Degeneracy Breaking, Modal Symmetry and MEMS Biosensors

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Abstract

This work is concerned with systems possessing cyclic symmetries. In particular, we concentrate on the case in which the medium possesses infinite order cyclic symmetry, while the boundary conditions (geometry) have cyclic symmetry of a lower order. We investigate the interactions between modes with cyclic symmetry of order M and geometries with underlying (intrinsic) cyclic symmetry of order N, for the cases where N=M and when N and M are incommensurate, that is, when N/M is not a natural number.
In a transversely isotropic material, waves from a point excitation source on a free surface propagate isotropically in the transverse plane, with the wavefronts forming concentric circles about the excitation. Given a concentric, infinite-cyclically symmetric finite geometry such as a disc or bar with cylindrical coordinates r,z,θ proscribed about the concentre, it follows that modes exist having the form of the first equation in Figure 1,where U is the displacement field and N is a positive integer - the modes with cyclic symmetry of order N. Note that, due to the general isotropy, that adding a phase constant to the angular term yields another, linearly independent, solution. Furthermore, given the form of the circumferential variation of the displacements, it is clear that for each N, there exists another mode of the form of the second equation in Figure 1.
such that the inner product of the two modes over one rotation is necessarily zero. These two special modes are orthogonal and degenerate. The principle of a degenerate mode sensor is to influence one, but not the other, of the degenerate mode pair, and to observe the resulting natural frequency split. This has many distinct advantages over a traditional QCM or resonant sensor, most notably common-mode rejection of temperature and ageing bias effects. Breaking of the modal degeneracy impedes performance in devices based on these principles. For this reason, it is important to gain insight into how degeneracy is broken by underlying symmetry.

Take, for example, the mode displayed in Figure 2. The underlying symmetry is infinite-cyclic; the mode order is 6, and the degeneracy is present. Contrast the situation with Figure 3. The underlying symmetry is order 8; the modal order is 2; and degeneracy is broken. We explain these results intuitively and demonstrate agreement using COMSOL Multiphysics®.
In the paper, an analytical model for these effects based on earlier work of Gallacher[1] and group theory is presented. A series of COMSOL Multiphysics® models are developed, using the Solid Mechanics and Piezoelectric Devices interfaces, and solved using Modal, Frequency Domain, and Time Dependent studies. Both propagating-wave and modal behaviours are considered, and the results compared to the analytical model. Based on the conclusions drawn,

ramifications for degenerate sensor design are considered and simple, practical rules formulated to best exploit modal degeneracy and mitigate frequency splitting caused by material-mode cyclic symmetry interactions.

Reference

[1] "Design, Fabrication and Testing of a Multi-axis Vibrating Ring Gyroscope", B.J.Gallacher, PhD Thesis, Newcastle University, 2002.

Figures used in the abstract

$$\mathbf{u}(r,z,\theta) = \Psi_N(r,z)e^{2\pi i N\theta} + \Psi_{-N}(r,z)e^{-2\pi i N\theta}$$

$$\overline{\mathbf{u}}(r,z,\theta) = \Psi_N(r,z)e^{i\pi\left(2+\frac{1}{2N}\right)N\theta} + \Psi_{-N}(r,z)e^{-i\pi\left(2+\frac{1}{2N}\right)N\theta}$$

Figure 1: Equations referenced in the abstract.

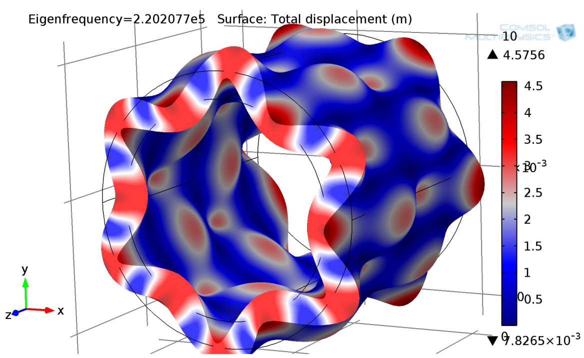

Figure 2: One of the degenerate pair of Order 6,5 - modes of a cylinder

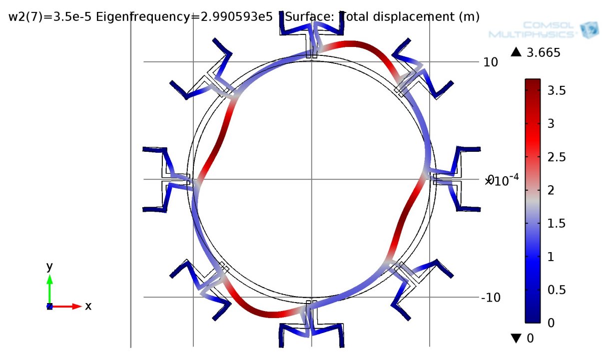

Figure 3: Order-2 flexural mode of a ring with symmetry broken by the suspension