# Topology Optimization of Thermal Heat Sinks

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COMSOL
CONFERENCE
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# Outline of presentation

- Introduction to topology optimization

- Thermofluid model
  - Heat sink topology optimization model
  - Fluid dynamics modeling
  - Heat transfer modeling

- Results
  - Optimized designs
  - Parameter studies

- Summary and outlook

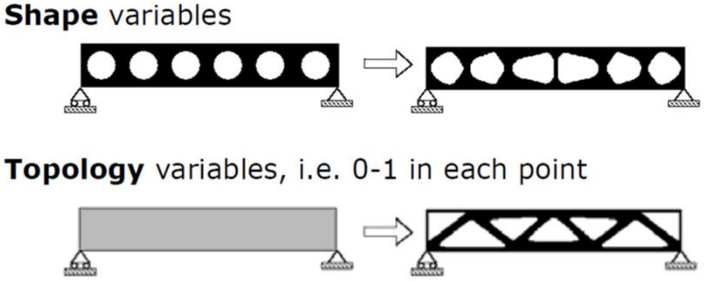
# Topology optimization – introduction

- What is topology optimization?
  - Originated in structural mechanics
  - Optimal material distribution in given design domain

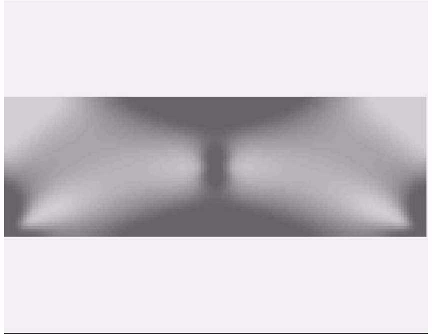
- Density-based topology optimization
  - Design variable field γ: 0 → solid; 1 → void
  - Problem relaxed to γ values between 0 and 1

  →Gradient-based optimization possible

- Advantages of topology optimization
  - Systematic design approach
  - Can yield unintuitive structures



Bendsøe et al. 2003



# Thermofluid topology optimization model

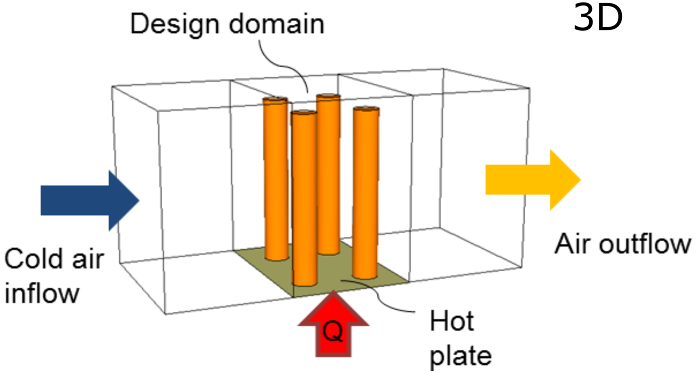
- Forced convection cooled heat sink

- Possible applications
  - CPU cooling
  - Thermoelectric generators

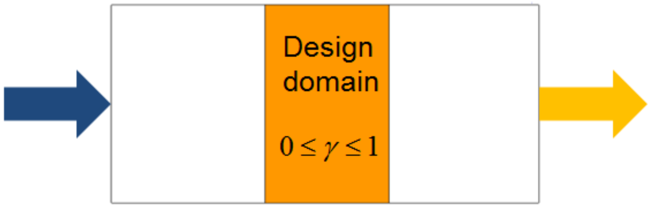
- Optimization objective
  - Minimization of solid plate temperature for prescribed pressure drop

- Topology optimization carried out with Optimization Module and LiveLink for MATLAB
  - Optimization method: MMA

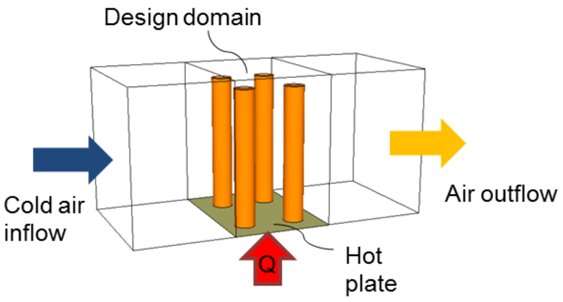
3D



2D



# Fluid dynamics modeling



- Implemented in CFD Module
- Assumptions
  - Stationary laminar flow
  - Incompressible fluid
  - 2D
- Incompressible Navier-Stokes and continuity equation

$$\rho_{fl} \cdot (\mathbf{u} \cdot \nabla)\mathbf{u} = -\nabla p + \mu(\nabla^2 \mathbf{u}) - \boxed{\alpha(\gamma)\mathbf{u}}$$

$$\nabla \cdot \mathbf{u} = 0$$

- Interpolation of artificial friction force
  - Fluid region $\gamma = 1 \Rightarrow \alpha = 0 \Rightarrow NS\ equation$
  - Solid region $\gamma = 0 \Rightarrow \alpha = \alpha_{\max} \approx \infty \Rightarrow \mathbf{u} = \mathbf{0}$

# Heat transfer modeling

- Implemented in Heat Transfer Module
- Heat transfer in fluid outside design domain

$$\rho_{fl} c_{fl} \mathbf{u} \cdot \nabla T_{fl} - \nabla \cdot (k_{fl} \nabla T_{fl}) = 0$$

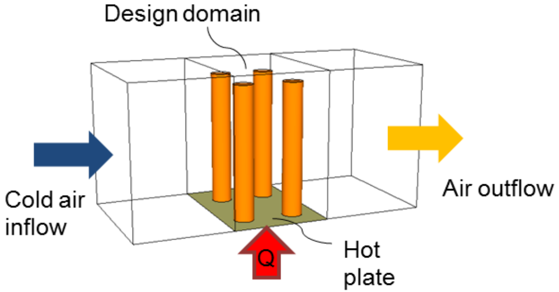
- Heat transfer modeling in design domain

$$\rho_{fl} c_{fl} \mathbf{u} \cdot \nabla T_{fl} - \nabla \cdot (\boxed{k(\gamma)} \nabla T_{fl}) = \boxed{\frac{h(\gamma)(T_s - T_{fl})}{dz_{fl}}}$$

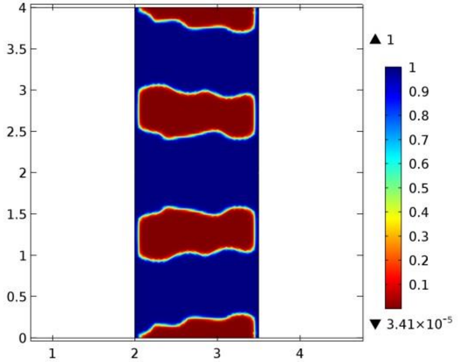
- Heat transfer in solid plate

$$-\nabla \cdot (k_s \nabla T_s) = \frac{q_{prod}}{dz_s} - \boxed{\frac{h(\gamma)(T_s - T_{fl})}{dz_s}}$$

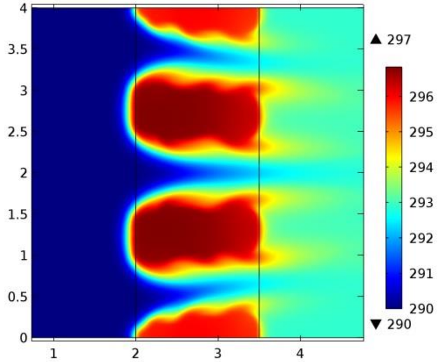
- Interpolation of
  - Thermal conductivity in design space
  - Out of plane heat transfer between solid and fluid layer



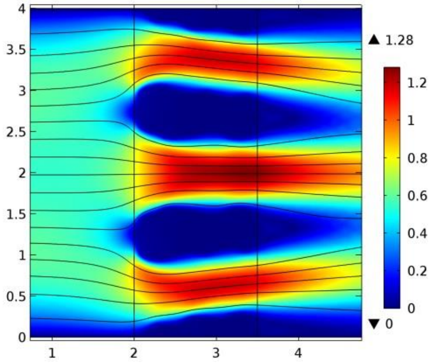
# Results optimization



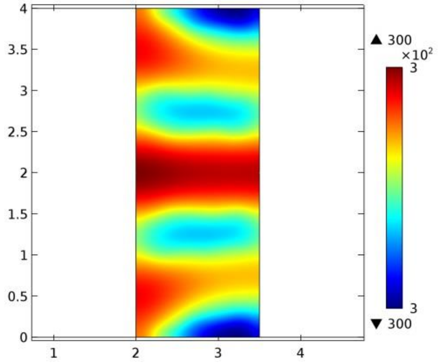
Design variable field



Fluid temperature (K)



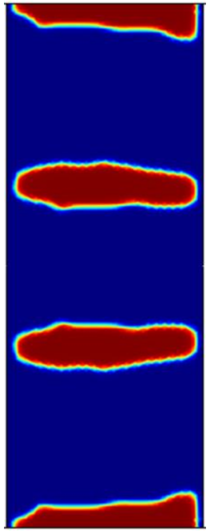
Velocity (m/s) and streamlines



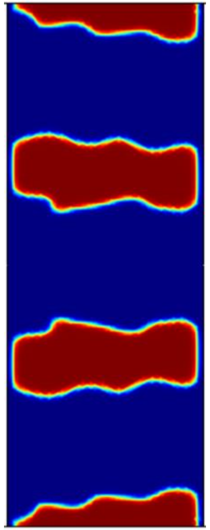
Solid plate temperature (K)

# Results – effect of pressure drop

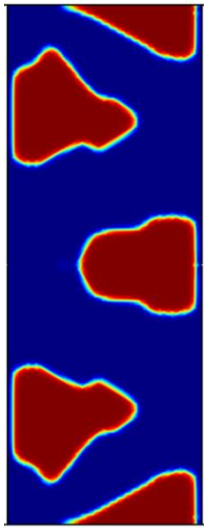
0.01 Pa



0.1 Pa



0.5 Pa

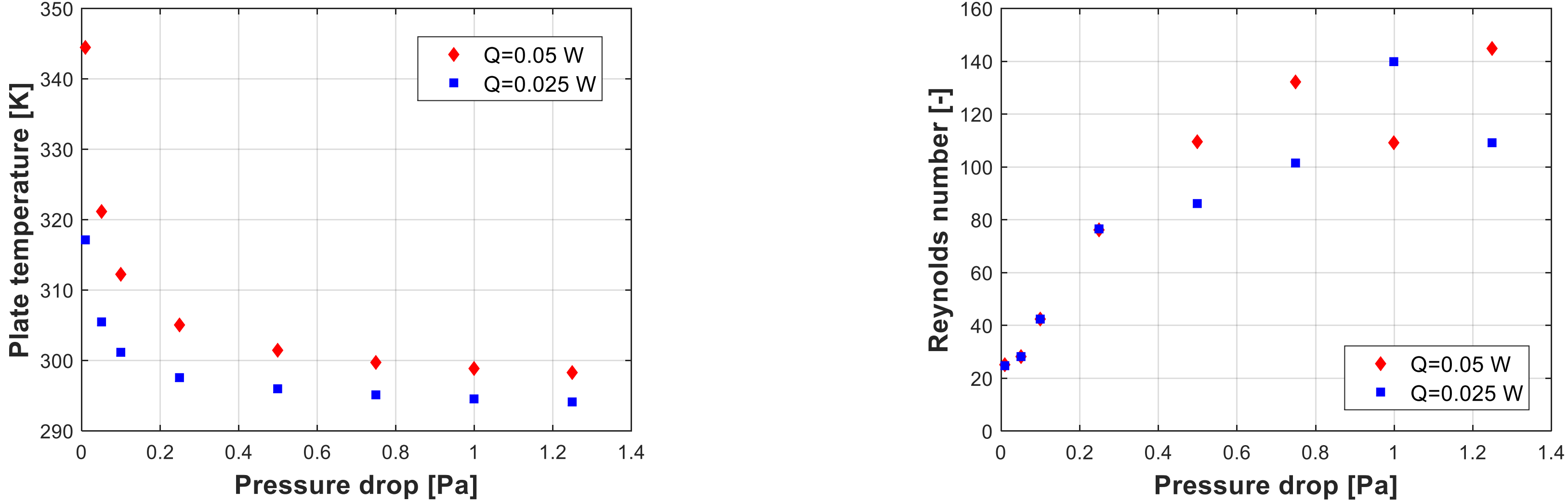


1.0 Pa

→ More and bigger fins with increasing Δp

→ Bumps on fins increase with increasing Δp

# Parameter studies – optimized structures



→Decreasing plate temperature with increasing Δp

→Generally Re increases with increasing Δp

→Re can decrease with Δp if new fin is added to design

# Summary and Outlook

Summary

- Thermofluid topology optimization model implemented in COMSOL Multiphysics
- Optimized designs and parameter studies presented

→ COMSOL Multiphysics allows for relatively straightforward implementation of topology optimization

Outlook

- Validation of 2D topology optimization
  - Full 3D optimization
  - Experimental comparison to standard structures

# Acknowledgments

This work was financed by the TOpTEn project sponsored through the Sapere Aude Program of the Danish Council for Independent Research (DFF – 4005-00320).

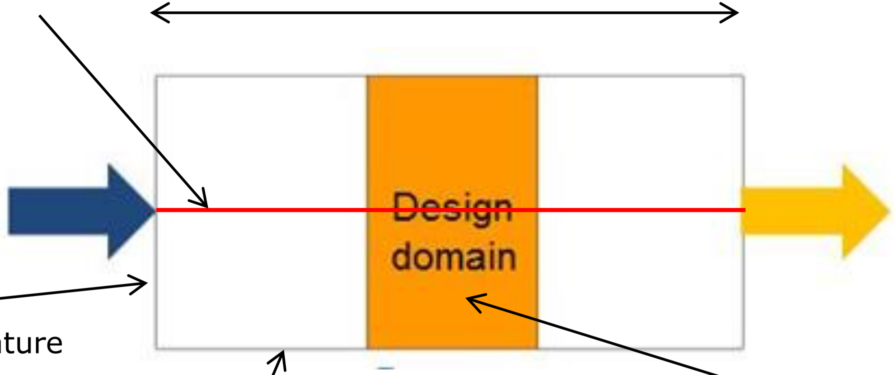
Sapere Aude

**DET FRIE FORSKNINGSRÅD**
DANISH COUNCIL FOR
INDEPENDENT RESEARCH

# Backup

# Thermofluid heat sink model II

## Boundary conditions thermofluid design layer



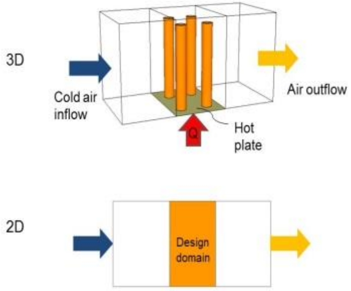
Prescribed system pressure drop

Symmetry condition

Fixed inlet temperature

No slip condition and thermal insulation at channel wall

Heat transfer between solid layer and thermofluid design layer



# COMSOL for topology optimization

- Advantages
  - Straightforward multiphysics modeling
  - MMA optimizer implemented
  - PDE filter can be easily added
  - Advanced post-processing tools available
- Drawback
  - Limited scalability for large scale optimization

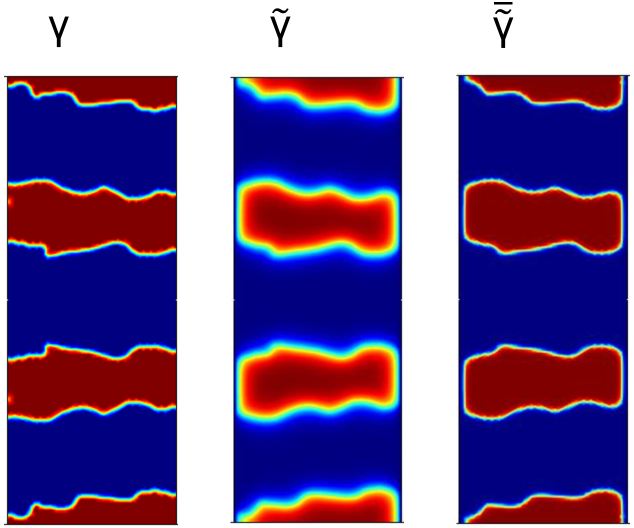
# Topology optimization – filter and projection

- Density filter needed to provide regularization for the optimization problem

- Implemented as PDE filter within COMSOL’s “Coefficient Form PDE interface”

$$-r^2\Delta\tilde{\gamma}+\tilde{\gamma}=\gamma$$

- Three-field topology optimization
  - Design variable: γ
  - Density filter (PDE) → $\tilde{\gamma}$
  - Smoothed Heaviside projection → $\bar{\tilde{\gamma}}$



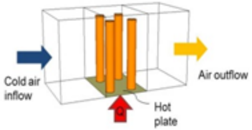
# Heat transfer modeling - complete

- Heat transfer modeling solid plate

$$-\nabla\cdot(k_s\nabla T_s) = \frac{q_{prod}}{dz_s} - \frac{h(\gamma)(T_s - T_{fl})}{dz_s}$$

- Heat transfer modeling fluid

$$\rho_{fl} c_{fl}\mathbf{u}\cdot\nabla T_{fl} - \nabla\cdot(k_{fl}\nabla T_{fl}) = 0$$



- Heat transfer modeling design space

$$\rho_{fl} c_{p,fl}\mathbf{u}\cdot\nabla T + \nabla\cdot(-k(\gamma)\nabla T) = \frac{h(\gamma)\cdot(T_s - T_{fl})}{dz_s}$$

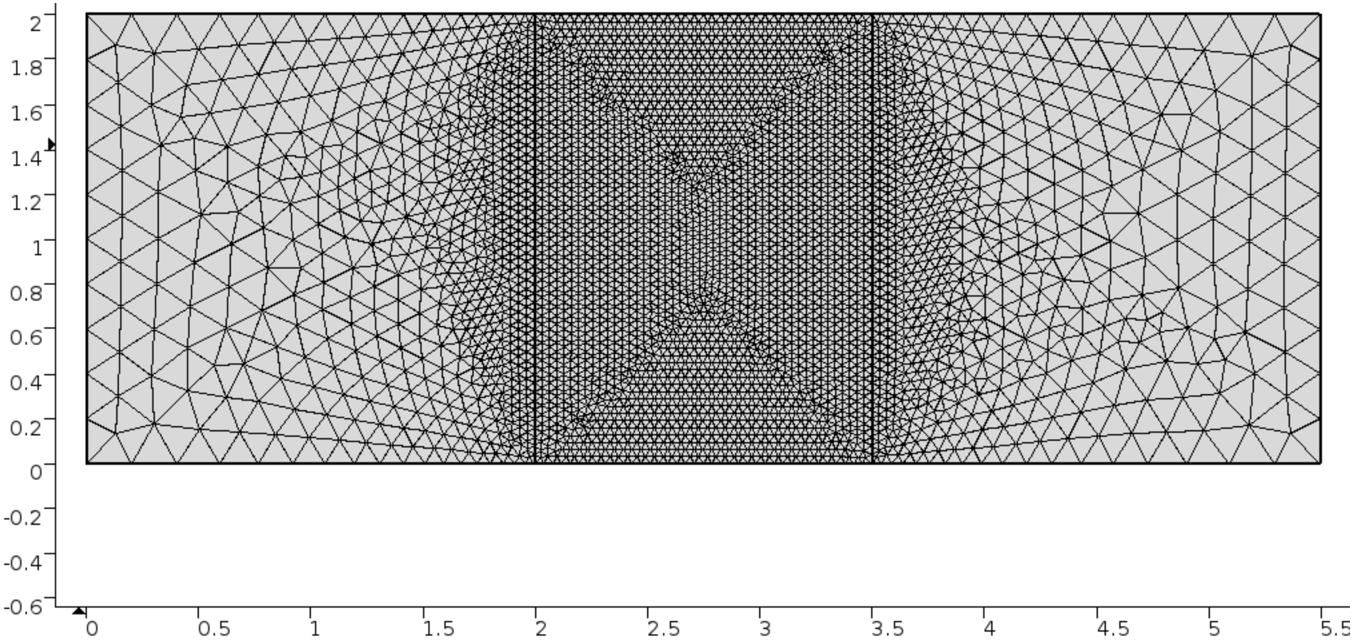
- Interpolation of
  - Out of plane heat transfer between solid and fluid layer

$$h(\gamma) = h_{\min}\frac{\gamma(C_h(1+b_h)-1)+1}{C_h(1+b_h\cdot\gamma)}$$

  - Thermal conduction in design space

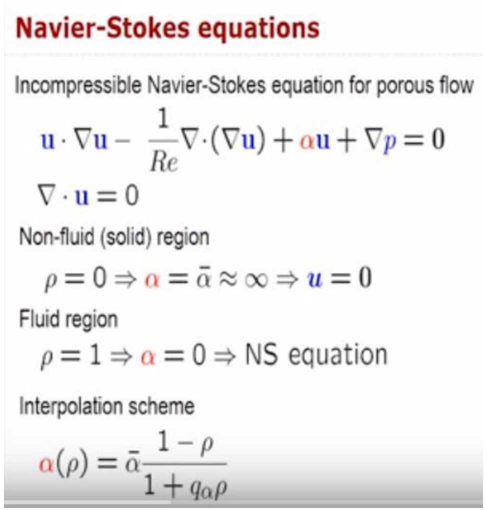
$$k(\gamma) = k_{fl}\frac{\gamma(C_k(1+b_k)-1)+1}{C_k(1+b_k\cdot\gamma)}$$

# Mesh – Fluid-thermal layer



# References

- Bendsøe et al. 2003 → TopOpt Buch



**Navier-Stokes equations**

Incompressible Navier-Stokes equation for porous flow

$$\mathbf{u}\cdot\nabla\mathbf{u} - \frac{1}{Re}\nabla\cdot(\nabla\mathbf{u}) + \alpha\mathbf{u} + \nabla p = 0$$

$$\nabla\cdot\mathbf{u} = 0$$

Non-fluid (solid) region

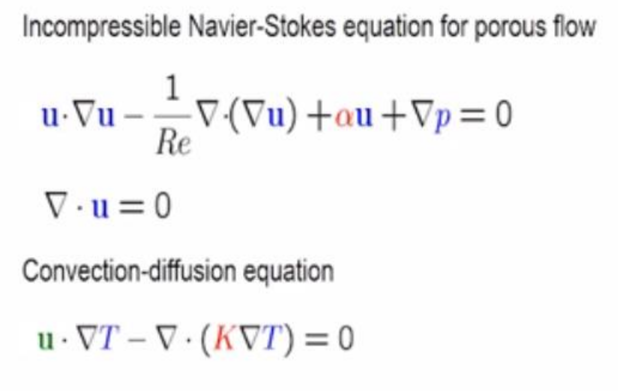
$$\rho = 0 \Rightarrow \alpha = \bar{\alpha} \approx \infty \Rightarrow u = 0$$

Fluid region

$$\rho = 1 \Rightarrow \alpha = 0 \Rightarrow \text{NS equation}$$

Interpolation scheme

$$\alpha(\rho) = \bar{\alpha}\frac{1-\rho}{1+q_\alpha\rho}$$



Incompressible Navier-Stokes equation for porous flow

$$\mathbf{u}\cdot\nabla\mathbf{u} - \frac{1}{Re}\nabla\cdot(\nabla\mathbf{u}) + \alpha\mathbf{u} + \nabla p = 0$$

$$\nabla\cdot\mathbf{u} = 0$$

Convection-diffusion equation

$$\mathbf{u}\cdot\nabla T - \nabla\cdot(K\nabla T) = 0$$