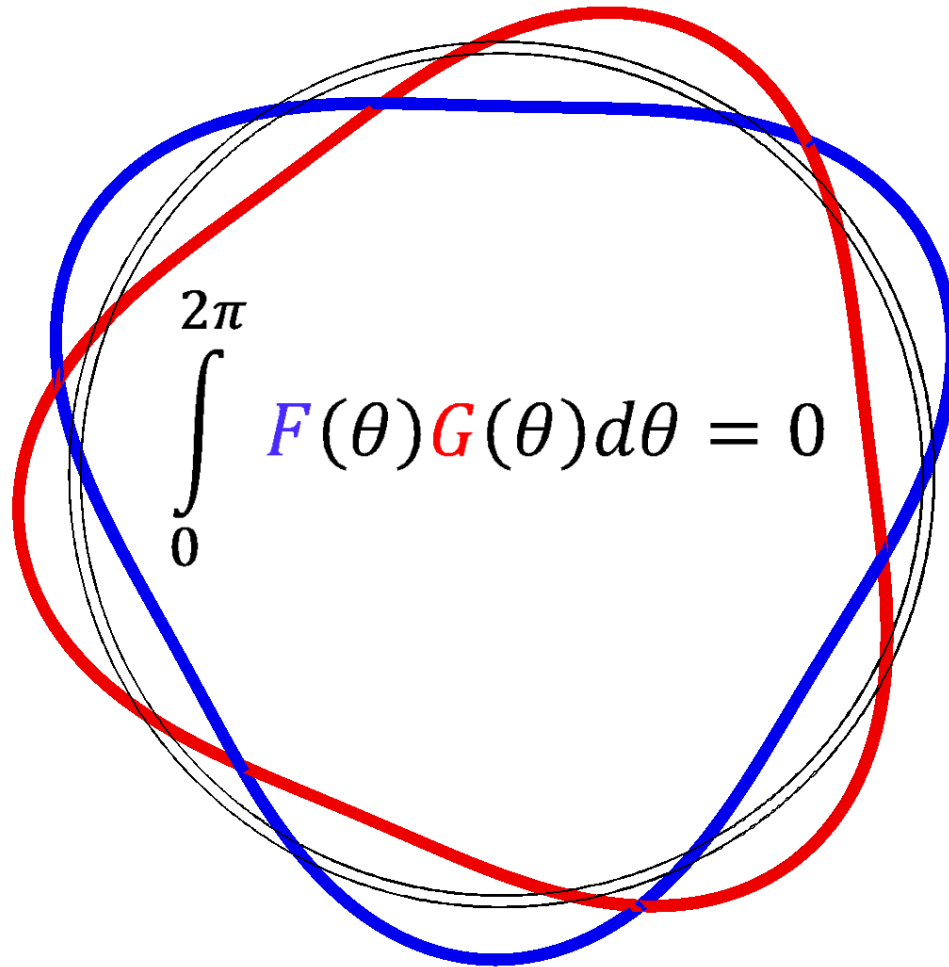


# **MODAL DEGENERACY AND SYMMETRY BREAKING IN MICROSYSTEMS**

COMSOL  
CONFERENCE  
ROTTERDAM2013

**HARRY T.D. GRIGG**  
**NEWCASTLE UNIVERSITY**  
**MICROSYSTEMS GROUP**

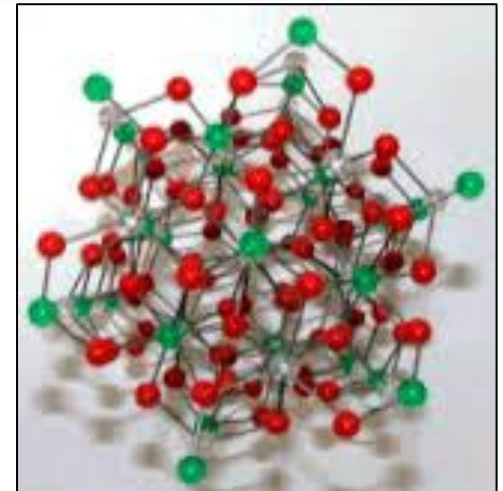

$$\int_0^{2\pi} F(\theta)G(\theta)d\theta = 0$$

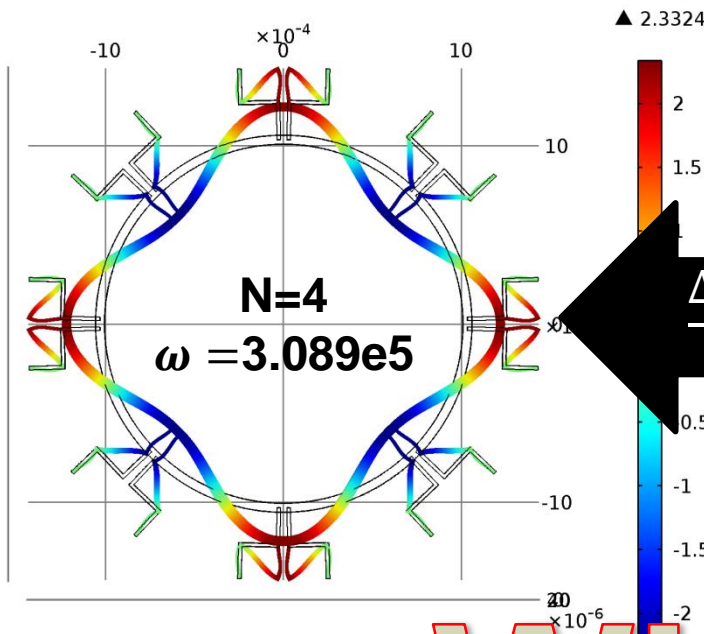
# GENERAL PRINCIPLES II

- **Breaking of the symmetry destroys the degeneracy**
  - The natural frequencies become distinct
  - Coupling and sensitivity weaken for gyroscopes
  - We have demonstrated and continue to develop biosensors based on an analyte-dependent mass loading breaking the symmetry and inducing a frequency split
- **The symmetry of the system can be broken by the constitutive relations (material properties) or by the boundary conditions**

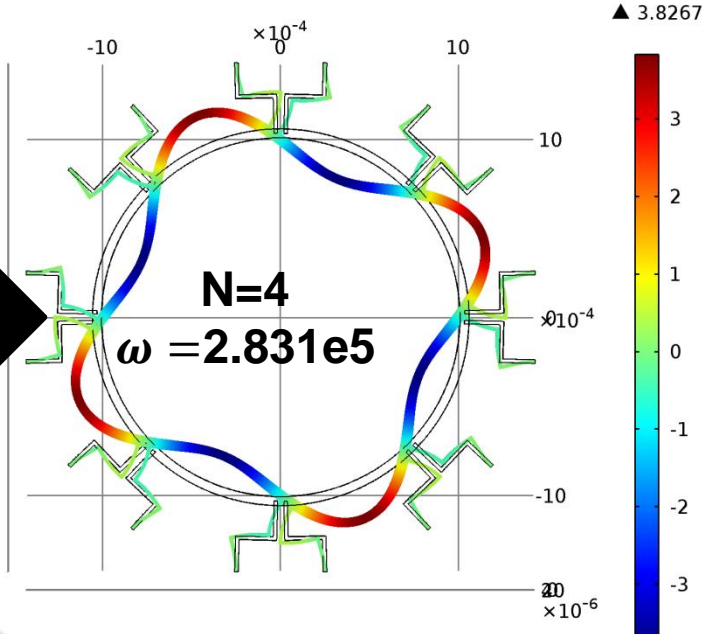
# MATERIAL SYMMETRY

- Even for geometries with cyclic symmetry of infinite order, if the underlying material is monocrystalline or otherwise anisotropic on the scale of the device
- MEMS devices often meet these conditions
  - Common monocrystalline substrates include Si, SiO, LiNbO<sub>3</sub>, etc.
  - The cut of the wafer is also relevant here
- There is no extant framework in the literature via which questions can be adequately addressed
- The variation of the material properties is readily expressed as a function of angle in a single plane via Bond matrix rotation or an equivalent

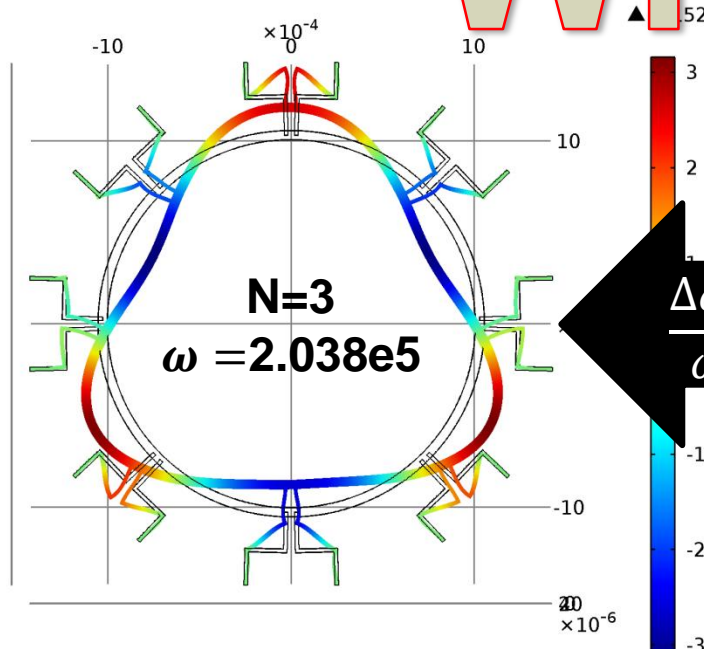




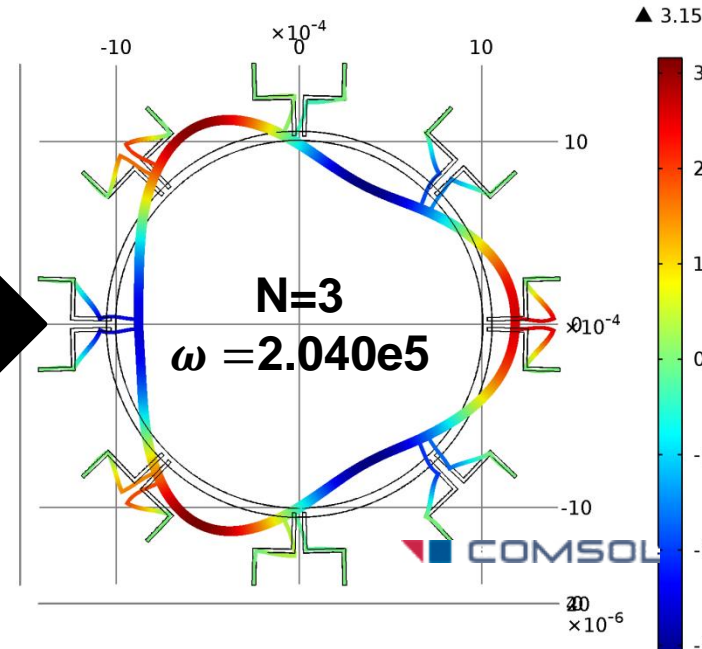
$\frac{\Delta\omega}{\omega} \approx 8\%$



Why?



$\frac{\Delta\omega}{\omega} \approx 0.1\%$



w2(4)=2e-5 Eigenfrequency=2.038445e5 Surface: u\*cos(atan2(y,x))+v\*sin(atan2(y,x)) (m)      w2(4)=2e-5 Eigenfrequency=2.040294e5 Surface: u\*cos(atan2(y,x))+v\*sin(atan2(y,x)) (m)

# THEORY

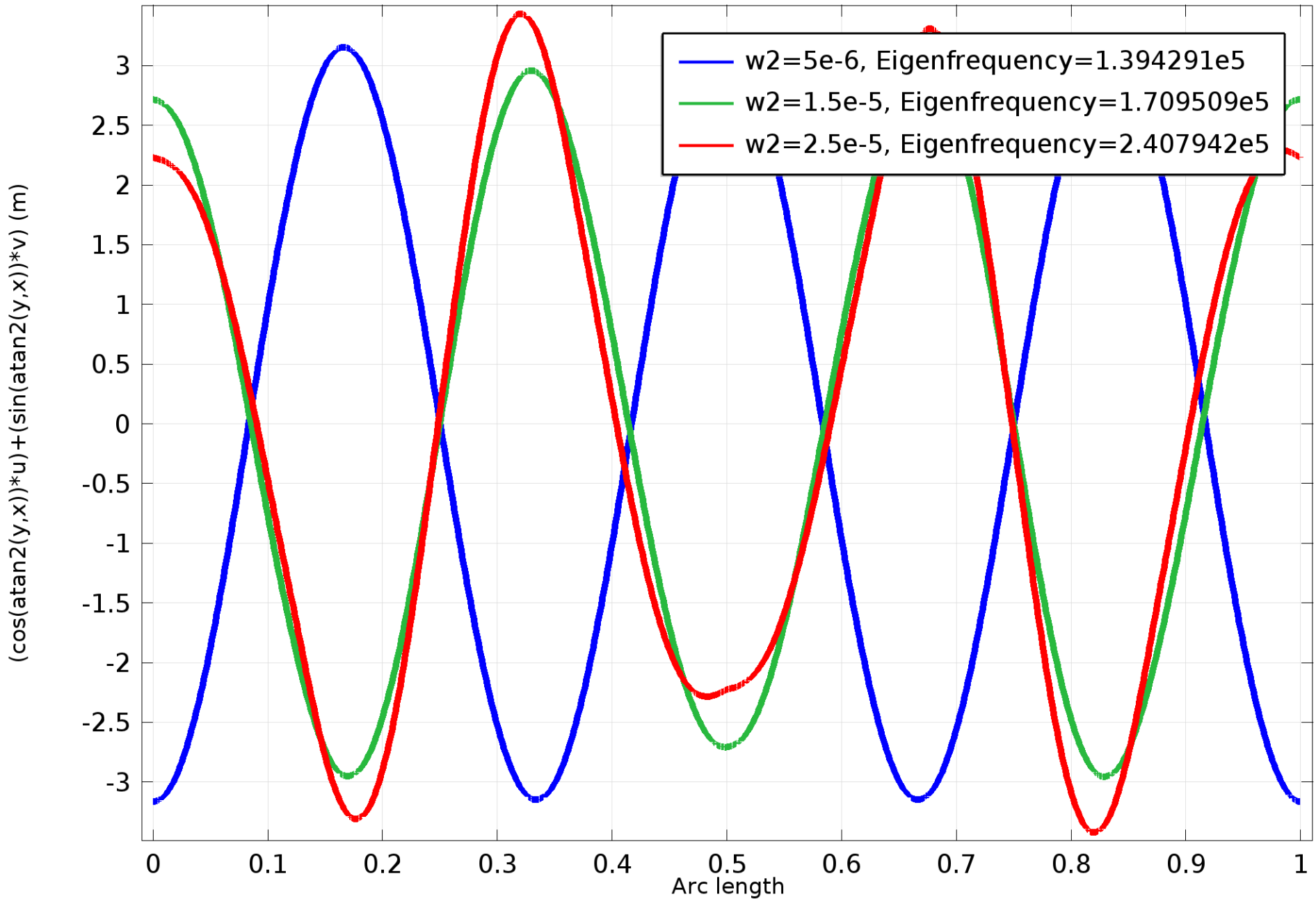
- Gallacher's 2003 thesis[1] addresses the problem from the perspective of Rayleigh's Quotient (RQ)

$$R = \frac{\langle \mathbf{u}, \mathbf{K}\mathbf{u} \rangle}{\langle \mathbf{u}, \mathbf{M}\mathbf{u} \rangle} \geq \omega^2$$

- Inner product has the form of an integral over the domain
- For a ring,  $\langle \mathbf{u}, \mathbf{K}\mathbf{u} \rangle = \int_0^{2\pi} EI(\mathbf{u}'')^2 + EAu(v^*)^2 d\theta$  ;  
 $\mathbf{u}_N = \cos(N\theta), \sin(N\theta)$  defines the degenerate (radial) pairs
- If the stiffness operator is perturbed by material properties, then the perturbation will manifest as a periodic variation in the Young's modulus
- Cyclic symmetry allows representation of the symmetry breaking as a Fourier series in the angular coordinate. A crystal with symmetry of order M will have Fourier components at M, 2M, and higher harmonics
- If N/M is not a natural number then the RQ of the system is **invariant** under the perturbation and **degeneracy is preserved** in this approximation

[1]: Design, Fabrication and Testing of a 3 Axis Vibrating Ring Gyroscope, PhD Thesis, Newcastle University, 2003

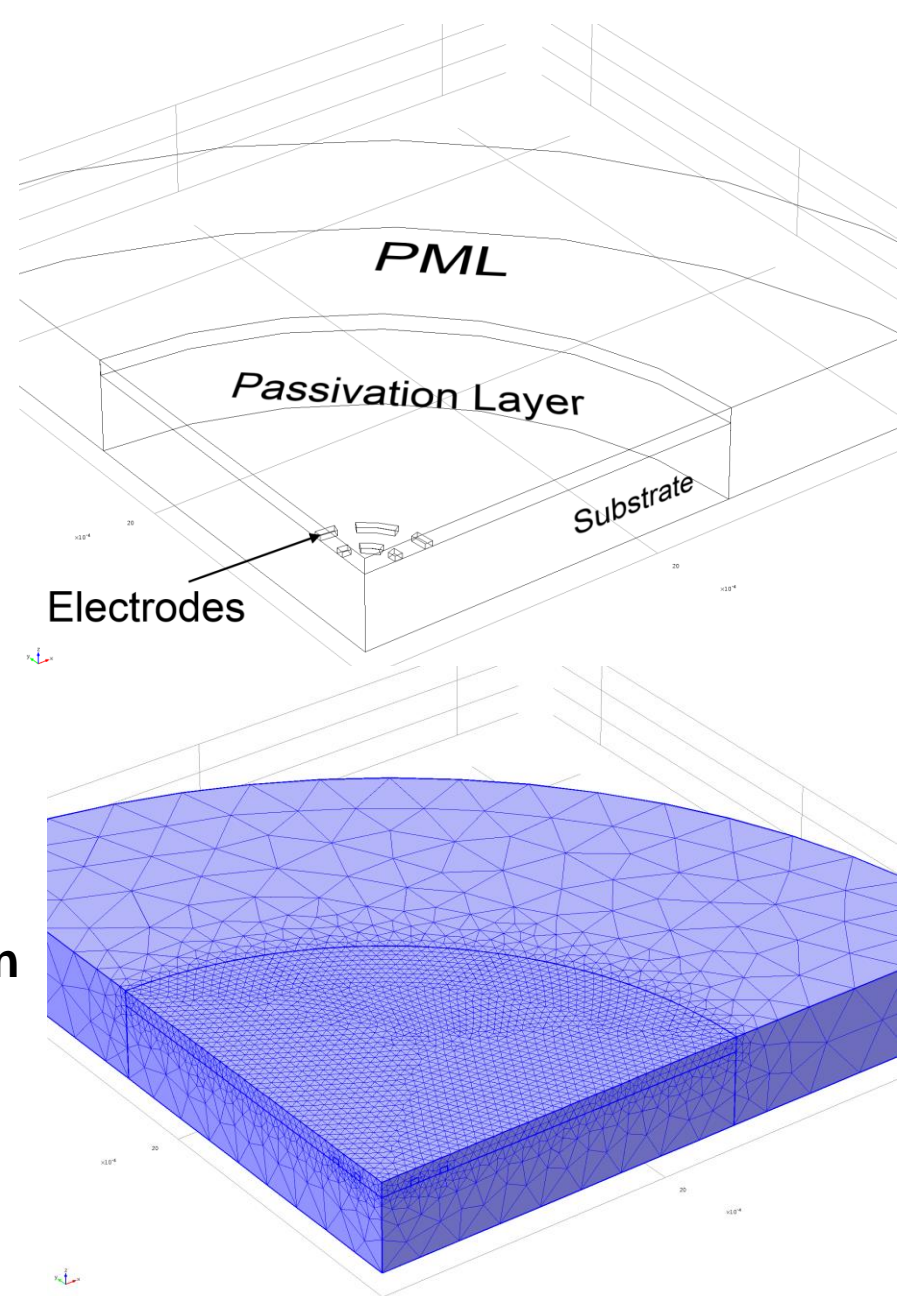
Line Graph:  $(\cos(\text{atan2}(y,x))*u)+(\sin(\text{atan2}(y,x))*v)$  (m)





# BIOSENSOR CONCEPT

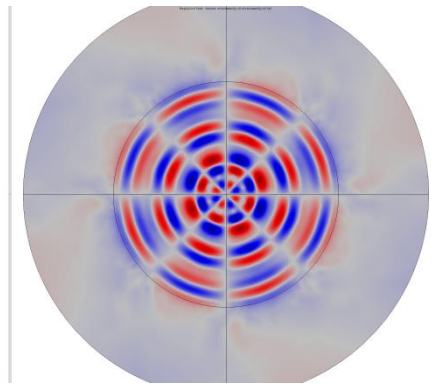
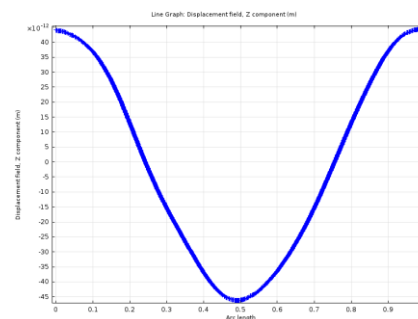
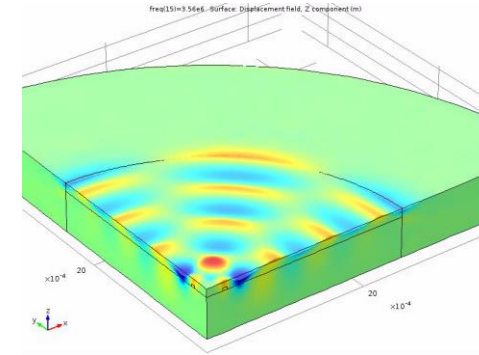
- Degenerate Rayleigh SAW device
- Novel concept developed by Professor Burdess and the team at Newcastle
- Piezoelectric anisotropic substrate overlain by isotropic layer – dispersive, anisotropic Rayleigh waves
- Modelled using Piezoelectric Devices interface, Frequency Domain and Time Dependent models
- PML used to simulate boundary absorber
- $N=4$  for this design





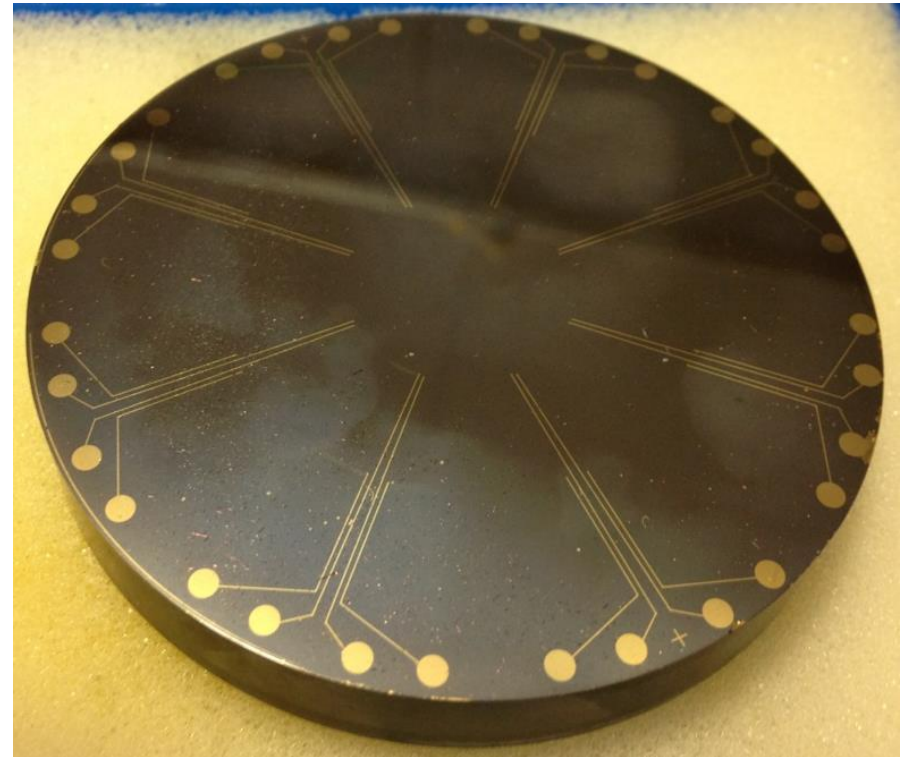
# BIOSENSOR RESULTS

- Frequency domain study yields outgoing wave front shapes
- By time symmetry, modes exist with the same spatial form
- By Fourier decomposing the displacements, the RR model yields the modal split
- Allows for selection of optimised wafer cut orientations and materials



# PROTOTYPING

- Prototypes fabricated by photolithography, CVD and RIE in-house
- Surface displacements mapped using our new UHF Polytec UHF-120 vibrometer
- First transduction prototype produced last week
- PZT Rayleigh device
- LiNbO<sub>3</sub> ongoing
- Other geometries also under investigation



# CONCLUSIONS

1. **Conservative symmetric systems with infinite order cyclic symmetry possess modes of vibration with lower order cyclic symmetry in degenerate pairs**
2. **By combining an analytical Rayleigh-Ritz approach with COMSOL models of complex wave propagation, information can be derived that is not accessible to either approach in isolation!**

**Thank you for listening!**