

# Simulation Tests of the Constitutive Equation of a Nonlinear Viscoelastic Fluid

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# Outline

- Rheometry of viscoelastic fluids
  - Measurements with a rotational rheometer
  - Motivations for FEM modeling
- Simulation of shear flow rheometry with COMSOL Multiphysics
- Results:
  - Normal force simulation
  - Rod climbing (Weissenberg effect) simulation

# Rheometry of viscoelastic fluids

- Shear flow tests:  
non-Newtonian flow
- Small Amplitude Oscillation Shear (SAOS) tests: loss and storage modulus, linear properties
- Large Amplitude Oscillation Shear (LAOS) tests: anharmonic analysis, nonlinear viscoelastic properties

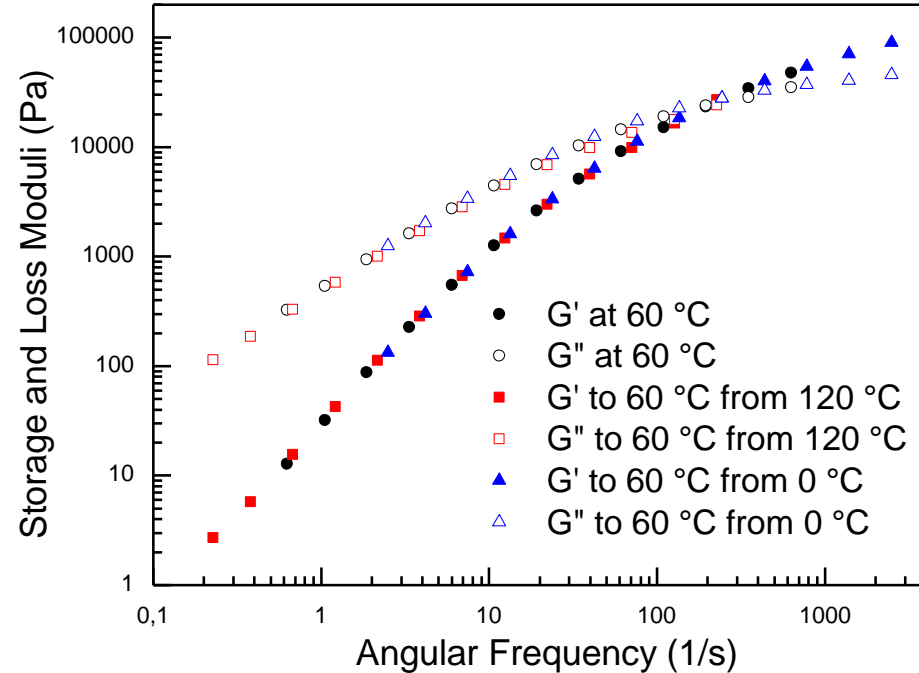
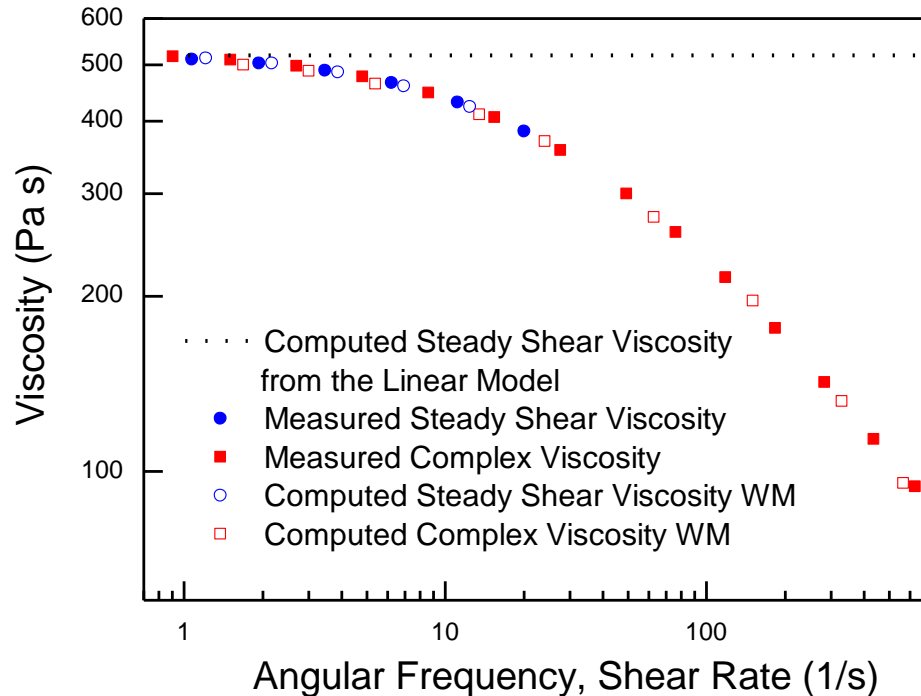


# Rheometry of viscoelastic fluids

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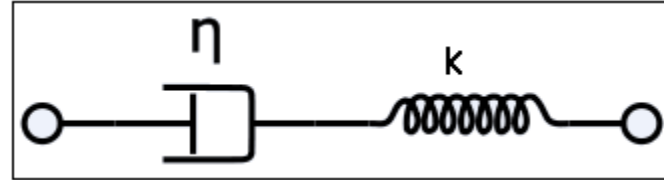


# Rheometry of viscoelastic fluids



# Rheometry of viscoelastic fluids

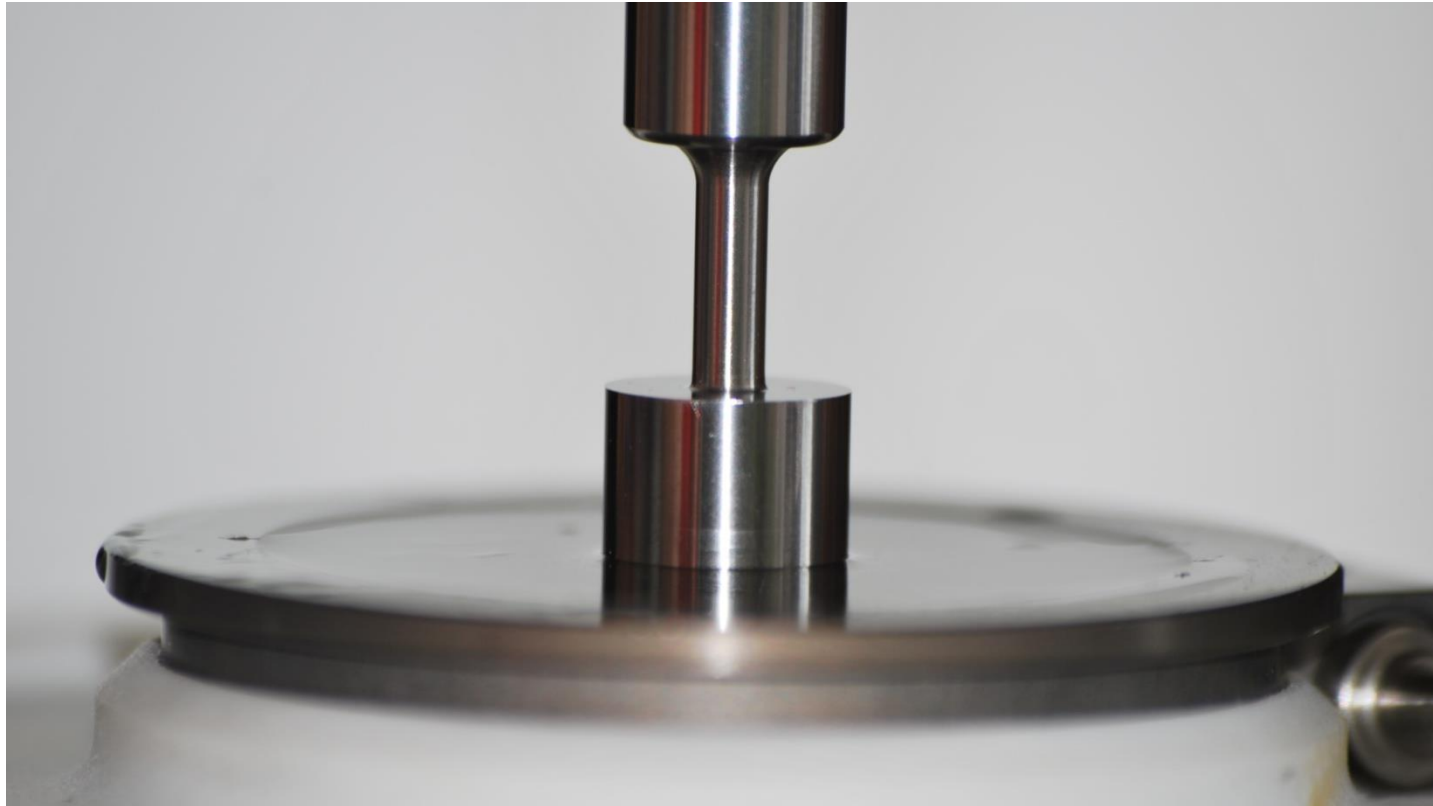
- Silicone oil, (Polydimethylsiloxane, PDMS)
- High viscosity: 100 – 2000 Pa\*s
- Viscoelastic fluid: 3 – 5 Maxwell elements for lumped parameters models



- Shear thinning, Cox-Merz rule, Nonlinear viscoelasticity
- Normal force measurements with CP geometry, Weissenberg-effect

# Rheometry of viscoelastic fluids

Weissenberg  
effect:



# Rheometry of viscoelastic fluids

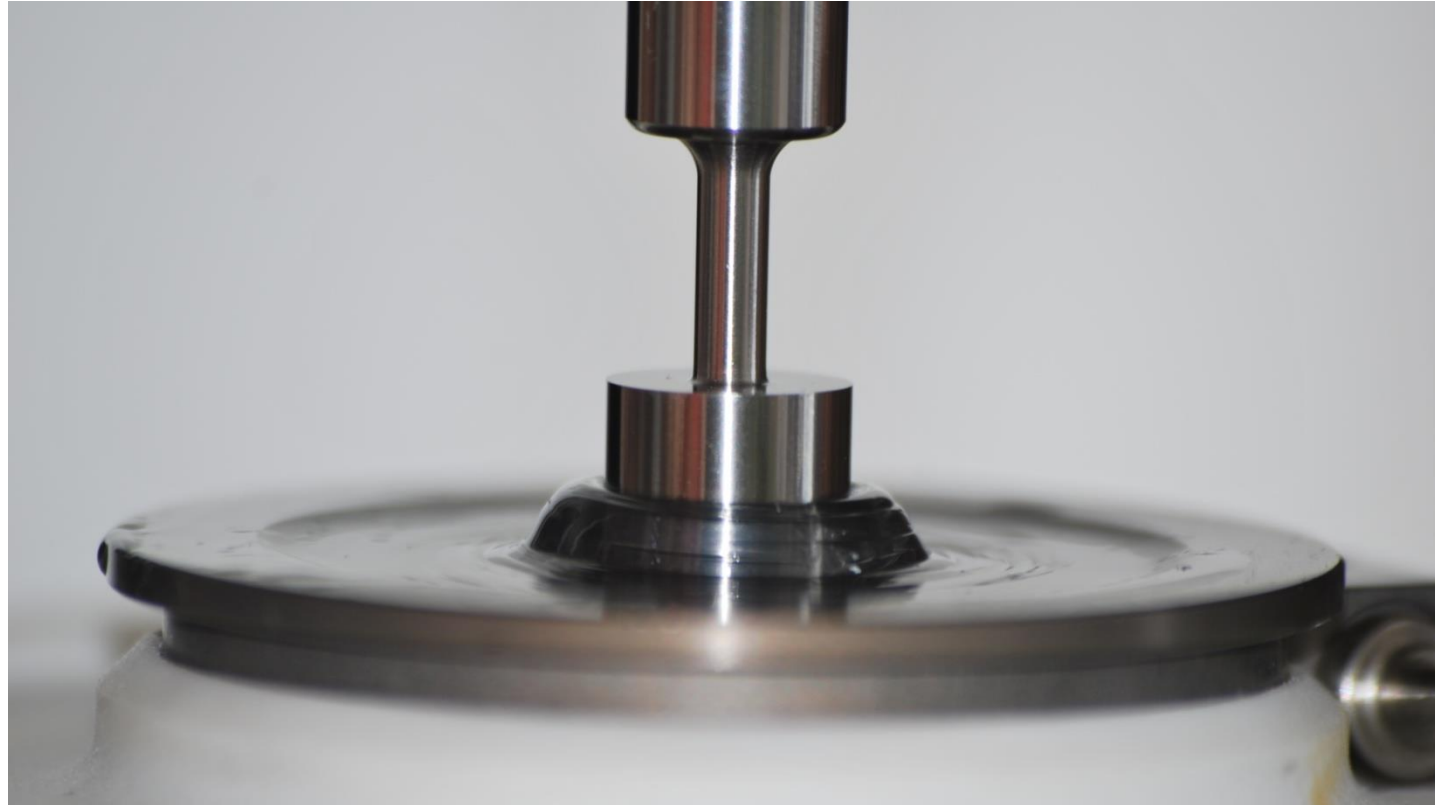
Weissenberg  
effect:





# Rheometry of viscoelastic fluids

Weissenberg  
effect:

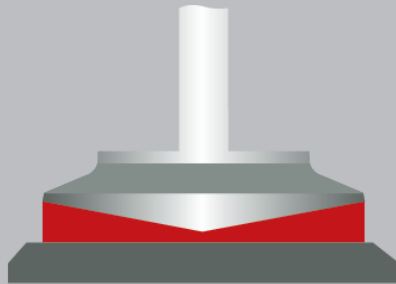


# Motivations for a FEM simulation

- Which 3D constitutive equation (e.g. UCM, Jeffreys, White-Metzner, Oldroyd, ...) is the best to model the fluid?
- How much is the effect of rod climbing on the measured viscosity?

# Simulation of shear flow with COMSOL

CP-model:



Cone-Plate  
(CP)

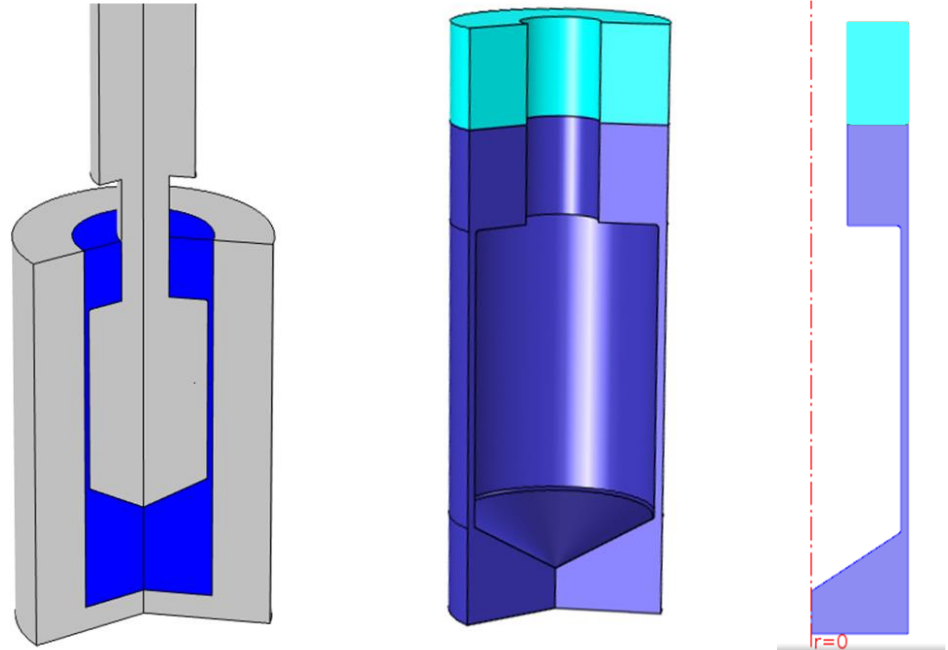
CC-model:



Concentric  
Cylinder (CC)

# Simulation of shear flow with COMSOL

- Swirl flow with two phases: silicone oil and air, surface tension and gravity included -> **2D axial sym., level set**
- Silicone oil: White-Metzner with 3 elements -> **2D PDE modes (cyl. coo.)**



# Simulation of shear flow with COMSOL

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**2D axial sym., level set**

- Silicone oil: White-Metzner with 3 elements ->

**2D PDE modes (cyl. coo.)**

$$\rho \frac{\partial \mathbf{u}}{\partial t} + \rho (\mathbf{u} \nabla) \mathbf{u} = \nabla [-p \mathbf{I} + \boldsymbol{\tau}] + \mathbf{F}$$

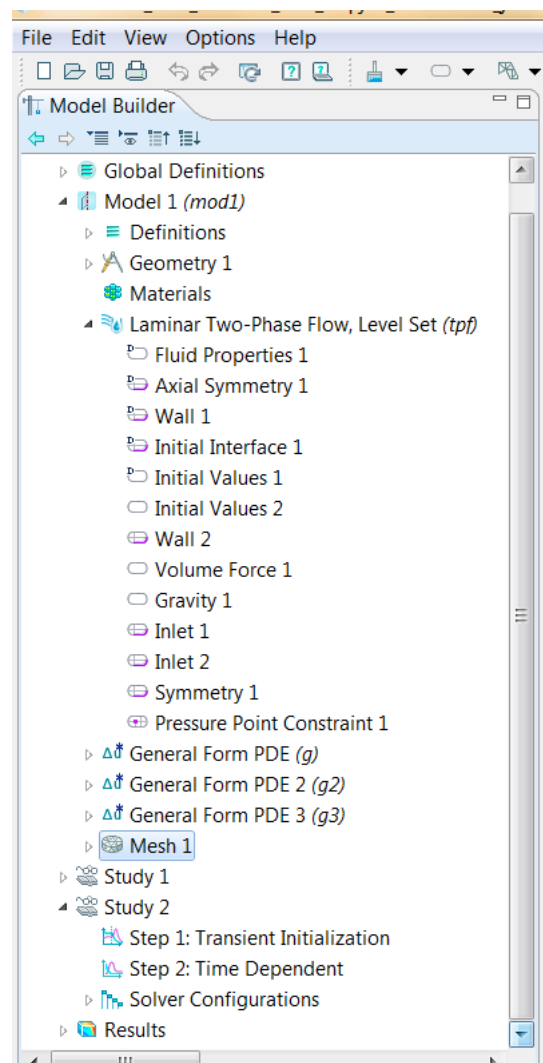
$$\boldsymbol{\tau} = \sum_{j=1}^n \boldsymbol{\tau}_j$$

$$\boldsymbol{\tau}_j + \frac{\eta_j (|\dot{\gamma}|)}{k_j} \nabla \boldsymbol{\tau}_j = -\eta_j (|\dot{\gamma}|) \cdot [\nabla \mathbf{u} + (\nabla \mathbf{u})^T]$$

$$\nabla \boldsymbol{\tau}_j \equiv \frac{\partial \boldsymbol{\tau}_j}{\partial t} + (\mathbf{u} \nabla) \boldsymbol{\tau}_j - [\boldsymbol{\tau}_j (\nabla \mathbf{u}) + (\nabla \mathbf{u})^T \boldsymbol{\tau}_j]$$

# Simulation of shear flow with COMSOL

- Boundary conditions
- Variable scalings
- Time dependent solution, initialization
- Ramping up the azimuthal velocity
- Ramping up the coupling (Volume Force)
- Many variables, large RAM



# Simulation of shear flow with COMSOL

- Reference simulation: Newtonian fluid, torque



# Results – CP

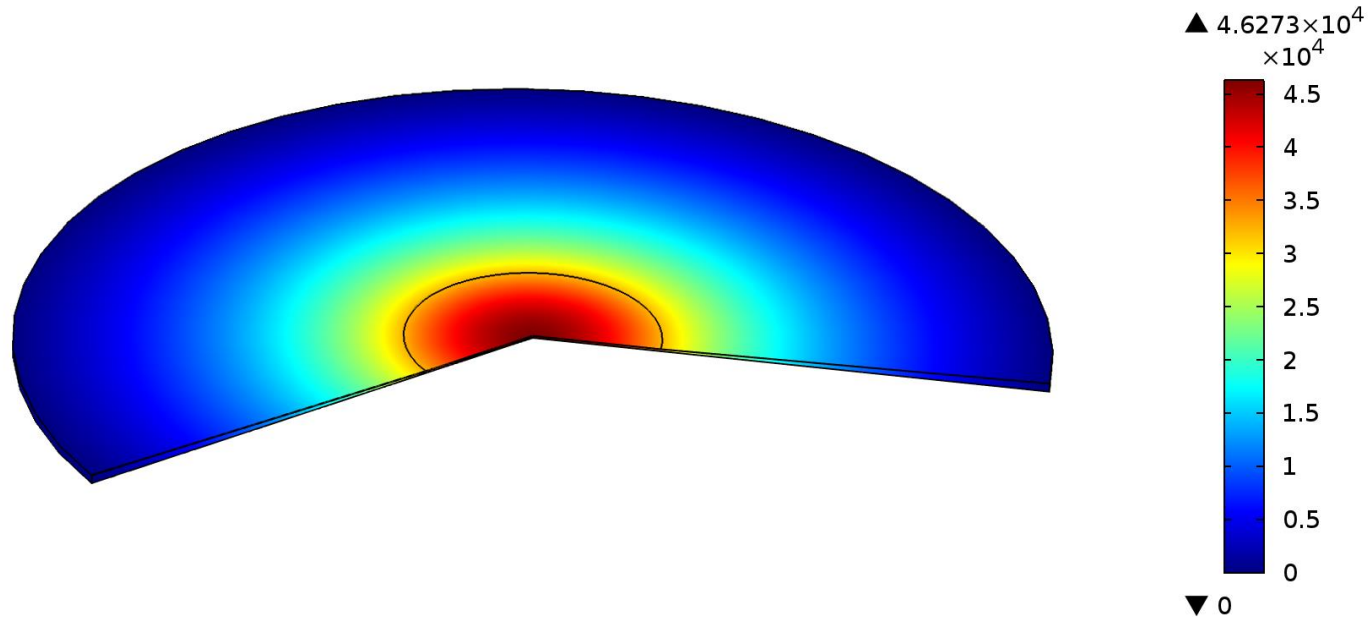
CP-model:



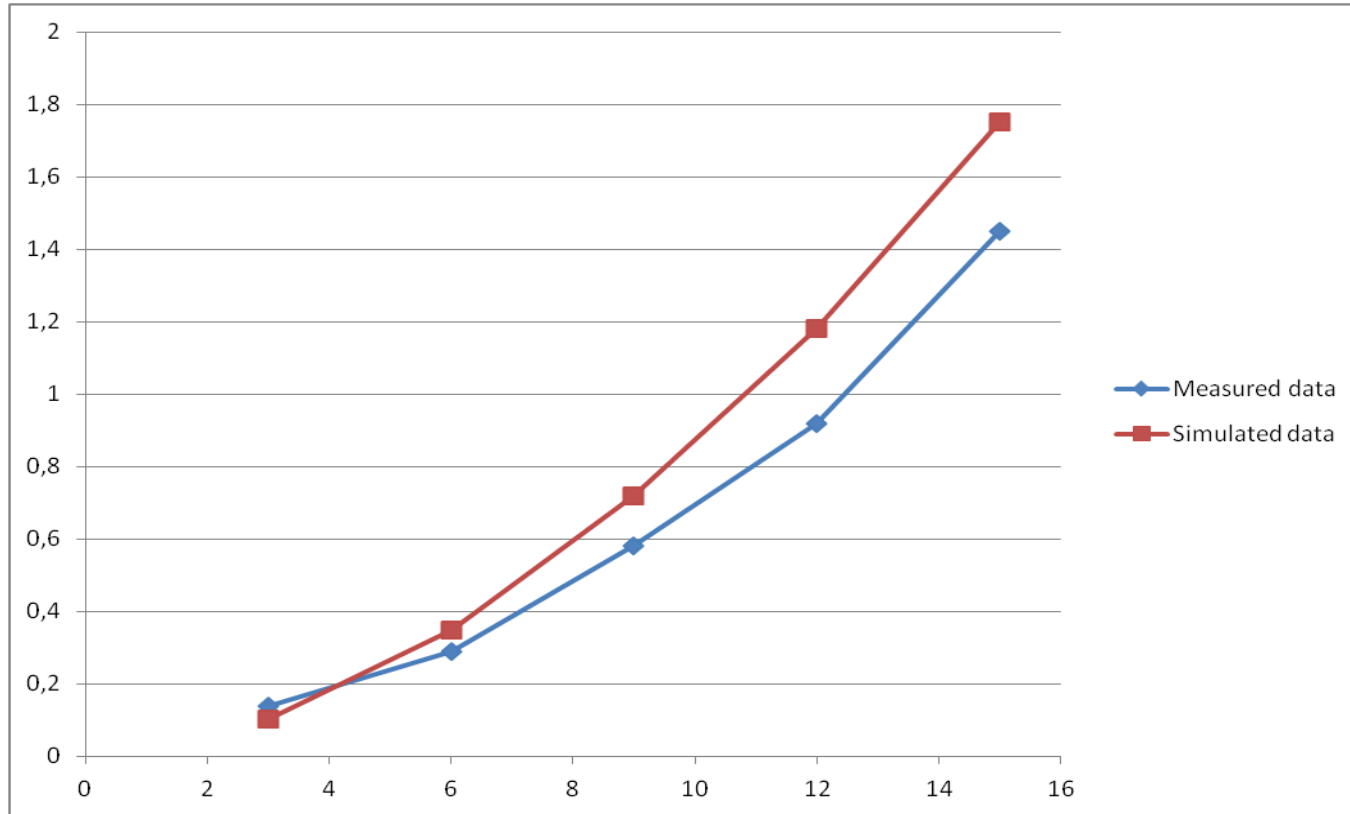


# Results – CP: Normal force

- Pressure distribution on the upper (conical) surface



# Results – CP: Normal force

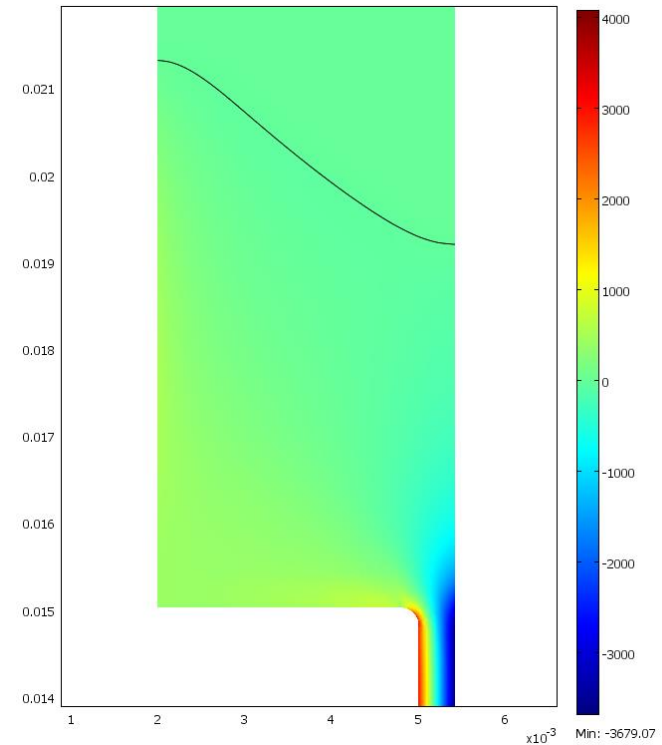
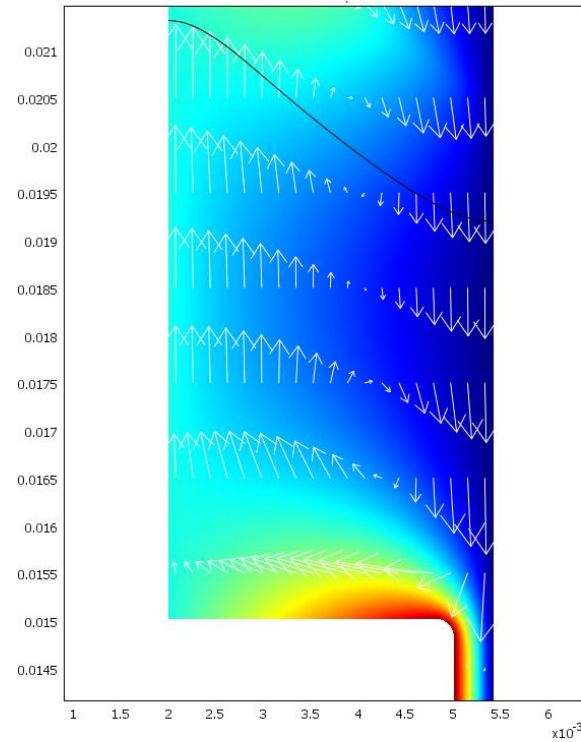
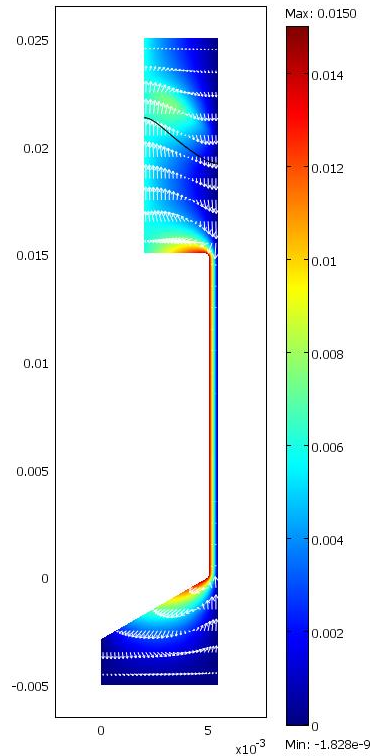


# Results – CC

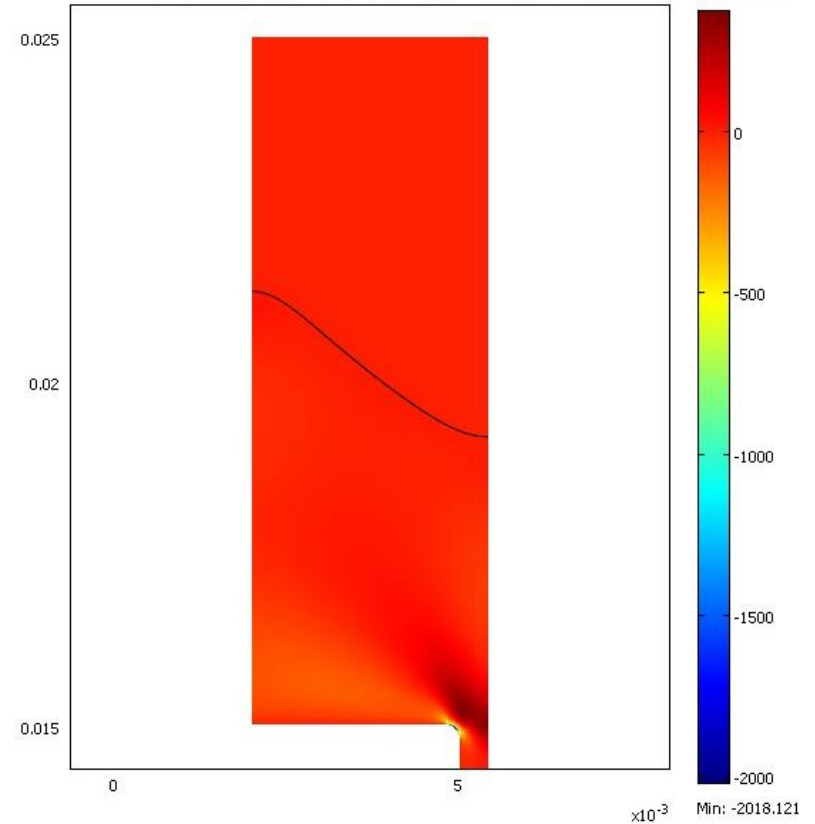
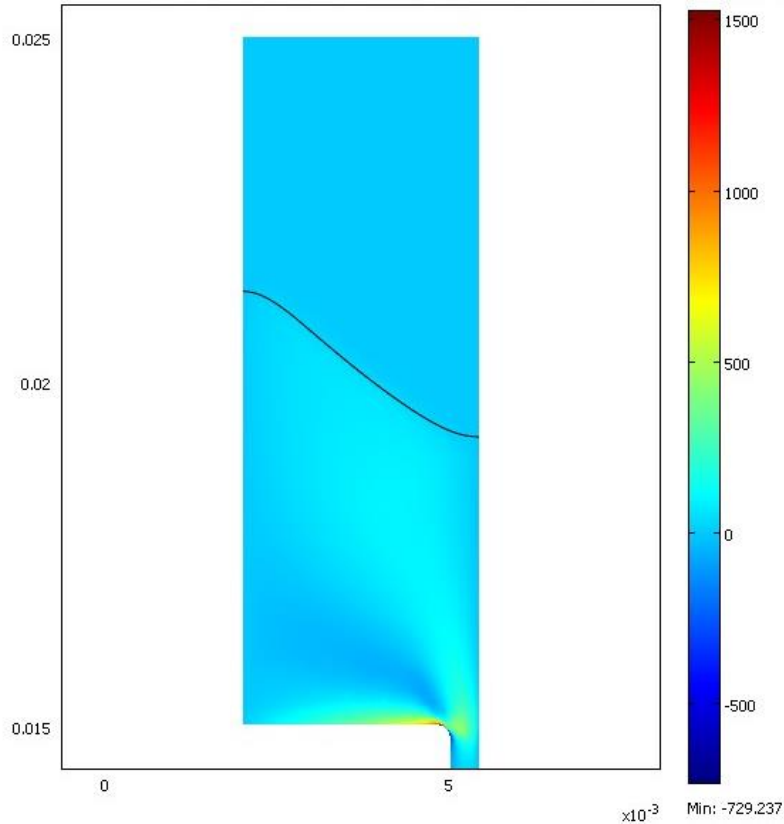
CC-model:



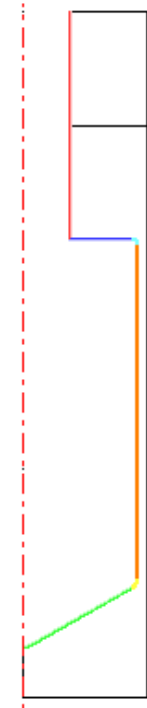
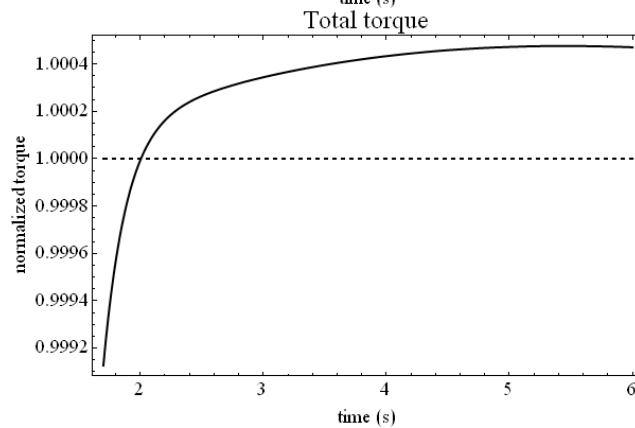
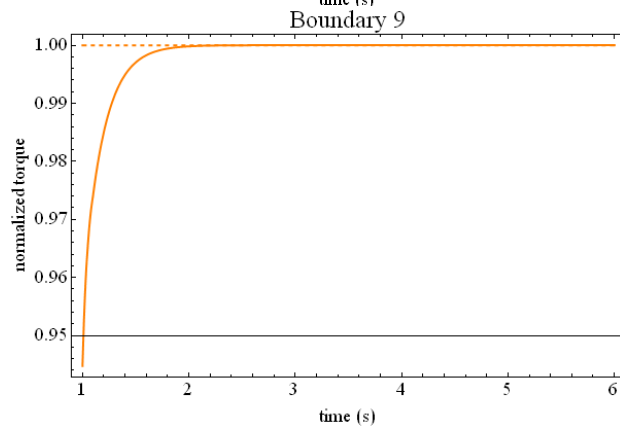
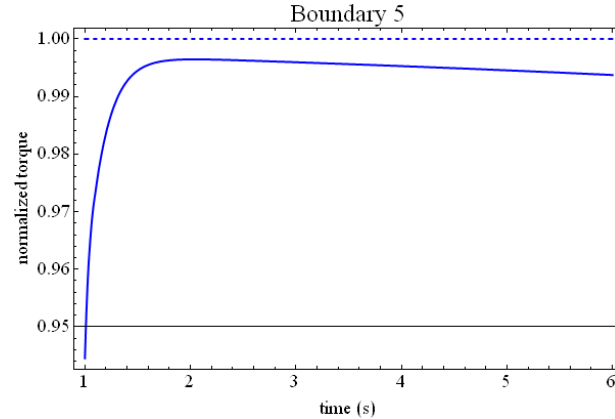
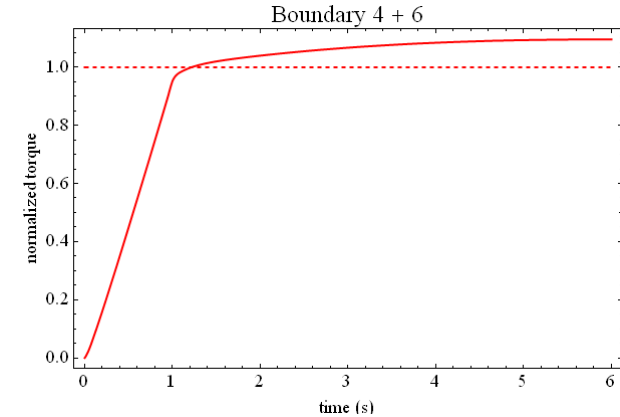
# Results – CC: Rod climbing



# Results – CC: Rod climbing



# Results – CC: Torque, viscosity



# Conclusions

- COMSOL is able to give the solution of this difficult problem
- Normal force values from CP simulation are in good agreement with measurements
- Rod-climbing in CC simulation is close to reality, computed changes in torque values mostly cancel, therefore viscosity measurements are not disturbed

Thank you for your attention!