

Electrical Scale-up of Metallurgical Process

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Outline



- Problem formulation
- Theory
- Model setup
- Simulation results
- Conclusions

Problem formulation



- Case: Study scale-up of generic metallurgical process for primary metal production
 - How is current distribution affected by changing the size of the geometry?
 - Change electrode system from 1 phase to 3 phase.

Theory



- Current distribution governed by Maxwell's equations.

$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon}$$

$$\vec{\nabla} \times \vec{E} + \frac{\partial \vec{B}}{\partial t} = 0$$

$$\vec{\nabla} \times \vec{B} - \frac{1}{c^2} \frac{\partial \vec{E}}{\partial t} = \mu \vec{J}$$

- Assumptions:
 - Harmonic electric field
 - Propagating in x-direction into an infinite half-plane of conducting media
- Non-dimensionalization: $x = L\tilde{x}$, $E = E_0\tilde{E}$

$$\frac{\partial^2 \tilde{E}}{\partial \tilde{x}^2} - 2i \left(\frac{L}{\delta}\right)^2 \tilde{E} + \left(2\pi \frac{L}{\lambda}\right)^2 \tilde{E} = 0 \quad (1)$$

Skin depth: $\delta = \sqrt{\frac{2}{\omega\sigma\mu}}$

- Different regimes for the electric field depending on the terms $(L/\delta)^2$ and $(2\pi L/\lambda)^2$:
 - Electromagnetic waves, $\lambda \ll L$
 - Alternating current (AC), $\lambda \gg L$
 - High frequency, $\delta < L$
 - Low frequency, $\delta \gtrsim L$
 - Direct current (DC), $\delta \gg L$ (approximation $L \lesssim \delta/5$)
- Approximation of (1) for alternating current

$$\nabla^2 \vec{E} = \frac{2i}{\delta^2} \vec{E}$$

Theory



- Cylindrical coordinates, $r \in [0, R]$

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial E}{\partial r} \right) - \frac{2i}{\delta^2} E = 0$$

- Current density:

$$J(r) = J_0 \left| \frac{J_0 \left((1-i) \frac{r}{\delta} \right)}{J_0 \left((1-i) \frac{R}{\delta} \right)} \right|$$

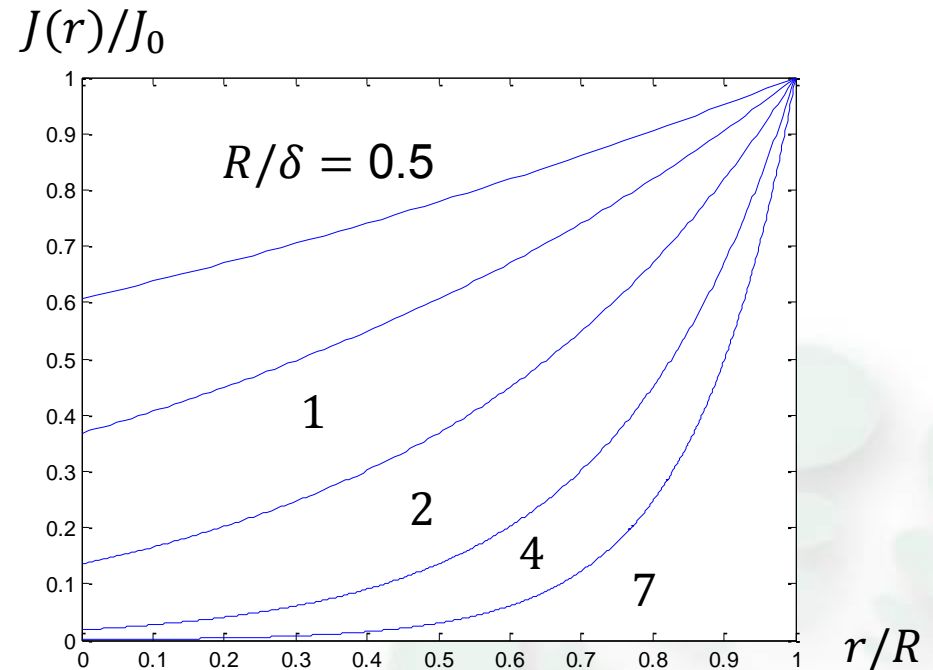
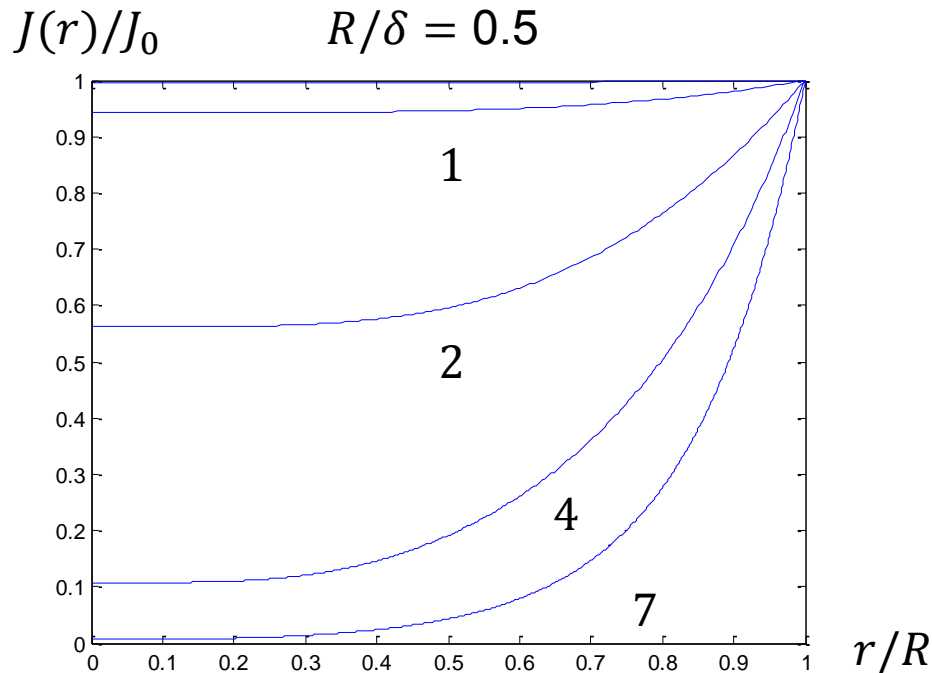
- $J_0(x)$ is zero order Bessel function of first kind
- Approximation yields: $J(r) \approx J_0 e^{-(R-r)/\delta}$

Theory

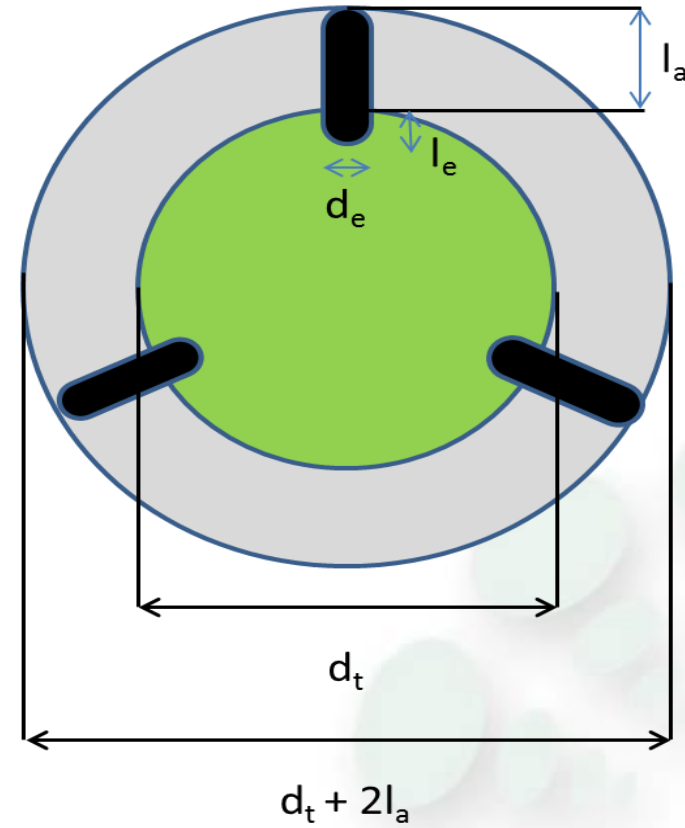
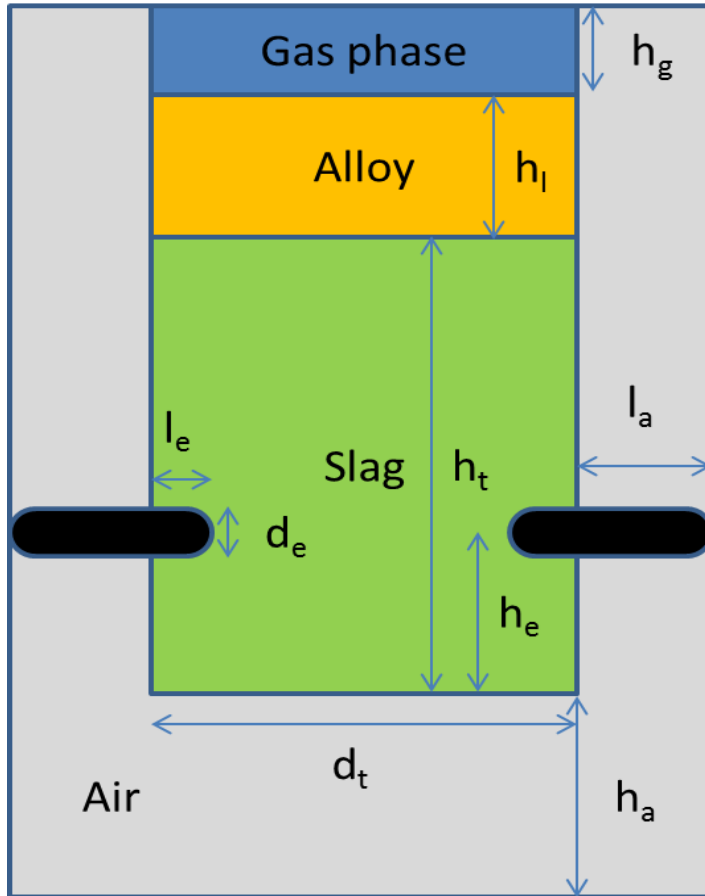


Exact solution

Approximate solution



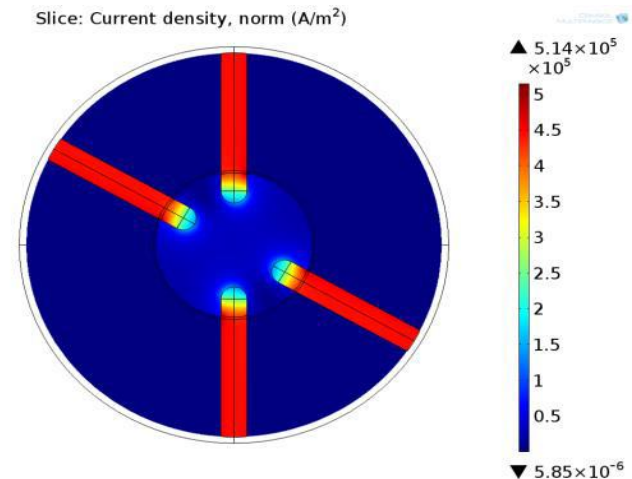
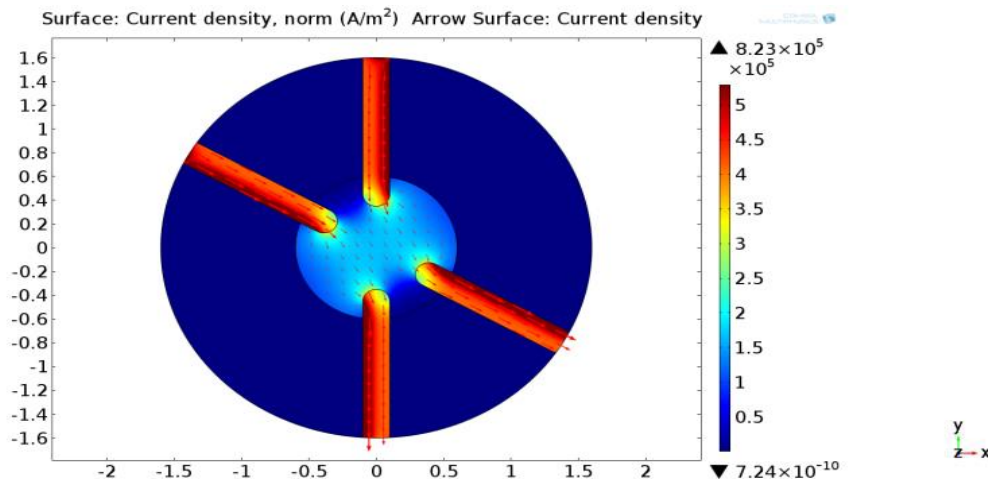
Geometry



2D Simulation Results



- 2D modeling a challenge due to infinite electrode plates

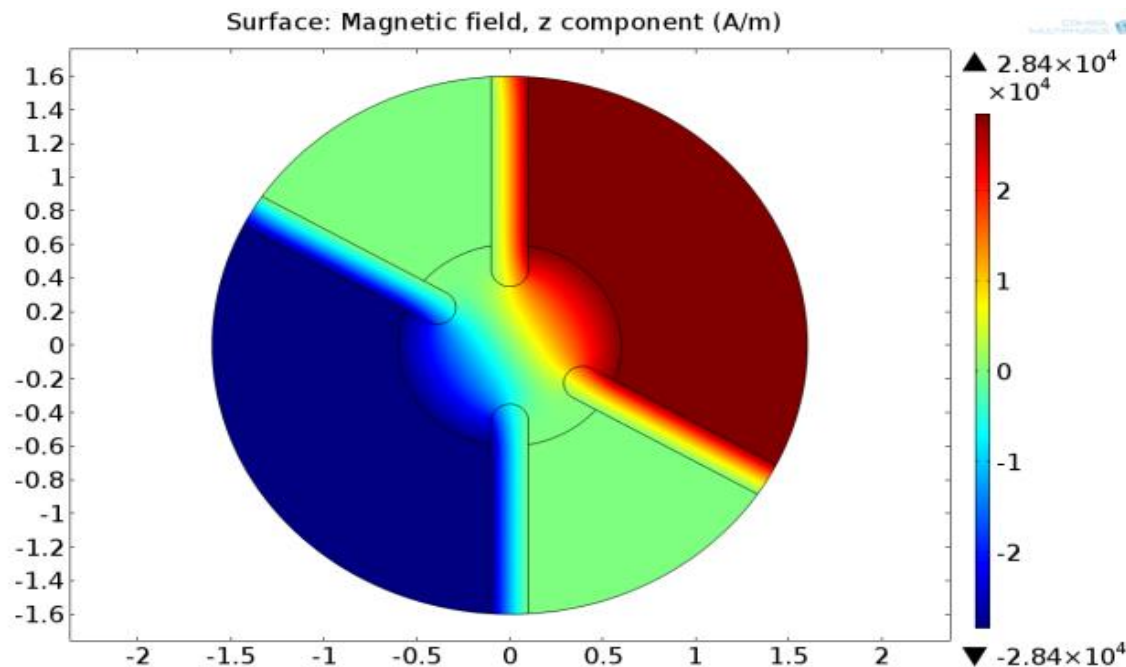


- Effect vanishes for 3D model

2D Simulation Results



- Due to Ampère's law magnetic field is proportional to the current and independent on distance

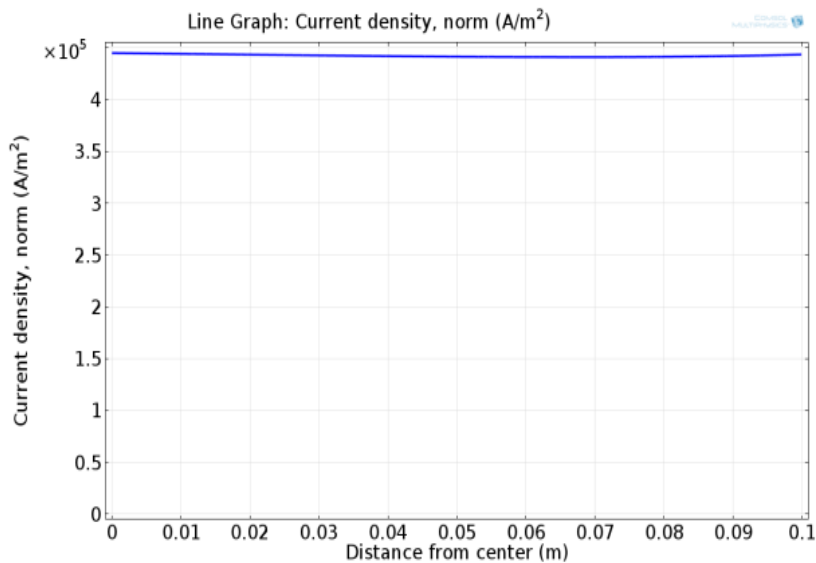


3D Simulation Results

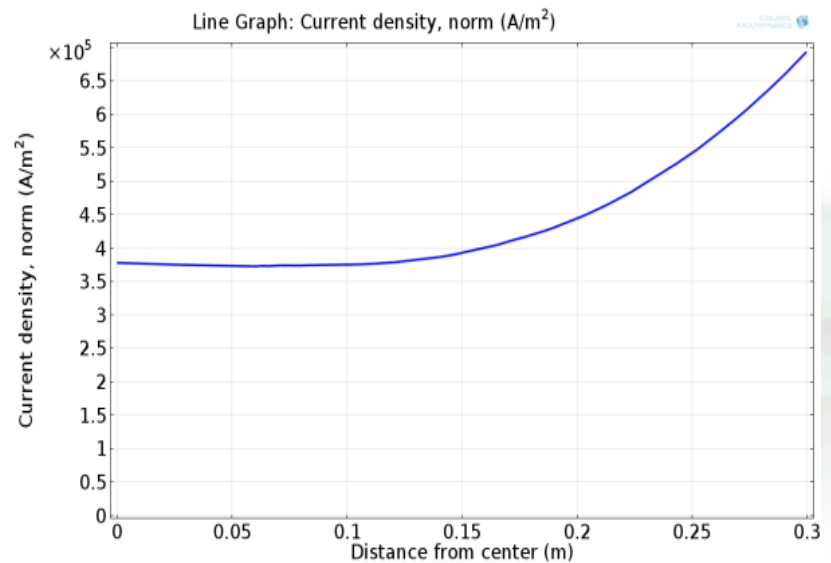


- Current density along electrode radius

$$(R/\delta)^2 \approx 0.5$$



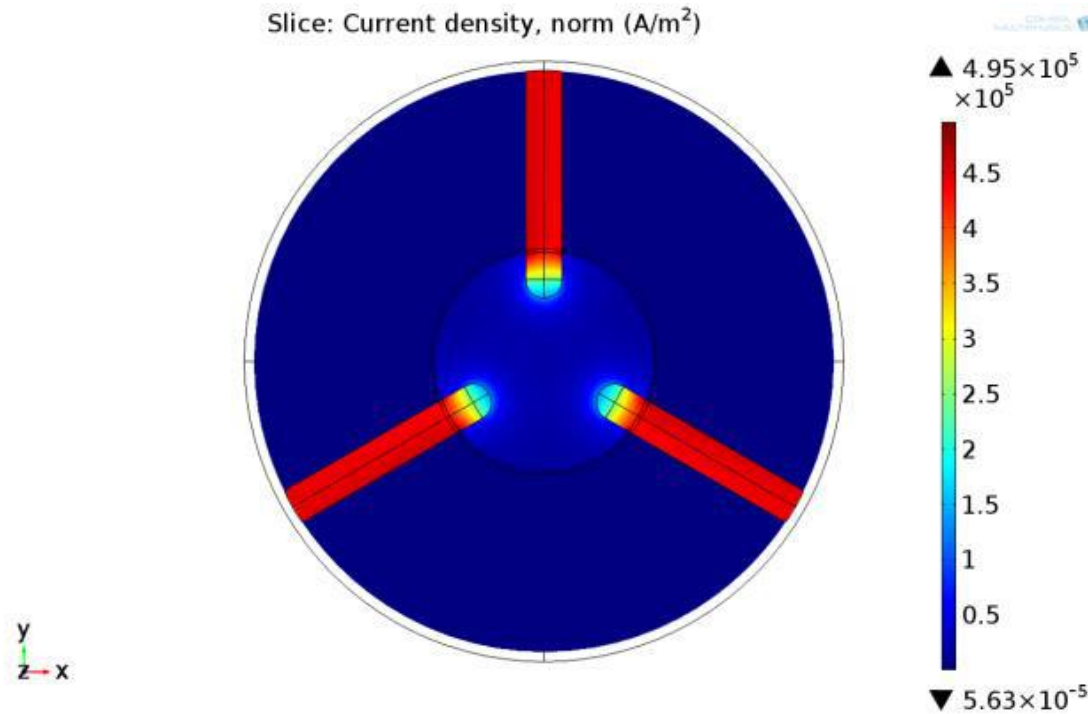
$$(R/\delta)^2 \approx 4$$



3D Simulation Results



- Resistive loss (current density) in slag is almost half compared to two 1 phase electrode pairs



Conclusions



- Shown three different regimes
 - Electromagnetic waves
 - Alternating current
 - Direct current
- Identified the parameter $(L/\delta)^2$ for electrical scale-up
 - DC approximation is reasonable for $L \lesssim \delta/5$
- 2D AC simulation show unreasonable strong proximity effects