Multiphysics Modelling of Sound Absorption in Rigid Porous Media Based on Periodic Representations of Their Microstructural Geometry

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Outline

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1. **Macroscopic model**
   - Parameters and effective functions
   - Acoustical characteristics
2. **Multi-scale approach**
   - Micro-scale level
   - Hybrid approach
3. **Examples**
   - Porous ceramics
   - Freely-packed spherules
4. **Conclusions**

Acoustics of porous media with rigid frame

Fluid-equivalent approach

An effective fluid is substituted for a porous medium. It is dispersive and substantially different from the fluid in pores.

Requirements: **(1)** open-cell porosity, **(2)** rigid (motionless) skeleton, **(3)** wavelengths significantly bigger than the characteristic size of pores.

- **Helmholtz equation** of linear acoustics:

$$\omega^2 \tilde{p} + c^2 \Delta \tilde{p} = 0, \qquad c^2 = \frac{K}{\rho}$$

$\tilde{p}$ – the amplitude of acoustic pressure, ω – the angular frequency, c, ρ, K – the speed of sound, density and bulk modulus of medium

- **Effective density** and **bulk modulus** for a porous medium:

$$\rho(\omega) = \rho_{\mathrm{f}}\, \alpha(\omega), \qquad K(\omega) = \frac{P_0}{1 - \dfrac{\gamma - 1}{\gamma\, \alpha'(\omega)}}$$

ρ_{f} – the density of fluid in pores, γ – the heat capacity ratio of fluid in pores, P_0 – the ambient mean pressure, $\alpha(\omega)$, $\alpha'(\omega)$ – the **dynamic (visco-inertial) tortuosity** and **"thermal tortuosity"**

Model parameters

Johnson-Champoux-Allard model (simplified)

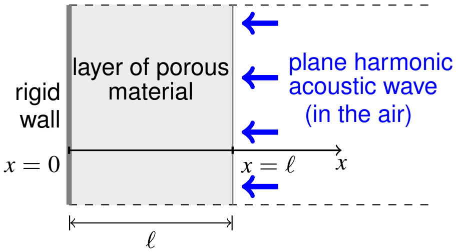
$$\alpha(\omega) = \alpha_\infty + \frac{\nu}{\mathrm{i}\omega}\frac{\phi}{k_0}\sqrt{\frac{\mathrm{i}\omega}{\nu}\left(\frac{2\alpha_\infty k_0}{\Lambda\phi}\right)^2 + 1}, \quad \alpha'(\omega) = 1 + \frac{\nu'}{\mathrm{i}\omega}\frac{\phi}{k_0'}\sqrt{\frac{\mathrm{i}\omega}{\nu'}\left(\frac{2k_0'}{\Lambda'\phi}\right)^2 + 1}$$

ϕ, α_∞, k_0, k_0', Λ, Λ' – purely geometric parameters of the skeleton
$\nu = \mu/\rho_\mathrm{f}$ – the kinematic viscosity of pore-fluid (μ – the dynamic viscosity)
$\nu' = \nu/\mathrm{Pr}$ (Pr – the Prandtl number of pore-fluid)

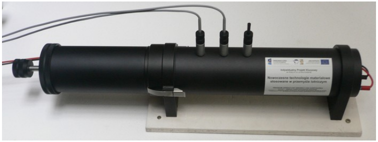
- **Parameters of the fluid in pores** (the density ρ_f, heat capacity ratio γ, viscosity μ, and Prandtl number Pr) and the **ambient mean pressure** P_0
- **Geometric parameters of the skeleton** of porous medium:

Symbol	*Unit*	*Parameter*
ϕ	[–]	porosity
α_∞	[–]	tortuosity
k_0	[m^2]	(static) viscous permeability
k_0'	[m^2]	“thermal permeability”
Λ	[m]	viscous characteristic length
Λ'	[m]	thermal characteristic length

Impedance and absorption of a porous layer



Impedance tube
for material testing



- Surface acoustic **impedance**:

$$Z(\omega) = \sqrt{\rho K}\,\frac{\exp(2\mathrm{i}\omega\ell\sqrt{\rho/K}) + 1}{\exp(2\mathrm{i}\omega\ell\sqrt{\rho/K}) - 1} = -\mathrm{i}\sqrt{\rho K}\cot\left(\omega\ell\sqrt{\rho/K}\right)$$

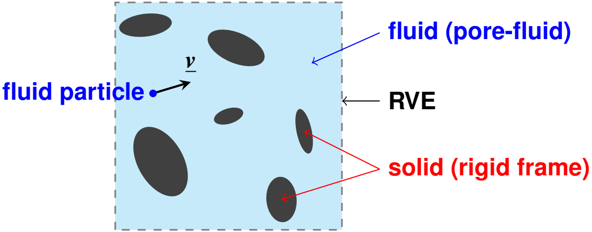
- Acoustic **absorption** and **reflection** coefficients:

$$A(\omega) = 1 - |R(\omega)|^2, \qquad \text{where} \qquad R(\omega) = \frac{Z(\omega) - Z_{\mathrm{f}}}{Z(\omega) + Z_{\mathrm{f}}}$$

(Z_{f} – the characteristic impedance of pore-fluid)

Outline

Small-velocity flow in a porous medium



Small fluctuations

The **velocity field** $\underline{v}$ describes ***small* fluctuations of fluid particles** around their initial (motionless) equilibrium state.
Fluid density, **pressure** and **temperature** are decomposed as follows:

$$\varrho = \varrho_0 + \tilde{\varrho}, \qquad p = p_0 + \tilde{p}, \qquad T = T_0 + \tilde{T}$$

$\tilde{\varrho}$, $\tilde{p}$, $\tilde{T}$ – **small fluctuations** of **density**, **pressure**, and **temperature**, respectively, around their constant, **equilibrium values**: ϱ_0, p_0, and T_0.

Hybrid approach

Micro-scale level: Solve **3 steady-state BVPs** on the micro-scale:

1. **Stokes flow** (steady, incompressible viscous flow) – then calculate:
   - static viscous permeability
   - viscous tortuosity at 0 Hz
2. **Steady heat transfer** – then calculate:
   - static thermal permeability
   - thermal tortuosity at 0 Hz
3. **Laplace problem** – then calculate:
   - parameter of tortuosity (tortuosity at ∞ Hz)
   - viscous characteristic length

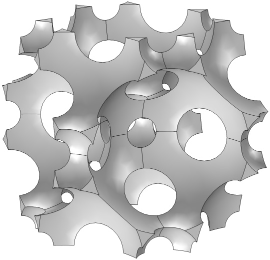
The **thermal characteristic length** and the **porosity** are determined directly from the **micro-geometry**. The thermal length is computed as the ratio of the doubled volume of fluid domain to the surface of skeleton walls.

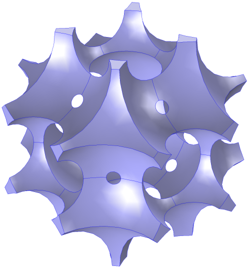
Macro-scale level: **Use the parameters** calculated (averaged) **from microstructure** for the **Johnson-Allard formulas** to compute the **dynamic tortuosity functions**, and then the **dynamic permeability functions**, and eventually, the **effective density** and **bulk modulus**.

Outline

Periodic skeleton cells with porosity 90%

- Both RVEs have **open-cell porosity of 90%.**
- Both RVEs are **cubic**, **periodic** and "**isotropic**" (identical with respect to three mutually-perpendicular directions).





- 7 pores per cell
- 3 types of pores

- 8 pores per cell
- 4 types of pores

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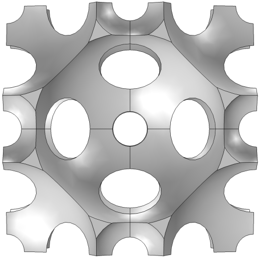

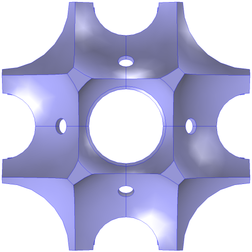

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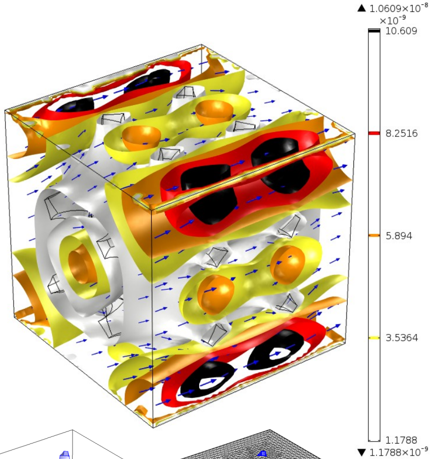
Incompressible flow through the periodic cell

- **No-slip boundary conditions** on the skeleton boundaries
- **Periodic boundary conditions** on the relevant pairs of cell faces
- The **local flow permeability (‘scaled velocity’) field** is computed in the **fluid domain**



Viscous permeability

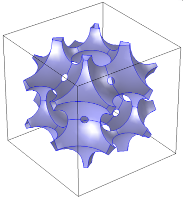
The static, macroscopic **permeability**:

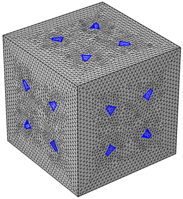
$$k_0 = 7.50 \times 10^{-10}\,\mathrm{m}^2$$

is obtained as the **fluid-domain average** of the computed field. It is consistent with the value:

$$k_0 = 7.13 \times 10^{-10}\,\mathrm{m}^2$$

found using the **inverse identification** procedure.

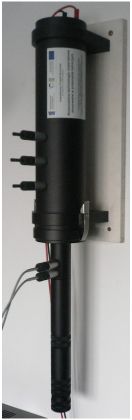




Testing freely-packed layers of spherules

Spherule: • diameter = 5.9 mm • volume = 107.54 mm^3 • mass = 0.3 g

- **Large Tube** (diameter = 100 mm) →
  Frequency range = 50 Hz to 1600 Hz
- **Medium Tube** (diameter = 63.5 mm) →
  Frequency range = 100 Hz to 3200 Hz
- **Small Tube** (diameter = 29 mm) →
  Frequency range = 500 Hz to 6400 Hz



Testing freely-packed layers of spherules

Spherule: • diameter = 5.9 mm • volume = 107.54 mm^3 • mass = 0.3 g

- **Large Tube** (diameter = 100 mm)

Layer:	L-41
Height [mm]:	41
No. of spherules:	1840
Porosity:	**app. 39%**

- **Medium Tube** (diameter = 63.5 mm)

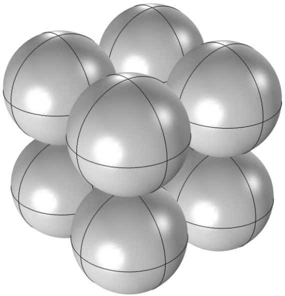
Layer:	M-41	M-106
Height [mm]:	41	106
No. of spherules:	708	1840
Porosity:	**app. 41%**	

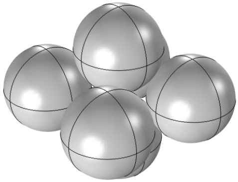
- **Small Tube** (diameter = 29 mm)

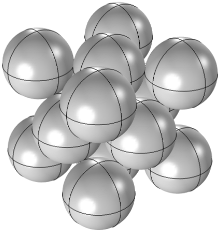
Layer:	S-41	S-106	S-200
Height [mm]:	41	106	200
No. of spherules:	147	380	710
Porosity:		**app. 42%**	

Periodic sphere packings and RVEs

SC	**BCC**	**FCC**
simple cubic	body-centered cubic	face-centered cubic







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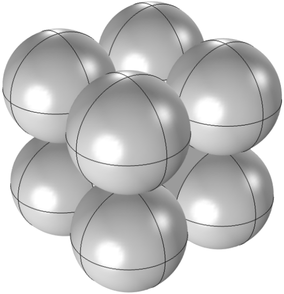
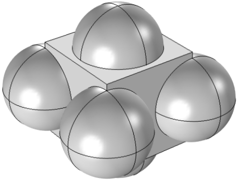
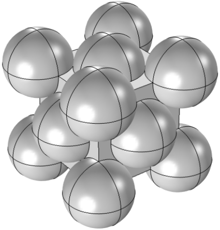




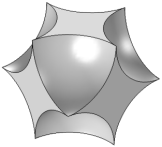


Packing type:	**SC**	**BCC**	**FCC**
number of spheres:	1	2	4
edge to diameter ratio:	1	$\frac{2}{\sqrt{3}} = 1.155$	$\sqrt{2} = 1.414$
edge length* [mm]:	5.90	6.81	8.34

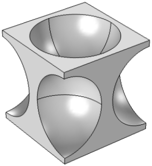
*for the sphere diameter 5.9 mm

Periodic sphere packings and RVEs

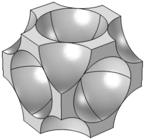
SC
simple cubic



BCC
body-centered cubic



FCC
face-centered cubic



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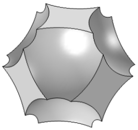
number of spheres:	1	2	4
edge to diameter ratio:	1	$\frac{2}{\sqrt{3}} = 1.155$	$\sqrt{2} = 1.414$
edge length* [mm]:	5.90	6.81	8.34
solid fraction:	$\frac{\pi}{6} = 0.524$	$\frac{\pi\sqrt{3}}{8} = 0.680$	$\frac{\pi\sqrt{2}}{6} = 0.740$
porosity [%]:	**47.6**	**32.0**	**26.0**

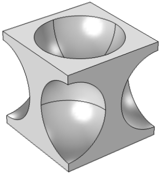
*for the sphere diameter 5.9 mm

Periodic sphere packings and RVEs (porosity 42%)

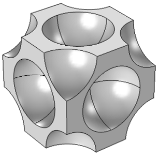
SC
simple cubic



BCC
body-centered cubic



FCC
face-centered cubic



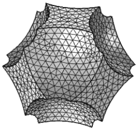
Packing type:	**SC$_{(42\%)}$**	**BCC$_{(42\%)}$**	**FCC$_{(42\%)}$**
number of spheres:	1	2	4
edge to diameter ratio:	0.960	1.218	1.534
edge length* [mm]:	5.66	7.19	9.05

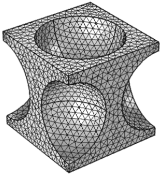
By shifting spheres the porosity is set to 42%.
This is the actual porosity of loosely-packed layers of spherules.

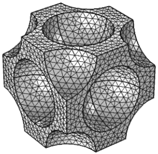
*for the sphere diameter 5.9 mm

Periodic sphere packings and RVEs (porosity 42%)

SC	**BCC**	**FCC**
simple cubic	body-centered cubic	face-centered cubic



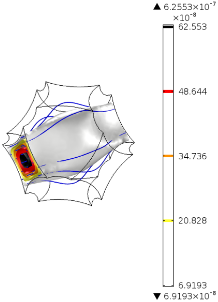


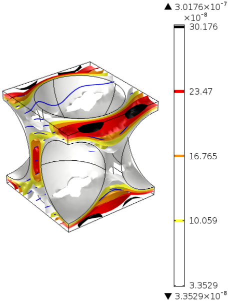


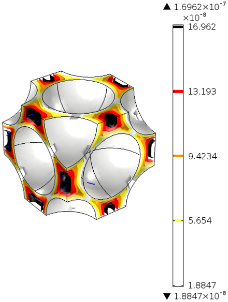
Packing type:	**SC**$_{\mathbf{(42\%)}}$	**BCC**$_{\mathbf{(42\%)}}$	**FCC**$_{\mathbf{(42\%)}}$
permeability [m^2]:	5.46×10^{-8}	4.52×10^{-8}	3.93×10^{-8}
thermal permeability [m^2]:	1.46×10^{-7}	8.03×10^{-8}	8.34×10^{-8}
tortuosity (at ∞ Hz):	1.5263	1.3245	1.3191
tortuosity at 0 Hz:	2.3052	1.9343	1.8371
thermal tortuosity at 0 Hz:	1.4438	1.3141	1.5238
viscous length [mm]:	0.9900	1.1054	1.1197
thermal length [mm]:	1.5573	1.4268	1.4230

Parameters for Johnson-Champoux-Allard model

SC	**BCC**	**FCC**
simple cubic	body-centered cubic	face-centered cubic







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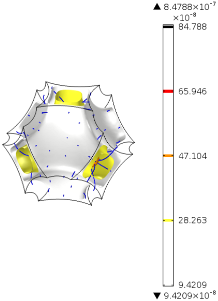

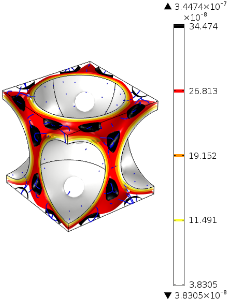

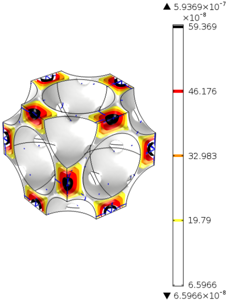

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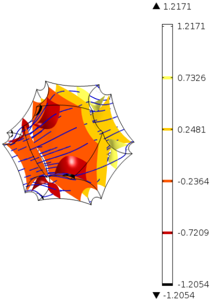

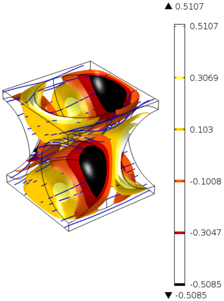

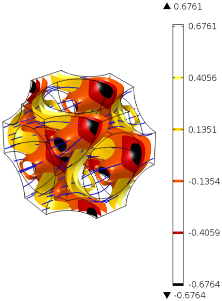

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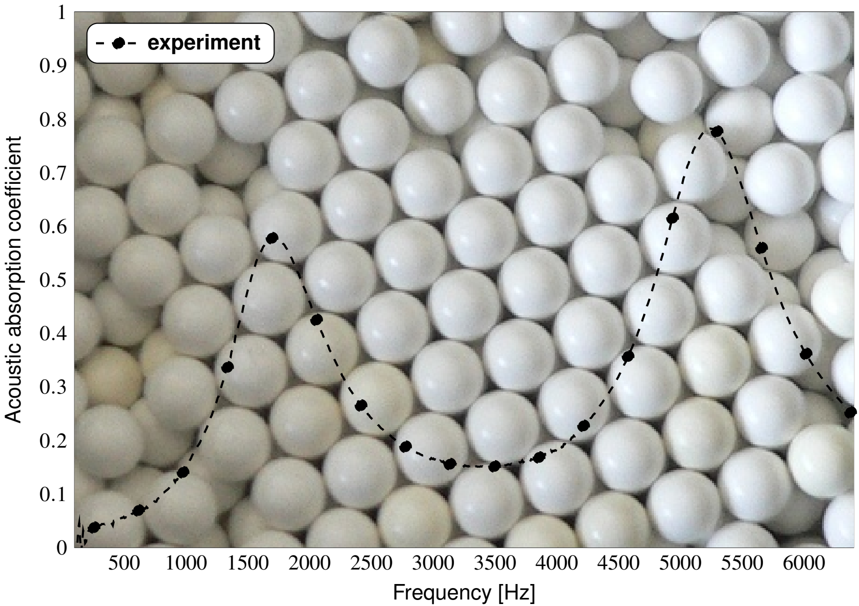
Spherules layer: porosity 42%, height 41 mm



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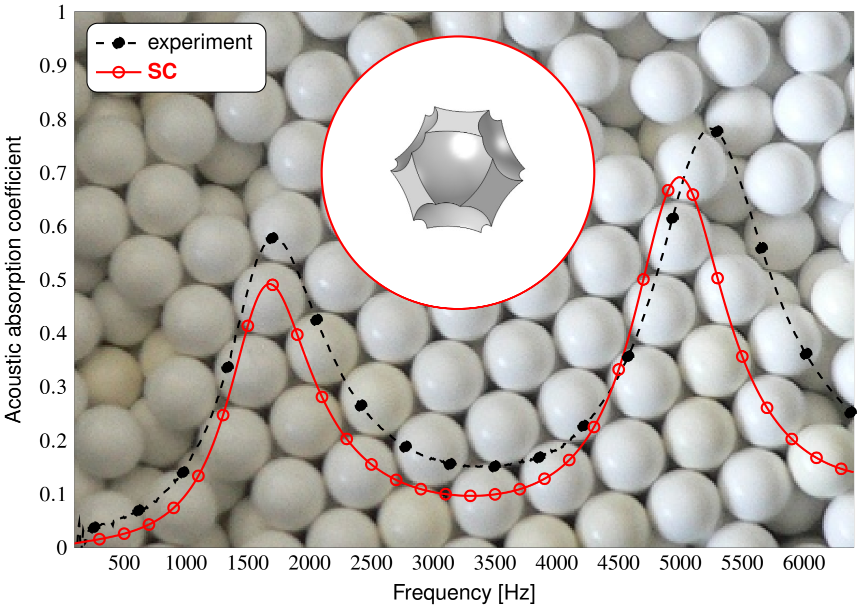

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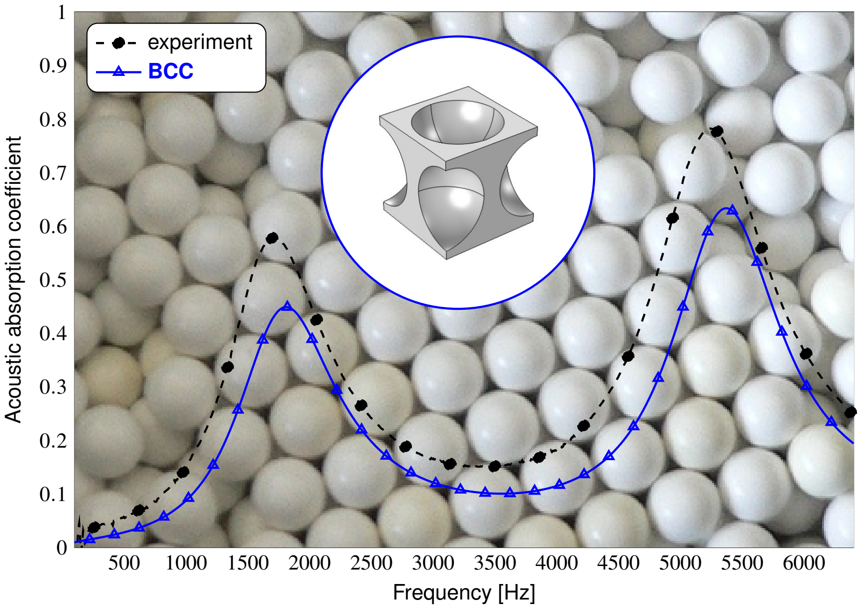

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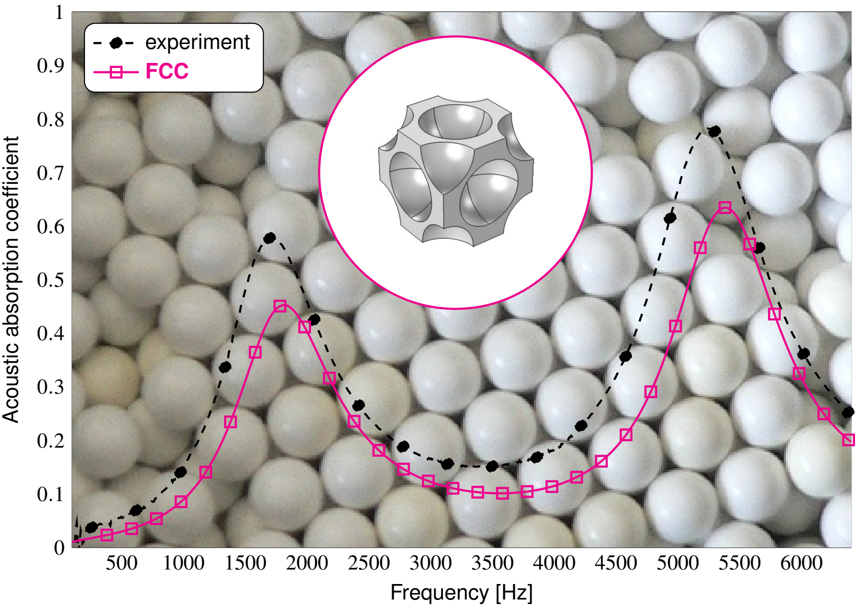

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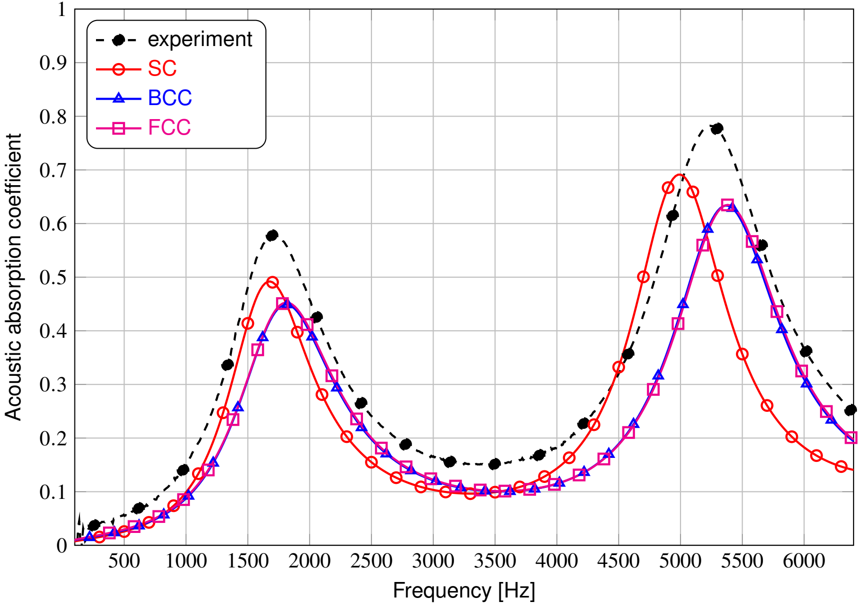

Outline

Conclusions

Modelling in COMSOL Multiphysics

- **Periodic boundary conditions**
- **Symbolic expressions and equation-based modelling**
- **LiveLink to MATLAB**

Microstructure representations

- Larger RVEs (i.e., containing **more pores, spheres, or fibres**, etc.) seem to be necessary
- **Random generation** of periodic cells should easily yield **better representation**
- **Periodicity conditions on lateral faces** may be substituted by **Neumann-like conditions**

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