract

Compressive strength is an important material property of advanced ceramics. In order to accurately characterize this property through uniaxial compression testing, a dumbbell-shaped geometry [1] was developed to ensure that compressive fracture and failure resided only in the reduced diameter gage section (Figure 1). However, the manufacturing of any sample can lead to machining defects which introduce fracture nucleation sites and the appearance of lower strength. The elastic properties of test specimens are also of interest. Popular non-destructive techniques to characterize these properties involve observing the interaction of acoustic waves with the sample, from which the elastic properties can be determined. The Resonant Ultrasound Spectroscopy (RUS) method characterizes the resonance peaks that arise in a swept acoustic frequency spectrum. The sample geometry and density are usually known or can be easily measured and the elastic tensor is then determined through solving the inverse problem by finding the match between a numerical model and the measured resonance spectrum.

The eigenfrequency study of solids included in the Structural Mechanics Module of COMSOL®, along with the LiveLink™ for MATLAB® provide a very powerful tool for solving this inverse problem. First, the 3D dumbbell geometry (Figure 1) was parameterized and constructed, allowing for variation in the measured sample geometry due to machining variability. The other model parameters (Young's modulus, Poisson's ratio, and density) were instantiated to best-guess values and the forward problem of determining eigenfrequencies is solved for using COMSOL's MUMPS solver. These eigenfrequency locations are then brought into MATLAB in which the Jonker-Volgenant algorithm [2] for solving the linear assignment problem of matching the appropriate modeled eigenfrequencies to measured frequencies is used to determine a fitness score for the current model. This process is then repeated through a direct search method with new model parameters until the score is minimized and the optimal solution is found.

Figure 2 shows a measured resonance spectrum for an alumina dumbbell specimen overlaid with the modeled modes of vibration. In both the COMSOL model and experimental spectrum several overlapping degenerate bending frequencies were identified which will diverge if material inhomogeneity or machining defects disrupt the axial symmetry. A response surface of model fitness to a measured alumina sample over possible Young's modulus and Poisson ratio values

shows a single distinct minimum which a direct search algorithm should easily find (Figure 3). A sensitivity analysis was also performed showing the effect of geometry and density variations on the minima location.

COMSOL and its interface with MATLAB have been shown as a capable tool for solving the inverse problem of RUS. With the ability to detect machining defects and extract elastic properties of compression specimens, researchers are able to accurately determine the qualities of advanced ceramic materials.

Reference

[1] Dunlay, W. A., C. A. Tracy, et al. (1989). “A proposed uniaxial compression test for high strength ceramics,” DTIC Document.

[2] Jonker, R. and A. Volgenant (1987). "A shortest augmenting path algorithm for dense and sparse linear assignment problems." Computing 38(4): 325-340.

Figures used in the abstract

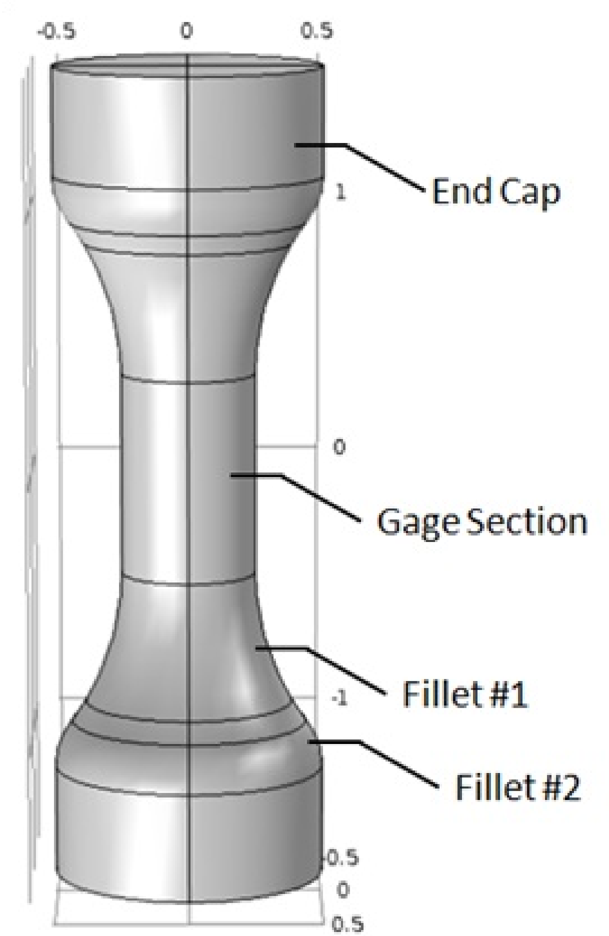

Figure 1: Dumbbell compression test specimen [1]. Scale is in millimeters.

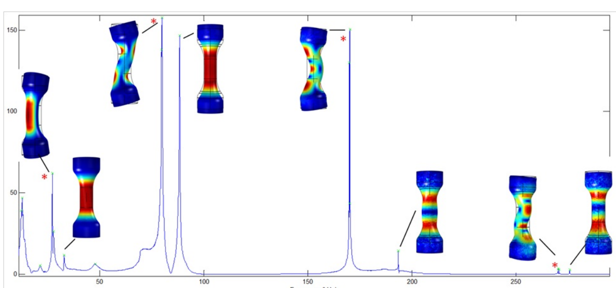

Figure 2: Measured resonance spectrum for an alumina dumbbell sample overlaid with the modeled vibrating modes. Large asterisks identify the degenerate bending frequencies that diverge if the axial symmetry is disrupted by inhomogeneities or machining defects.

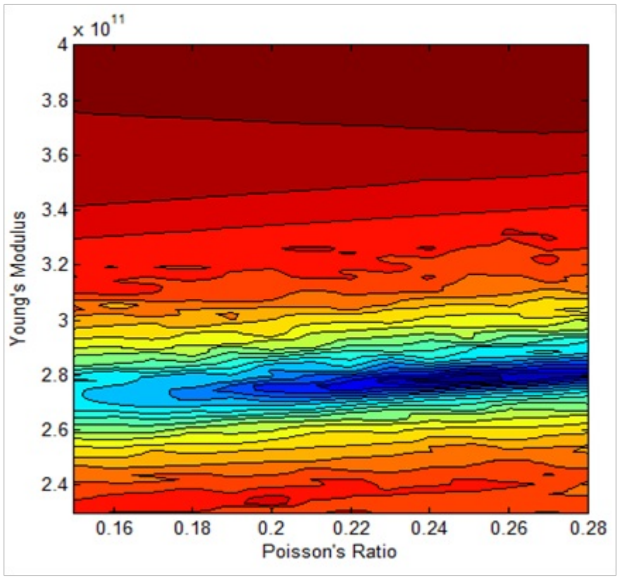

Figure 3: Response surface of model fit to experimental for an alumina dumbbell sample.