Hydrodynamics of Viscoelastic Jets in Extensional Flow

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Introduction:

The extensional flow of viscoelastic flow is encountered during fibre spinning processes. A variety of polymers such as polyesters, polyacrylonitrile, rayon, acrylic, spandex etc. are manufactured using this process. In the wet-spinning process, polymer solution is extruded in the form of jet, through a porous plate called spinneret, into a bath having a suitable environment which allows solidification of the polymer to form a fibre. This fibre is drawn by rollers at the exit of the bath, which exerts an extensional stress on the jet, thereby orienting the polymer chains along the fibre axis..

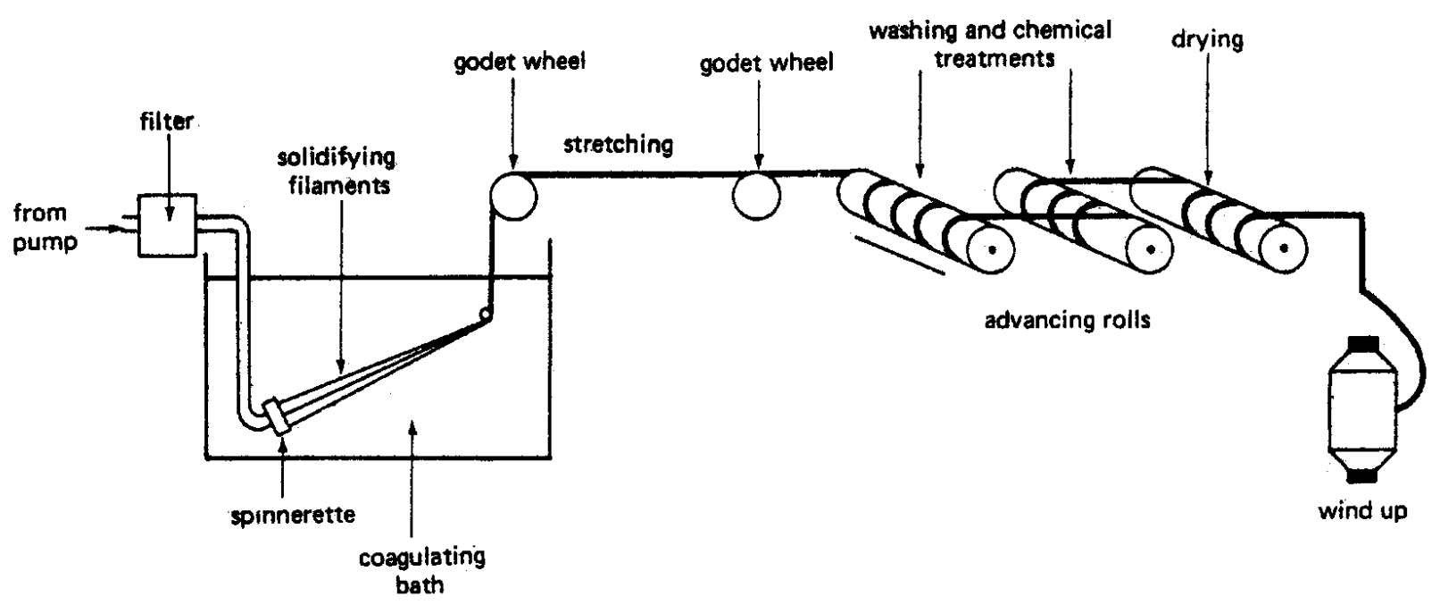


Figure 1: A typical wet-spinning process

This is essential for improving mechanical strength of the fibre. Hydrodynamics of polymeric jets involves extensional flow of viscoelastic fluids. Understanding of hydrodynamics is important for optimizing the process parameters. COMSOL Multiphysics is ideal suited to address these complexities. In the present work, we have studied velocity profile, interface profile, stress distribution in a cylindrical jet of a viscoelastic fluid in a bath of a Newtonian fluid. The jet is subjected to different extensional strain measured in terms of the draw ratio (ratio of the velocities at outlet and at the inlet) . The jet-fluid is a modeled as an Oldroyd-B fluid. Isothermal operation is considered. The COMSOL simulation is conducted in a 2D axisymmetric geometry. Equation of continuity and motion are solved. The profile of the jet is evaluated using the phase-field model. Simulations are performed for different values of the exit jet velocity, Reynolds number and the Weisenberg number

Computational Method

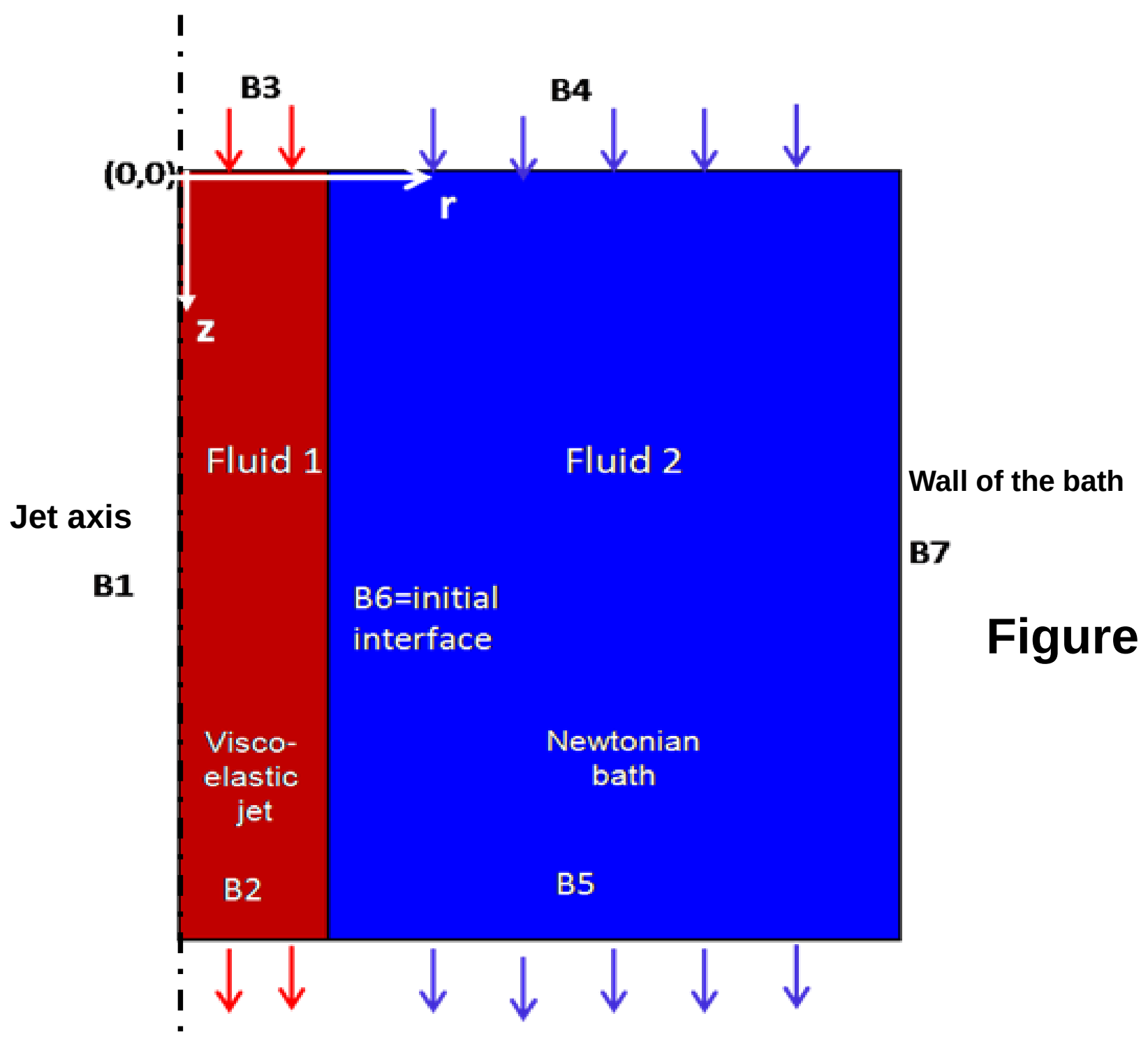


Figure 2: Modelling domains of jet and bath fluids

The geometry is modeled as 2D-axisymetric, with inner domain as jet and outer domain as bath. The outer boundary of the bath is a solid wall. The jet fluid consists of a concentrated solution of polymer (viscosity η_p) in a solvent (viscosity η_s) while bath fluid consists of very dilute solution of polymer. Both fluids enter the system from the top and exit from bottom.

We use unsteady laminar two phase flow, phase field model to simulate the flow and use steady state PDE mode to derive extra stress tensor.

We have solved dimensionless forms of equation of continuity for incompressible fluid i.e.

$$\bar{\nabla}.\bar{u} = 0$$

and equation of motion ,

$$\mathrm{Re}\left(\frac{\partial\bar{u}}{\partial t} + (\bar{u}.\bar{\nabla})\bar{u}\right) = \bar{\nabla}.\left[-\mathrm{p}\bar{\bar{\mathrm{I}}} + \mu\left(\bar{\nabla}\bar{\mathrm{u}} + \bar{\nabla}\bar{\mathrm{u}}^{\mathrm{T}}\right) + \bar{\bar{\mathrm{T}}}\right] + \bar{\mathrm{F}}_{\mathrm{g}} + \bar{\mathrm{F}}_{\mathrm{ST}}$$

where $\bar{\mathrm{F}}_{\mathrm{g}} = \rho\bar{\mathrm{g}}$ (gravitational force), $\bar{\mathrm{F}}_{\mathrm{ST}} = \sigma\left(\bar{\bar{\mathrm{I}}} - \bar{\mathrm{n}}\bar{\mathrm{n}}\right)\delta$ (surface tension force $\bar{\bar{\mathrm{T}}}$ is an extra stress contribution satisfying OldroydB constitutive equation. which exists here due to the extra stress contribution satisfying following Oldroyd-B constitutive relation :

$$\bar{\bar{T}} + Wi\left[(\bar{u}.\bar{\nabla})\bar{\bar{T}} - \left(\bar{\nabla}\bar{\mathrm{u}}.\bar{\bar{\mathrm{T}}} + \bar{\bar{T}}.\bar{\nabla}\bar{\mathrm{u}}^{\mathrm{T}}\right)\right] = \mu_p[\bar{\nabla}\bar{\mathrm{u}} + \bar{\nabla}\bar{\mathrm{u}}^{\mathrm{T}}]$$

where $Re = \frac{RU_{in}\rho}{\eta}$ (R is characteristic length and U_{in} is characteristic velocity) , $\mu_s = \frac{\eta_s}{\eta}$,
$\eta = \eta_s + \eta_p$, $Wi = \frac{\lambda U_{in}}{R}$ (λ is polymer chain relaxation time).

The motion of the fluid interface separating the two phases is computed by solving the phase field equation,

$$\frac{\partial\phi}{\partial t} + \bar{u}.\bar{\nabla}\phi = \bar{\nabla}.\frac{\gamma\lambda}{\varepsilon^2}\bar{\nabla}\Psi$$

The boundary condition include symmetry at the jet axis, $p_0 = 0$ (no viscous stress) at boundaries B3 and B4, normal outflow velocity at B2 and B5, initial interface at B6 and wall (no slip) at B7.

The following constraints for the extra stress tensor are added on the boundaries.
For wall B7:

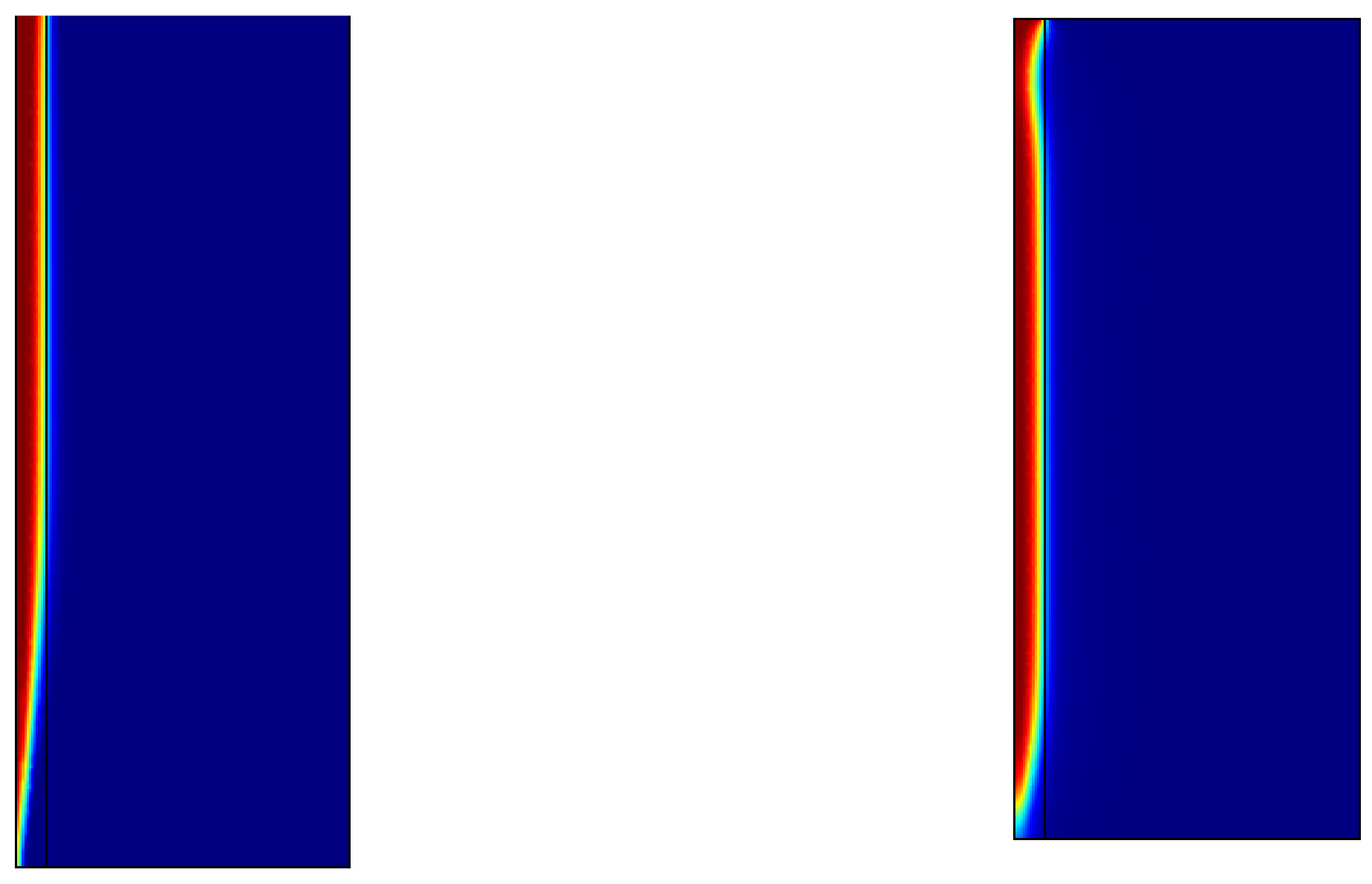
$$(\bar{\bar{\mathrm{T}}}.\hat{n}).\hat{n}\text{=0}$$

The components of extra stress tensor, obtained from the solution of relevant PDE are added in the domains 1 and 2 as well as on the boundaries in the weak form along with the stabilizing terms.

Results:

Conditions used for simulation:

1. Fixed properties of the bath fluid:Re_2=0.01, Wi_2=0.001, μ_{s2}= 0.99
2. Initial velocity of both domains: u_z=0.01
3. Pulling velocity at the outlet of the jet is varied and inlet velocity and pulling stress σ_{zz} are computed as functions of time
4. Parameter varied: Wi_1, μ_{s1}, Re_1



Re_1=0.1, Wi_1=0.01, μ_{s1}= 0.99

$u_{in} = 0.77, u_{out} = 49.9, \sigma_{zz} = -247$

No jet swell, jet stable at higher velocity, sharp cone at end

Re_1=0.1, Wi_1=0.2, μ_{s1}= 0.59

$u_{in} = 0.00929, u_{out} = -0.299, s_{zz} = -1.529$

jet swell, jet unstable at higher velocity, rounded end at end

Figure 3: Comparison between steady state shapes of Newtonian and viscoelastic jet

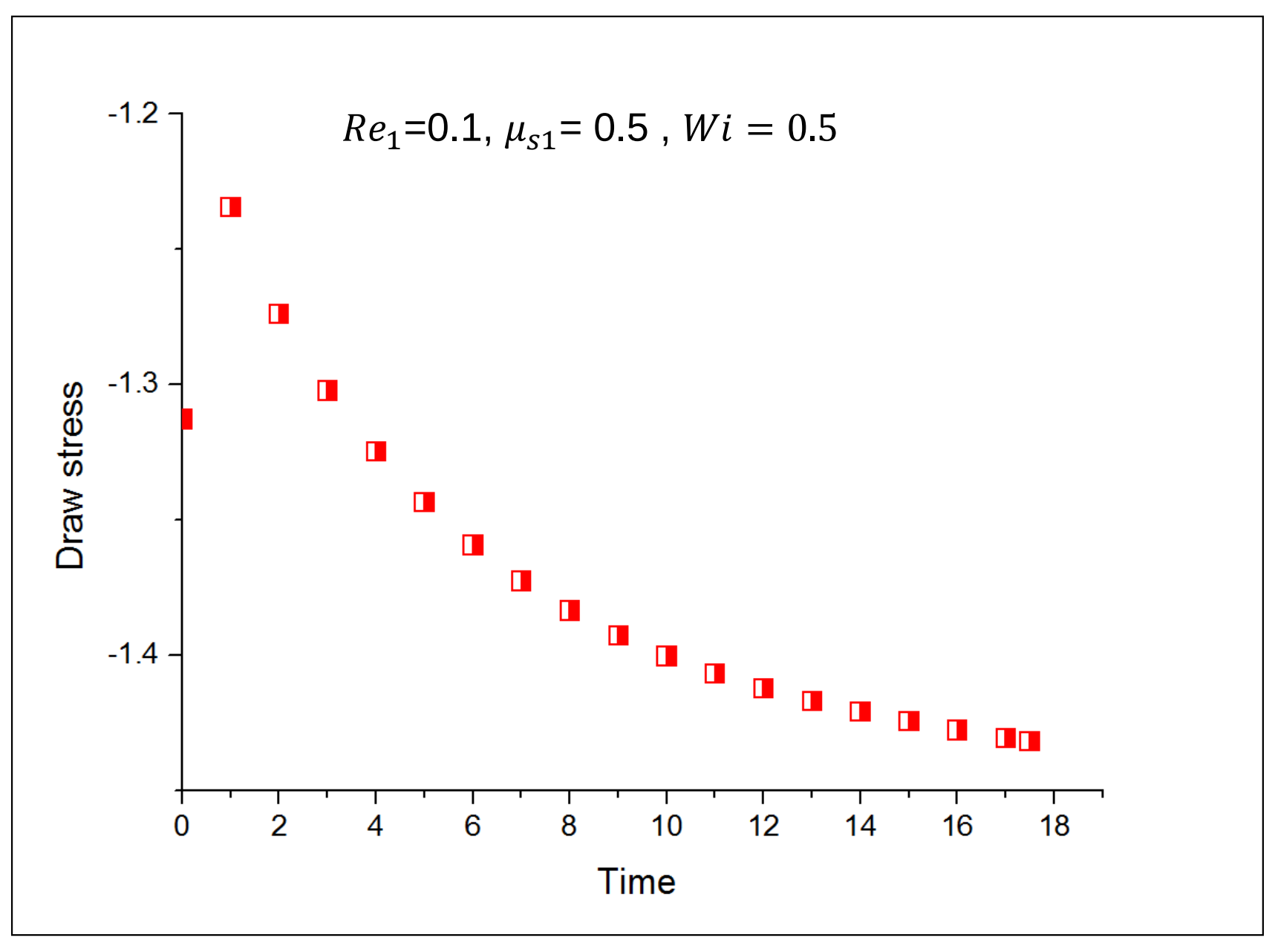


Figure 4: Stress transience during spinning (draw ratio = 28, pulling velocity= 0.3)

Initial stress is high due to sudden motion. Stress decreases due to reduction in velocity gradient and goes through a minimum and again increases as more and more jet fluid experiences the extensional stress.

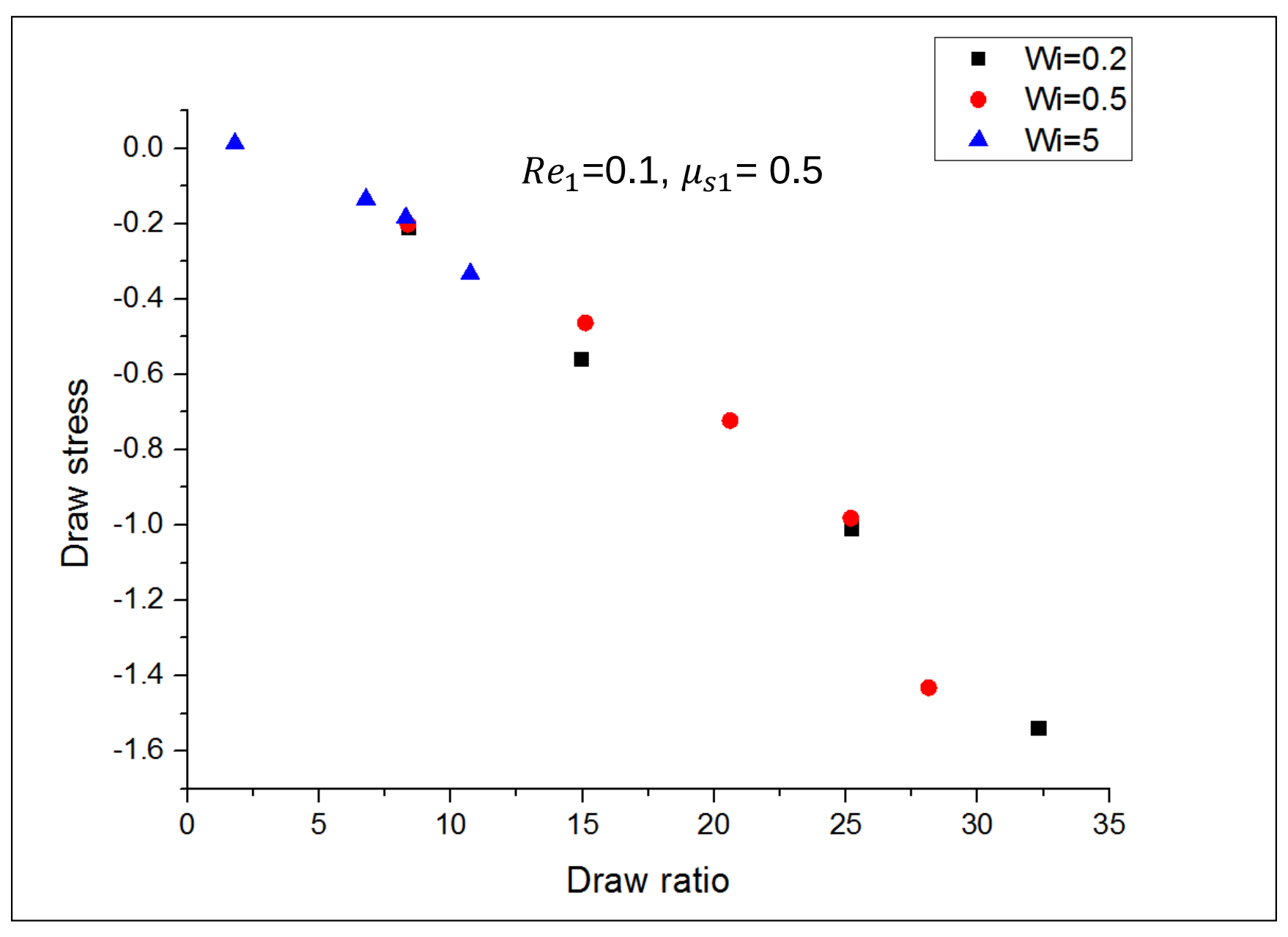


Figure 5: Plot of draw stress (s_{zz}) at the outlet of the jet

Draw stress increases with increasing in draw ratio as expected and is independent of Weissenberg number

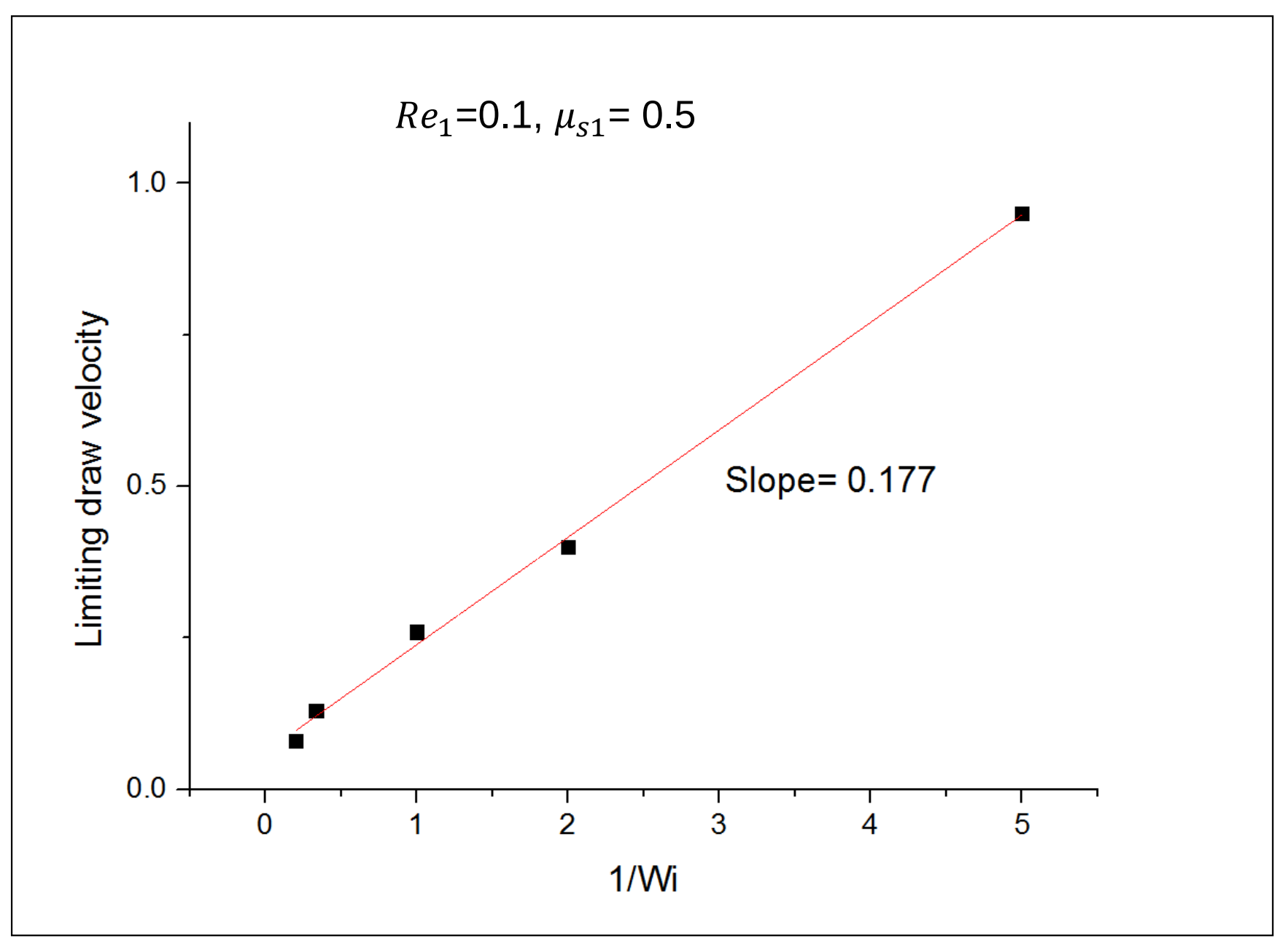


Figure 6: Plot of draw stress (s_{zz}) at the outlet of the jet

As Weissenberg number increases, limiting draw velocity decreases (the velocity beyond which the jet breaks). This happens because of slow response of polymer chains to the extensional strain rate.

Conclusion:

1. Weissenberg number is important in determining the stability of the viscoelastic jet during spinning process. Jet stability decreases with increase in Weissenberg number

2. Once jet is stable, the dimensionless draw stress depends only on draw ratio and is independent of either Weissenberg number or Reynolds number

References:

COMSOL, Flow of Oldroyd-B Viscoelastic Fluid, Application ID 4383, Downloaded from: https://www.comsol.co.in/model/flow-of-oldroyd-b-viscoelastic-fluid-4383 (downloaded date: 07-07-2015)

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