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REALIZING TOMORROW'S TECHNOLOGY

COMSOL CONFERENCE
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Scattering of Electromagnetic Waves

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Overview

- **Motivation**
- **Theory**
  - Cross Section Calculations
  - Optical Theorem
  - Radiation Force
- **Use of COMSOL Multiphysics**
- **Results**
- **Summary**

Motivation

- Comparison with analytical solution for EM scattering
  - Dielectric, metal and magnetic materials
- Provide methodology for other users
- Extend solutions to sizing applications
  - Atmospheric
  - Droplets in fluids
  - Cell nuclei

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Theory

- **Scattering - charge excitation and re-radiation of electromagnetic energy**
- **Elastic scattering - frequency of scattered wave equals incident wave**
- **Total wave:**

$$\mathbf{E} = \mathbf{E}_{inc} + \mathbf{E}_{sca} \qquad \mathbf{H} = \mathbf{H}_{inc} + \mathbf{H}_{sca}$$

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Extinction Cross-Section

$$\sigma_{ext} = \sigma_{abs} + \sigma_{sca}, \quad \left[m^2\right]$$

Optical theorem:

$$\sigma_{ext} = \frac{4\pi}{k} \operatorname{Im}\{f(0)\} / E_{inc}$$

In COMSOL:

$$\mathbf{E}_{far} = \lim_{r\to\infty} r\mathbf{E}_{sca}$$

Direct calculation:

$$\sigma_{ext} = \frac{4\pi}{kE_{inc}} \operatorname{Im}\{E_{far}(0)\}, \quad \left[m^2\right]$$

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Radiation Force

From radiation cross section:

$$\mathbf{F} = \frac{1}{c}\sigma_{pr}\mathcal{P}_{inc}, \quad [N]$$

Integrating Maxwell's stress tensor over particle surface:

$$\hat{\mathbf{T}} = \frac{1}{2}\mathrm{Re}\left[\mathbf{E}^*\mathbf{D} + \mathbf{H}^*\mathbf{B} - \frac{1}{2}\left(\mathbf{E}^* \cdot \mathbf{D} + \mathbf{H}^* \cdot \mathbf{B}\right)\hat{\mathbf{I}}\right], \quad [Pa]$$

Electric flux density:

$$\mathbf{D} = \varepsilon E$$

Magnetic flux density,

$$\mathbf{B} = \mu \mathbf{H}$$

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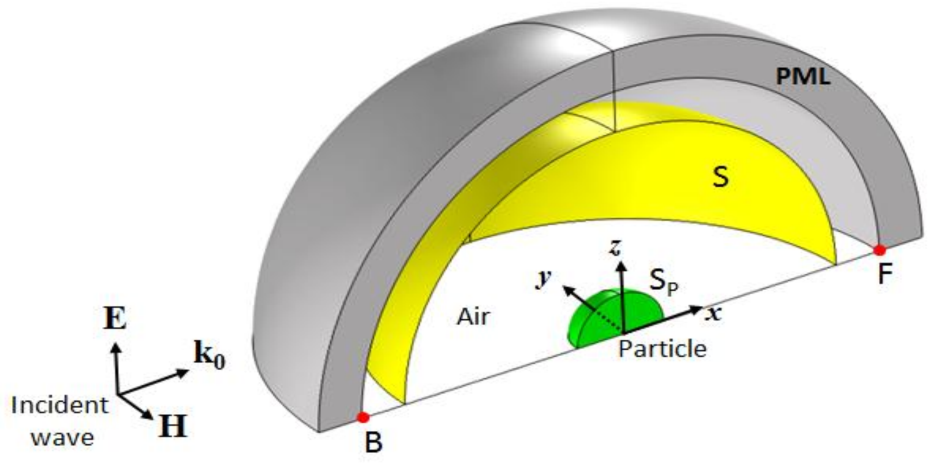
Use of COMSOL



- **Far-field calculations are done on the inner boundary of the PML domain**
- ***S* is used to calculate total scattered energy**
- **Incident plane wave travels in positive x-direction**
- **Electric field polarized along the *z*-axis**

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Use of COMSOL Multiphysics

Quantity	Expression in Comsol	Unit
Q_{loss}, Eq. 11	emw.Qh	W/m^3
W_{abs}, Eq. 12	intop1_Vp(emw.Qh)	W
W_{sca} , Eq. 13	intop2_S(-(emw.relPoavx*nx+emw.relPoavy*ny+ emw.relPoavz*nz))	W
σ_{ext} , Eq. 14	-4*pi/emw.k0*imag(emw.Efarz*1[m])/E0, evaluated at point F	m^2
σ_b ,	4*pi*(emw.normEfar*1[m]/E0)^2, evaluated at point B	m^2
$\langle\cos\theta\rangle$, Eq. 18	intop2_S(-(emw.relPoavx*nx+emw.relPoavy*ny+emw.relPoavz*nz)*(-nx))/P0/sig_sca	1
F , Eq. 21	intop3_Sp(emw.dnTx+ emw.dnTy+ emw.dnTz)	N

The following integration coupling operators are used in the table:

intop1_Vp() : domain integration operator over particle volume

intop2_S() : boundary integration operator over imaginary surface S

intop3_Sp() : boundary integration operator over particle surface S_P

The following Electromagnetic Waves (emw) interface predefined variables are used in the table:

emw.Qh: total power dissipation density, W/m^3

emw.relPoavx, emw.relPoavy, emw.relPoavz: time average Poynting vector components of the scattered field, W/m^3

emw.Efarz: far-field variable, z-component, V/m

emw.dnTx, emw.dnTy, emw.dnTz: Maxwell upward stress tensor components, Pa

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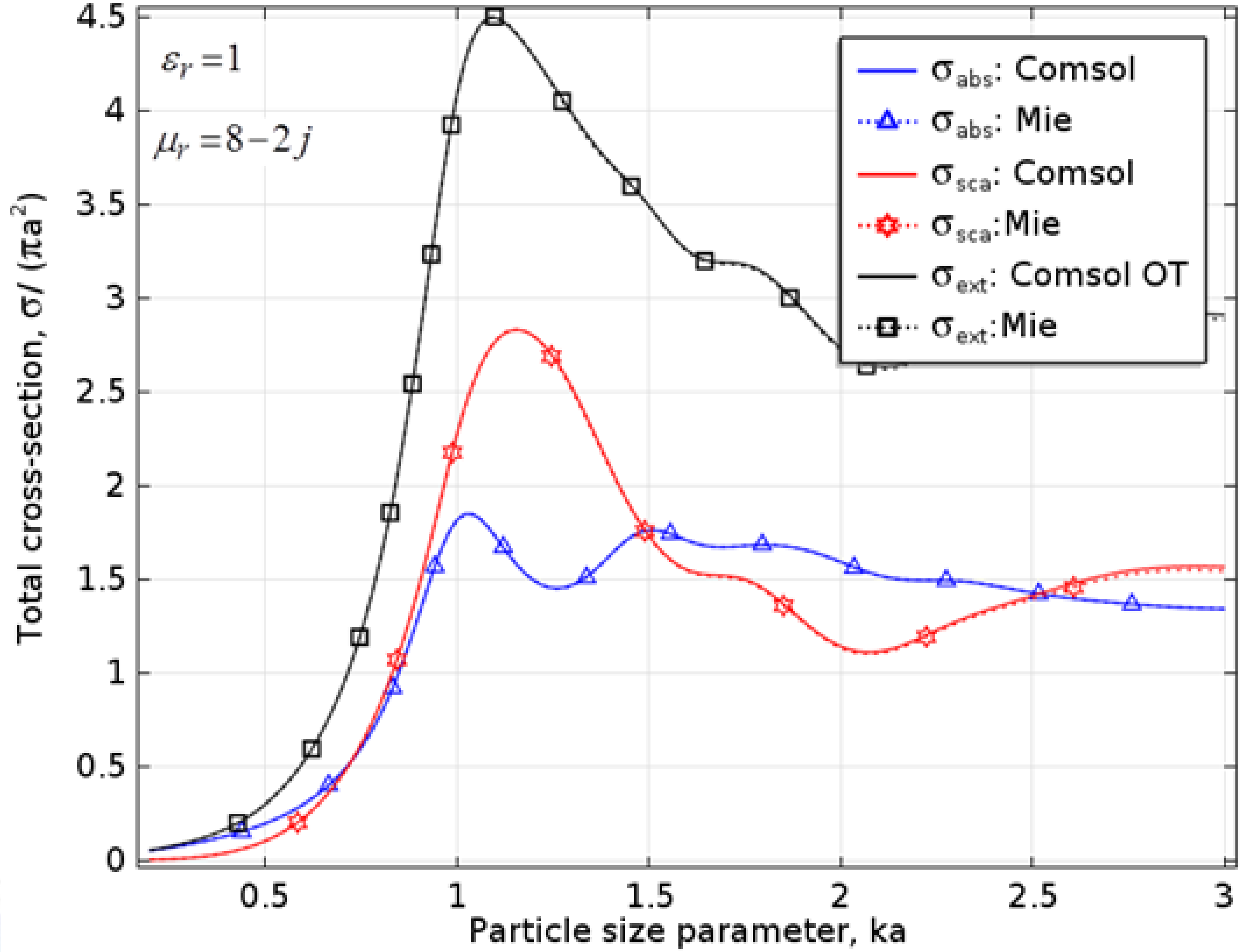
Results - Magnetic Particle



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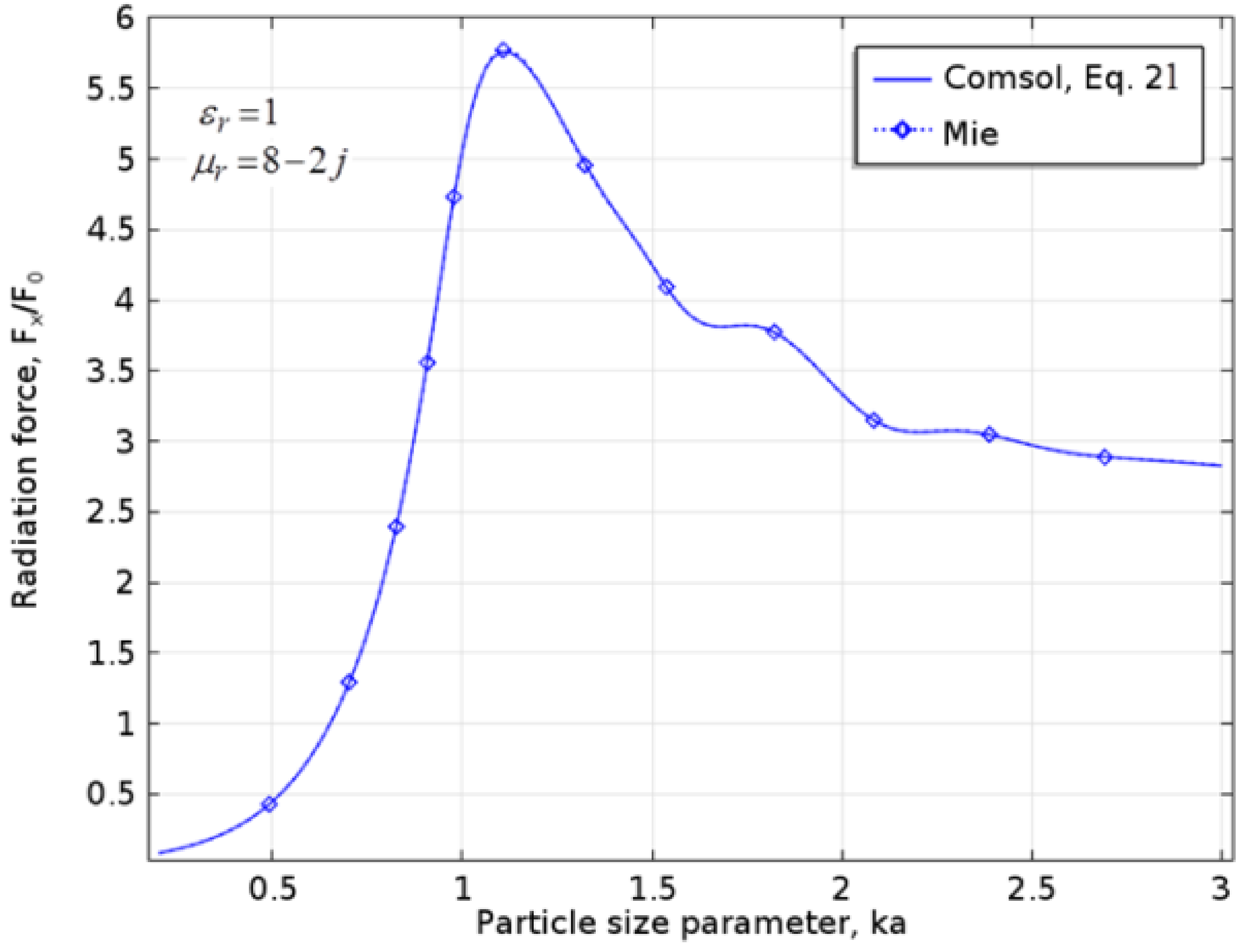

Summary

- Scattering solution for EM waves calculated for three particle types – metal, dielectric and magnetic
- Paper fully describes method for others to use
- Results compared with Mie solution
- Excellent agreement for cross sections and radiation force

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