Numerical Solution of the Natural Convection Around a Horizontal Cylinder Subjected to Non-uniform Radiative Heat

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Abstract: A two-dimensional steady mathematical model of the natural convection around a horizontal cylinder is solved numerically using COMSOL Multiphysics. The cylinder receives non-uniform heat from a radiating panel, which is assumed to be sufficiently far away so that the flow around the panel does not interfere with the cylinder. Changes in temperature and velocity profiles are monitored for varying heating rates and sample sizes, or, equivalently, the Rayleigh number Ra. It is found that COMSOL Multiphysics agrees well with the numerical results in the literature, which deal with isothermal and uniform heating scenarios. Variable flow properties and reradiation effects are found to have an appreciable effect on the temperature and flow around the horizontal cylinder subjected to non-uniform heating. It is found that larger cylinders are more likely to support a flame than smaller cylinders when exposed the same heating rate.

Keywords: Natural convection, Free Convection, Thermal Radiation, Horizontal Cylinders, View Factors.

1. Introduction

The laminar natural convection flow around a horizontal cylinder, which is subjected to non-uniform heat, is studied numerically. The cylinder receives heat from a radiating panel (Figure 1), which results in a buoyant flow. The primary reason for this study is to gain an insight into a flame-front preheating vegetation during a wildfire [1]. The possibility of attaining flammability conditions, which occur at around 300ºC for wood-based fuels, is of particular interest. This is explored for a variety of cylinder diameters and heating rates from the hot panel.

There are many industrial applications, including refrigeration, ventilation and the cooling of electrical components, for which the present study may also be applicable [2, 3].

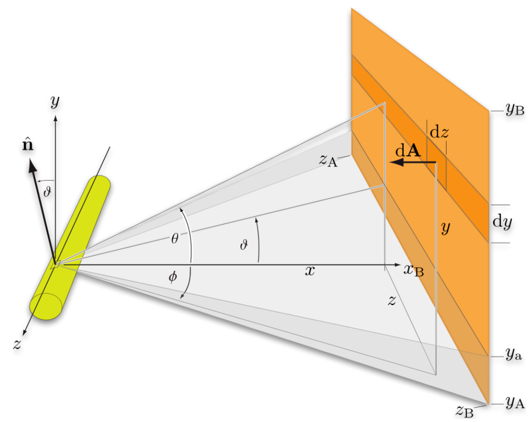

Figure 1. Schematic of cylinder heated by the hot radiating panel.

2. Governing Equations

Conservation equations for mass, momentum and energy, as well as the equation of state, hold in the air domain. The heat conduction equation applies for the solid cylinder domain. Two important parameters of the study are the Prandtl and Rayleigh numbers. If variable flow properties are assumed then the governing equations are as follows

$$\begin{aligned} \rho c_p \mathbf{u}\cdot\nabla T &= \nabla\cdot\lambda\nabla T \\ \rho\mathbf{u}\cdot\nabla\mathbf{u}+\nabla P &= \mathrm{Pr}\,(\nabla\cdot\boldsymbol{\tau}+\mathrm{Ra}\,\rho\,T\,\mathbf{j}) \\ \nabla\cdot\rho\mathbf{u} &= 0 \\ \rho &= \frac{1}{1+A_T T} \\ \nabla\cdot\lambda_s\nabla\Theta &= 0. \end{aligned}$$

The heat flux boundary condition is illustrated in Figure 2. It incorporates terms for solid and air conduction, reradiation, and a term that uses the geometry of the setup to calculate the view factor from the panel to the cylinder [4].

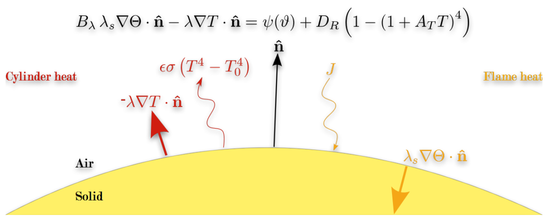

Figure 2. Heat flux boundary condition with non-uniform heating.

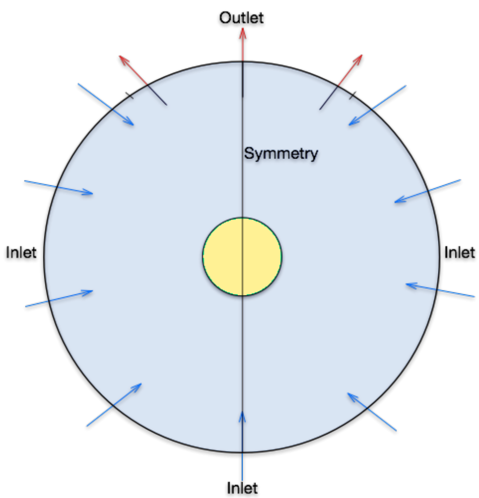

Figure 3. Inlet and outlet conditions with symmetry line.

Other conditions are the usual no-slip and continuity of temperature as well as inlet and outlet conditions for the outer domain (Figure 3). These conditions describe an `infinite' outer domain.

3. Radiating panel view factor

The radiating panel is modelled as a rectangle of finite width and height. However it is assumed to be much larger than the cylinder. This seems a reasonable assumption, as one would expect a flame-front during a wildfire to be larger than the vegetation in its path. Geometric arguments are used to calculate the heat received by the cylinder from the panel.

A visible element on the panel subtends the solid angle [4]

$$\mathrm{d}A = -\mathrm{d}\mathbf{A} \cdot \mathbf{r}/4\pi r^3$$

about a point on the cylinder (Figure 1). This means that the energy flux per unit surface area arriving at a point on the cylinder is given by

$$\mathrm{d}J = 4J_0 \frac{\hat{\mathbf{n}} \cdot \mathbf{r}}{r}\,\mathrm{d}A = \frac{J_0}{\pi} x_\mathrm{B} \frac{y\cos\vartheta - x_\mathrm{B}\sin\vartheta}{(x_\mathrm{B}^2 + y^2 + z^2)^2}\,\mathrm{d}y\,\mathrm{d}z.$$

Integrating this equation along the panel's width and height gives the radiative power per unit area received at a point on the cylinder as

$$J = \frac{J_0}{\pi} x_\mathrm{B} \int_{y_\mathrm{A}}^{y_\mathrm{B}} \int_{z_\mathrm{A}}^{z_\mathrm{B}} \frac{y\cos\vartheta - x_\mathrm{B}\sin\vartheta}{(x_\mathrm{B}^2 + y^2 + z^2)^2}\,\mathrm{d}z\,\mathrm{d}y.$$

In the case of a panel which has a much larger width compared to its height, the radiative power equation reduces to

$$J = \frac{J_0}{2}\left(\cos(\theta_\mathrm{A} - \vartheta) - \cos(\theta_\mathrm{B} - \vartheta)\right).$$

This formula is based on the assumption that the angles to the top and bottom of the cylinder (Figure 1) are between 0 and π so that the bottom and top of the cylinder are both visible. If this is not the case then the following equation for the nondimensional radiative power per unit area received holds

$$\psi(\vartheta) = \frac{1}{2}[\cos(\min\{\pi, \max[0, \theta_\mathrm{A} - \vartheta]\}) - \cos(\max\{0, \min[\pi, \theta_\mathrm{B} - \vartheta]\})].$$

This term then describes the heat received from the radiating panel by a point in the cylinder and is incorporated in the heat flux boundary condition (Figure 2).

4. Use of COMSOL Multiphysics

The nondimensional governing equations, together with the boundary conditions, are solved using COMSOL Multiphysics 3.5a. A stationary analysis is carried out using the Weakly Compressible Navier-Stokes, Convection and Conduction, and Heat Transfer by Conduction COMSOL Multiphysics 3.5a applications to solve the natural convection model with an UMFPACK direct linear system solver.

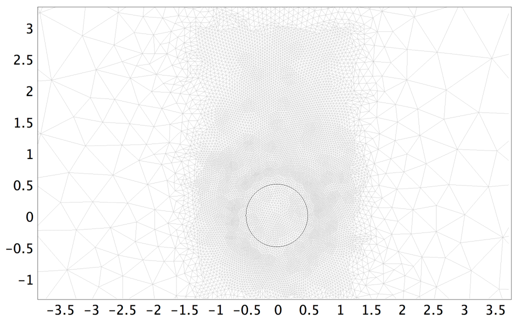

Figure 4. Typical adaptive mesh with about 30,000 elements.

The stationary analysis was carried out using adaptive mesh refinement (Figure 4) with about 30,000 elements spanning the combined solid and air domains. The governing equations were nondimensionalised before they were inserted into the relevant application [5].

4. Results

Test problems with isothermal and uniform heating conditions were carried out in order to verify the accuracy of the model. Results were found to be consistent with the literature [6, 7, 8]. This can be seen in Table 1 where the Nusselt number is the ratio of convective to conductive heat transfer across a boundary. The Nusselt number gives an ideal measurement to compare numerical results as it takes into account both heat and flow effects. The table shows a good agreement with the benchmark results of Saitoh et al. [7], where results were also found to be in good agreement with Kuehn and Goldstein [6] and Wang et al. [8].

Ra		$\theta = 0°$	90°	180°	$\overline{\text{Nu}}$
10^3	Present	3.782	3.363	1.217	3.011
	Saitoh et al. [7]	3.813	3.374	1.218	3.024
10^4	Present	5.960	5.389	1.533	4.809
	Saitoh et al. [7]	5.995	5.410	1.534	4.826
10^5	Present	9.752	8.813	1.984	7.939
	Saitoh et al. [7]	9.675	8.765	1.987	7.898

Table 1. Local and average Nusselt numbers for isothermal circular cylinder compared with the benchmark results of Saitoh et al. [7].

Reradiation effects and variable air properties are found to have a marked effect on the flow and temperature profile around the horizontal cylinder (Figure 5 and Table 2 respectively). The assumption of constant flow properties results in an overestimation of the temperature profile.

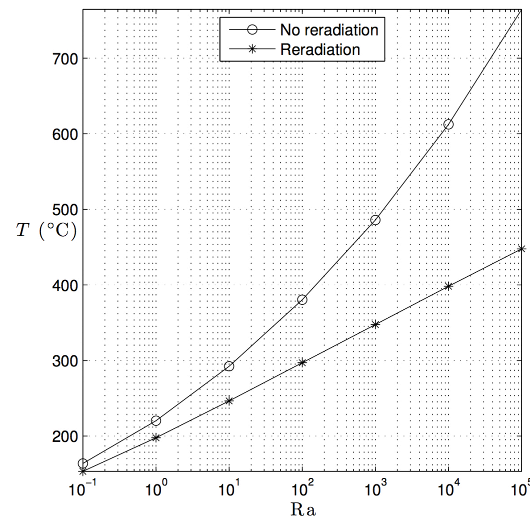

Figure 5. Surface temperature at 90º, showing the difference when reradiation is incorporated.

l (mm)	Ra (2 S.F.)	$\overline{\text{Nu}}$	$\overline{\text{Nu}}_{\text{const}}$	% difference
1	1.4×10^2	1.044	1.359	30
2	2.3×10^3	1.432	1.932	35
4	3.7×10^4	2.050	2.841	39
8	5.9×10^5	3.050	4.495	47
12	3.0×10^6	3.911	6.021	54
16	9.4×10^6	4.707	7.365	56

Table 2. Percentage change in average Nusselt numbers between constant and variable flow properties.

The experimental setup of Cohen and Finney [9] is used for a numerical study of their results. It is found that smaller cylinders attain lower temperatures when exposed to the same heating rate as larger cylinders (Figure 6), which is consistent with the experimental results in [8]. This suggests that larger fuels are more likely to support flames in a wildfire.

However, Figure 8 shows that the natural convection flow around larger cylinders (or, equivalently, cylinders exposed to larger heating rates) is stronger. The stronger flow around the larger and hotter cylinder would convect the pyrolysate vapour more quickly, which would dilute a potentially flammable mixture with air.

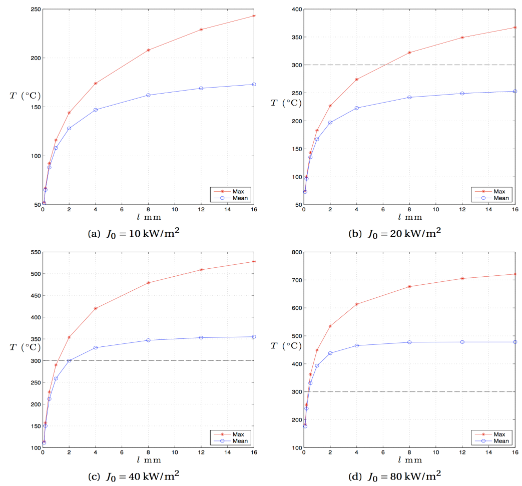

Figure 6. Cylinder temperatures for various cylinder sizes and heating rates.

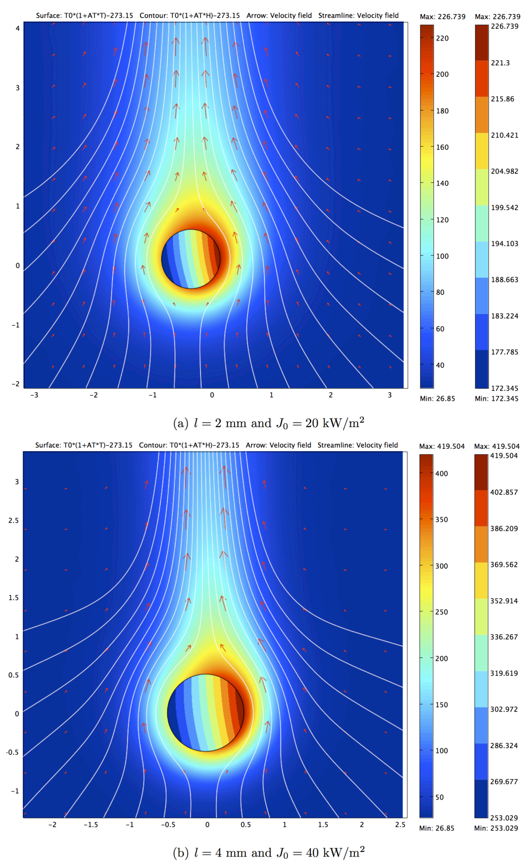

Figure 7. Flow and temperature profiles.

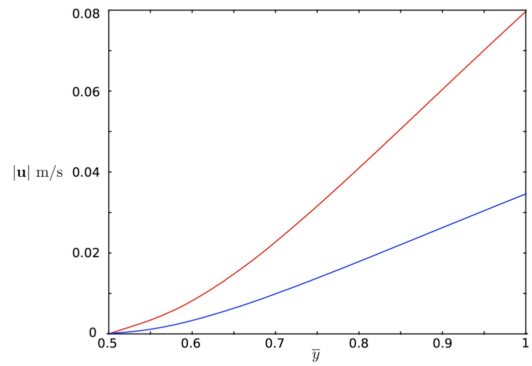

Figure 8. Flow speed against vertical position above cylinder for 2 mm (blue) and 4 mm (red) cylinders.

8. Conclusions

The natural convection model for the flow around a horizontal cylinder subjected to non-uniform heating from a radiating panel has been solved numerically using COMSOL Multiphysics 3.5a. The results suggest that though the temperatures are greater for larger cylinders, the flow is stronger too. Hence the flammable vapour that builds up around the cylinder is convected away due to the buoyant flow. This would tend to dilute the mixture thus reducing the possibility of ignition, although none of these issues in the progress towards flaming are examined here. However, it is envisaged that the strongly nonlinear increase in the rate of pyrolysis with temperature is likely to dominate, compared with rising buoyant flow, the chances of the production of a flammable mixture.

9. References

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