Flexible Numerical Platform for Electrical Impedance Tomography

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Introduction: electrical imaging

Metallic electrodes for current injection & potential measurements
How to take into account contact impedances ?

Complete Electrode Model [1] in Electrical Impedance Tomography [2]

$$\begin{cases} -\nabla\cdot(\sigma\nabla v) = 0 & \forall x \in \Omega \\ v + z_e \sigma \nabla v\cdot \mathbf{n} = v_e & \forall x \in E_e \\ \int_{E_e} \sigma\nabla v\cdot\mathbf{n}\, d\Gamma = i_e & \forall e = 1\ldots E \\ \sigma\nabla v\cdot\mathbf{n} = 0 & \forall x\in\partial\Omega\setminus\bigcup_{e=1}^{E} E_e \end{cases}$$

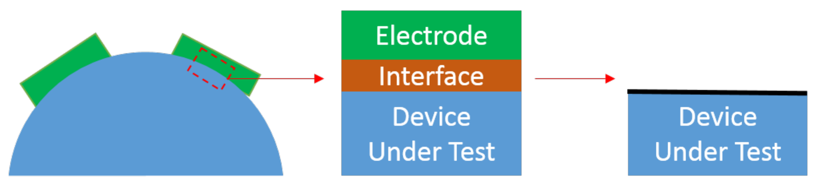

Figure: Electrode & interface as a boundary condition

Literature solutions

1. Custom-built finite element method (FEM) -based librairies, e.g. EIDORS [3]
Development & debugging costs, simplistic computation assumptions, augmented stiffness matrix
2. Model the thickness of electrodes within generalist FEM packages, e.g. COMSOL [4]
Huge number of uninformative degrees of freedom, how to recover sensitivity patterns ? [5]

Methods

Data predictions

Single NEUMANN boundary condition

Couple both current injection & contact impedance on a boundary

$$-\mathbf{n}\cdot\mathbf{j} = \frac{1}{z_e\|E_e\|}\left(\int_{E_e} v\, d\Gamma + z_e i_e - \|E_e\| v\right)$$

Electrode potentials deduced during post-processing

Sensitivity analysis

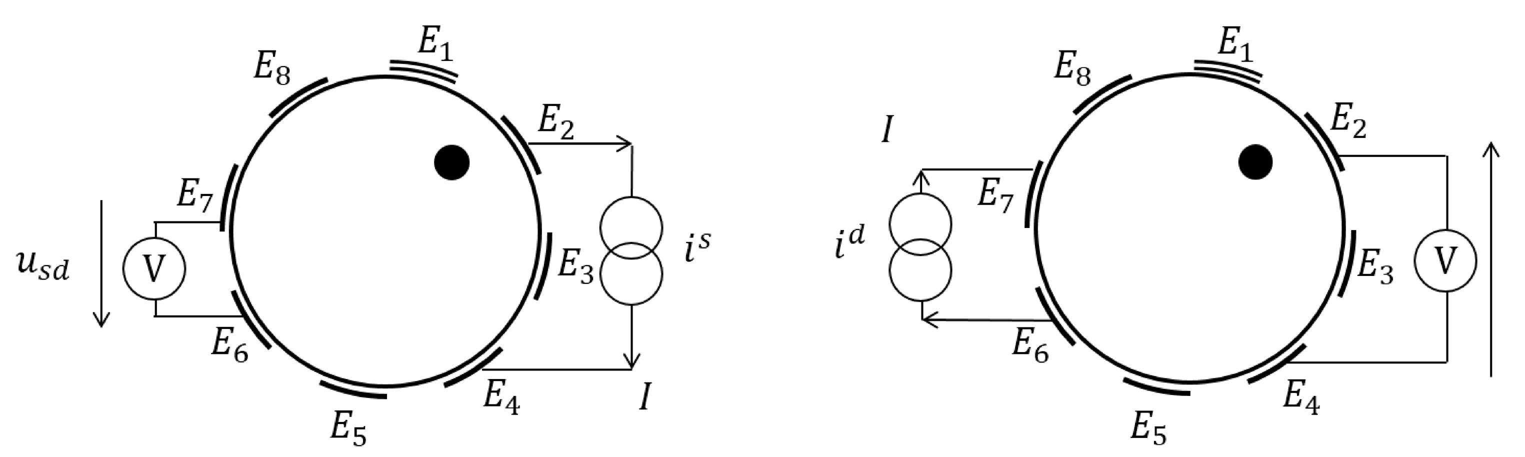

Figure: Dual configurations for sensitivity analysis: (left) source; (right) detector [6]

$$\frac{\partial u_{sd}}{\partial\sigma_c} = \frac{-1}{I}\int_{\Omega_c} \nabla v(i^s)\cdot\nabla v(i^d)\, d\Omega$$

VORONOÏ cells as control volumes [7]

Nodal approximation of electric fields

$$[\mathbf{J}]_{sd,n} = \frac{\partial u_{sd}}{\partial\sigma_n} = \frac{-\|\Pi_n\|}{I}\nabla\mathbf{v}_n(i^s)\cdot\nabla\mathbf{v}_n(i^d)$$

Parameter estimation : absolute or differential

Inverse problem operation

1. Comsol optimization module [8]

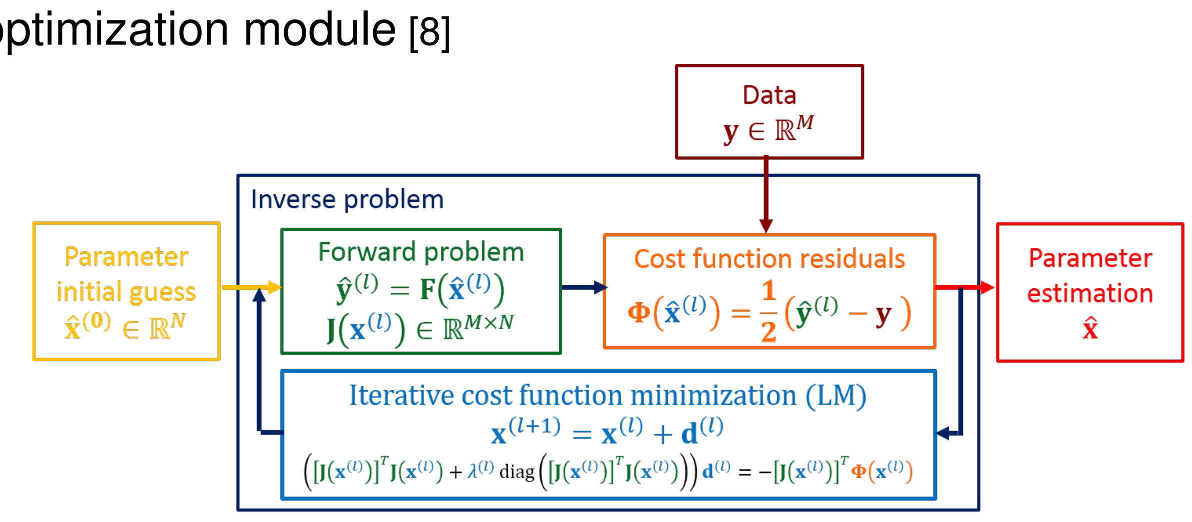

Figure: Absolute inversion in EIT using COMSOL : forward problem & cost function residuals are handled by a first component ; the optimization is operated by a second component, e.g. through a LEVENBERG - MARQUARDT algorithm

2. Any EIT inversion algorithm [2,9]
can rely on the proposed forward solver to reconstruct conductivity maps

Results

Forward solver

Verified versus analytic solutions, Test cases in 2D & 3D ($1\ \mathrm{mA}$, $1\ \mathrm{S\cdot m^{-1}}$)

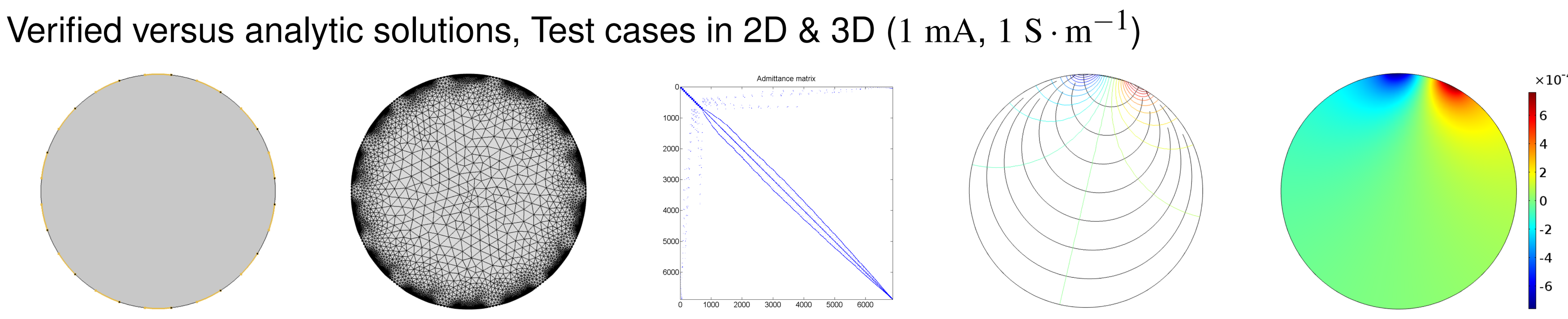

Figure: 2D test case: electrodes as boundaries, corresponding mesh, associated stiffness matrix, equipotentials and current density streamlines for the first projection

Influence of electrodes on the potential distribution, edge effects due to contact impedances

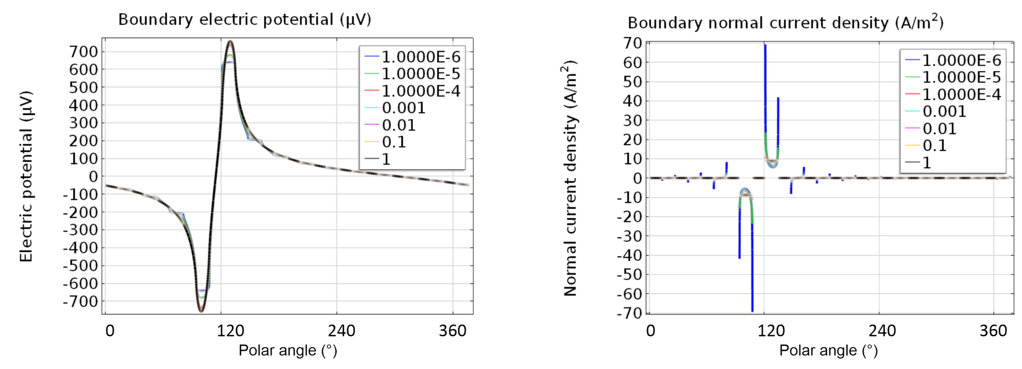

Figure: Boundary potential and normal current density, for varying z_e

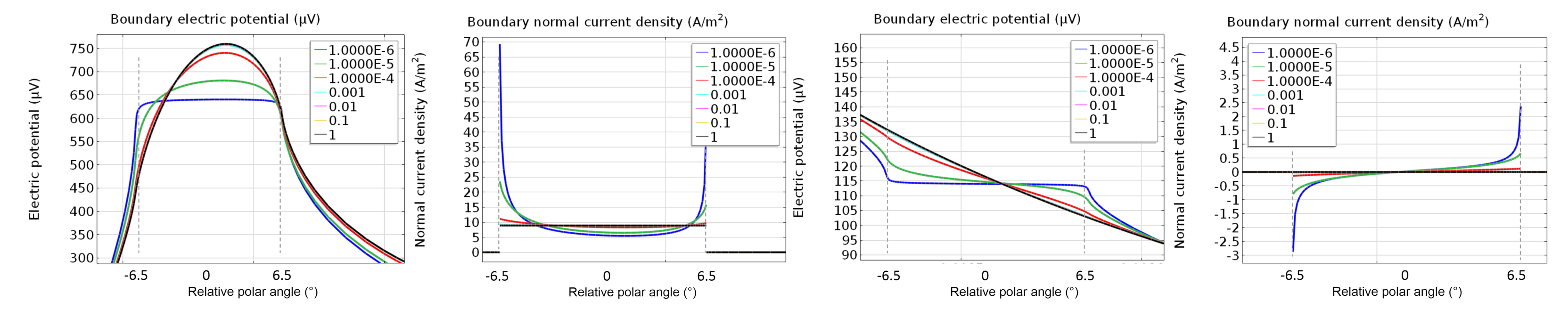

Figure: Boundary potential and normal current density: (left) source electrode; (right) detector electrode

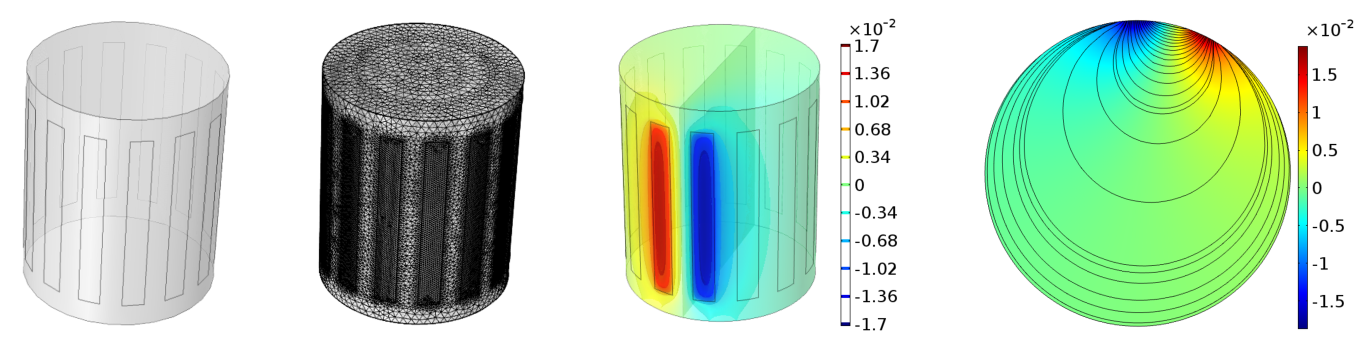

Figure: 3D test case: geometry, mesh, 3D isopotentials, cross-section

Benchmarking: EIDORS versus COMSOL

Figure of merit (FoM): maximum of relative error

Test case	FoM Node potentials	FoM Electrode potentials	CPU time (EIDORS / COMSOL)*
2D	10^{-10}	10^{-12}	26 s / 10 s
3D	10^{-4}	10^{-7}	330 s / 550 s

* CPU time encompasses only assembly of the stiffness matrix and solving in EIDORS while it also includes meshing in COMSOL

Reconstructions

Absolute inverse conductivity problem
Discontinuous GALERKIN elements (inverse mesh) within COMSOL optimization module

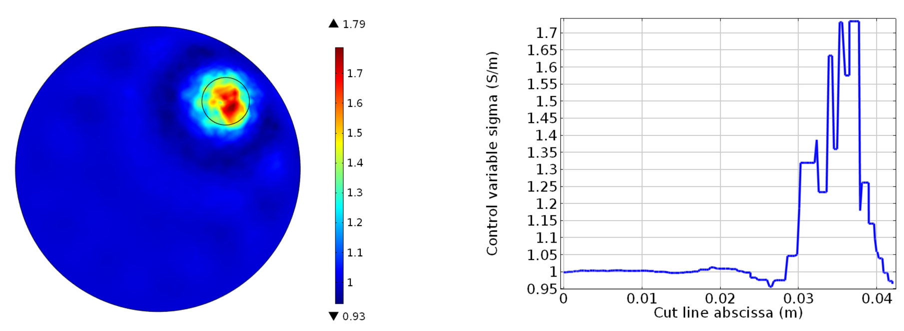

Figure: 2D absolute reconstruction on synthetic data: estimated conductivity map & profile along first diagonal

Differential inverse conductivity problem
Simplices for sensitivity computations inside COMSOL, parameter estimation in Matlab [9]

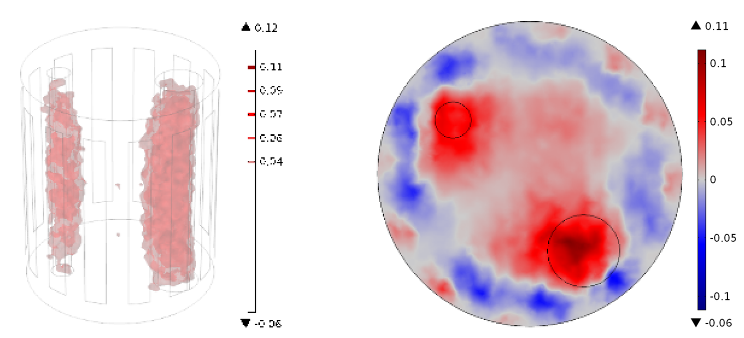

Figure: 3D differential reconstructions on *in vitro* data [10]: thresholded isosurfaces & cross-sectional map of estimated conductivity variation

Discussion et Perspectives

Versatile forward solver

Consistent numerical approximation with previous developments
Complete framework for EIT, with inverse problem capabilities
Extensions to other electromagnetic situations
e.g. tDCS, EEG, DBS (forward & inverse problems)

Future developments towards

Incorporation of advanced regularization strategies
Adaptive forward / inverse meshing schemes
Multispectral capabilities
Reducing CPU time for inverse problems & 3D models

References

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