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Identification and Analysis of Low-Frequency Cogging Torque Component in Permanent Magnet Machines

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Introduction

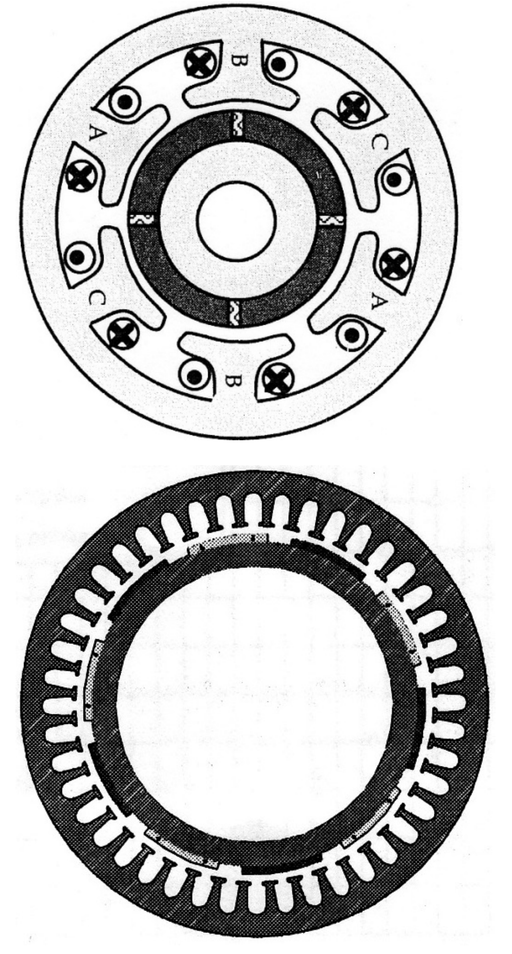
- Cogging torque ripple in PM machines characterized by a relatively high frequency (i.e. LCM of pole and slot numbers).
- FE calculations show a previously unaddressed low frequency modulation of cogging torque ripple that cannot be explained within the current formulation.
- The formulation was extended to allow an understanding and description of the modulation.
- Modulation frequency and amplitude estimates are shown consistent with FE results.

Introduction

- Background
  - Cogging torque occurrence & reduction means
  - Standard formulation
- Observed low frequency modulation
- Extended formulation
- Comparison with FE results.
- Conclusion

Background – Cogging Torque Occurrence

- Many configurations of PM machines
- Most use slotted stator iron-core structure with protruding teeth/shoes
- PM's on rotor interact magnetically with stator teeth/shoes, which causes rotor to align at preferential low energy positions relative to the teeth/shoes – cogging torque
- Generates torque fluctuations, which cause vibrations, noise, speed variations, startup and low speed operation difficulties.



Inner rotor radial flux PM machines with surface magnets & shoes

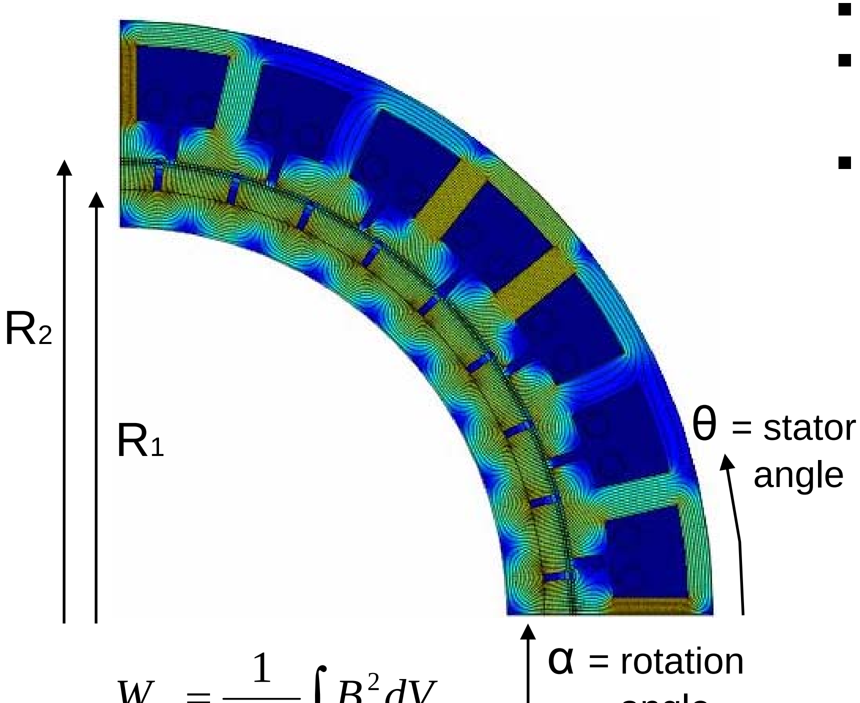
Background – Two Main Principles Underlying Cogging Torque Reduction

- Minimize abruptness of pole-teeth attraction
  - Magnet shaping
  - Teeth skewing
  - Pole skewing

- Use individual pole-teeth attractions to offset or at least minimally add to each other
  - Utilize shoes
  - Optimize pole-to-teeth number ratio
  - Pair shoes and poles of different widths
  - Teeth/shoe notching

Background – Standard Formulation

1. General Expressions



$$W_g = \frac{1}{2\mu_0}\int B_g^2 dV$$

$$W_g(\alpha) = \frac{1}{4\mu_0} L_A (R_2^2 - R_1^2)\int_0^{2\pi} G^2(\theta) B_r^{'2}(\theta-\alpha)d\theta$$

$$T(\alpha) = -\frac{\partial W_g(\alpha)}{\partial \alpha}$$

2. Fourier Expansions

- L_A is generator length (i.e. airgap length)
- G is dimensionless relative airgap permeance function – periodic with slot periodicity
- B_r' is PM remanence flux density – periodic with pole periodicity (prime notation is to allow B_r extension to include fringing fields)

$$G_0^2(\theta) = \sum_{n=0}^{\infty}\left[G_{anNs}\cos nN_S\theta + G_{bnNs}\sin nN_S\theta\right]$$

G_0 is standard (un-extended) form of G

N_S is the number of slots

$$B_r'^2(\theta-\alpha) = \sum_{n=0}^{\infty}\left[\begin{array}{l} B_{anNp}\cos nN_P(\theta-\alpha) \\ + B_{bnNp}\sin nN_P(\theta-\alpha)\end{array}\right]$$

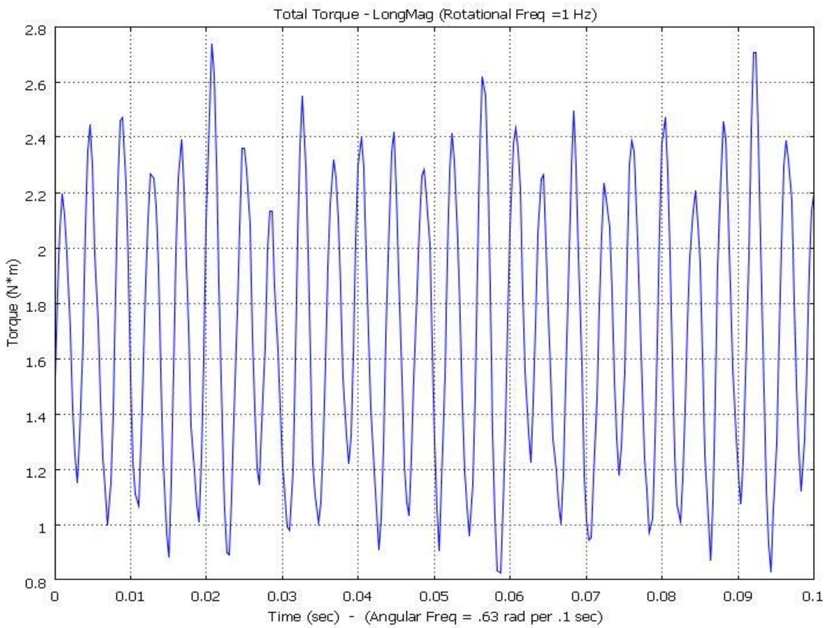
N_P is the number of poles

$$W_{g0}(\alpha) = \frac{\pi L_A}{4\mu_0}(R_2^2 - R_1^2)\sum_{n=0}^{\infty} G_{anNL} B_{anNL}\cos nN_L\alpha$$

$W_{g0} = W_g$ when $G = G_0$

$N_L = \mathrm{LCM}\{N_S, N_P\}$

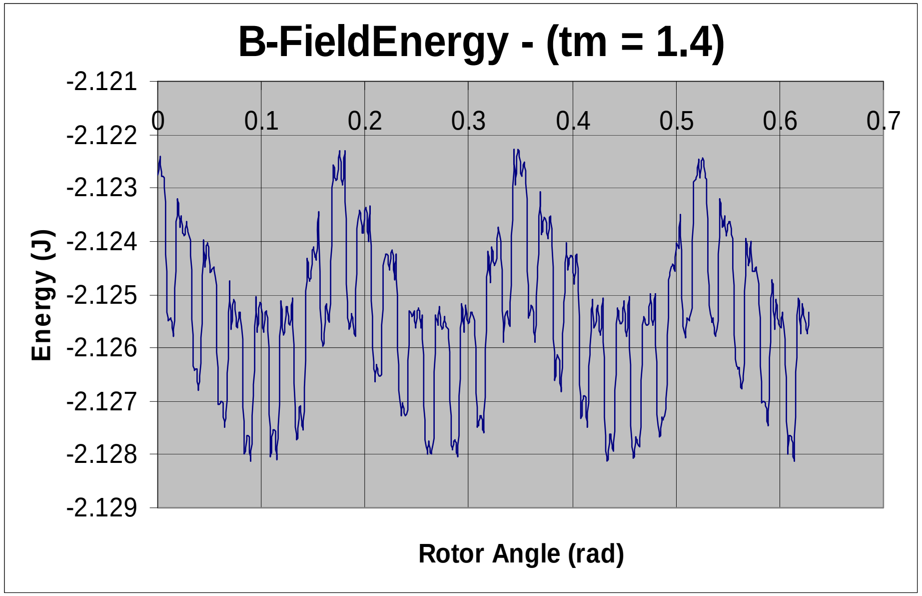
Background – Standard Formulation

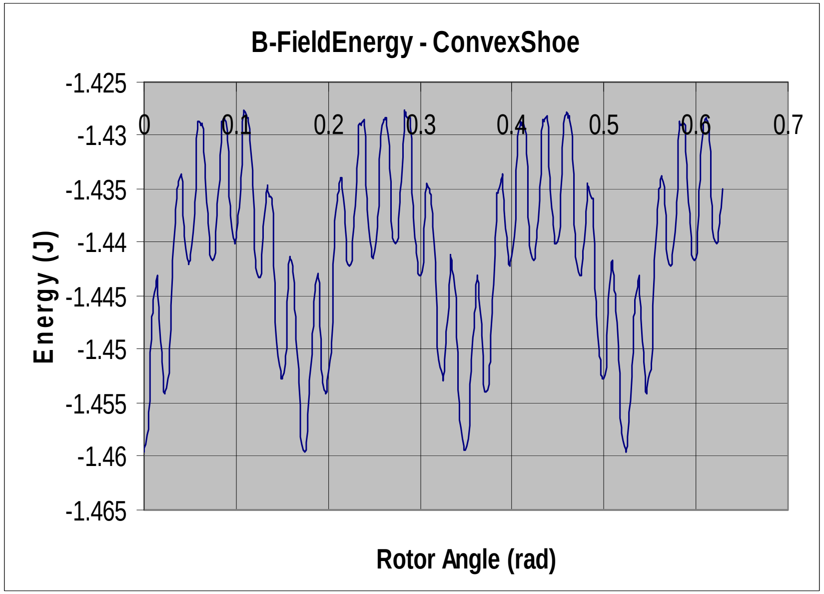


~25.2 cycles of torque ripple over a period of 0.63 radians yields 252 cycles in 2π radians (Model shown on prior slide – built using approach that is combo of "Generator in 2-D" & "Generator with Mechanical Dynamics and Symmetry" in AC/DC model library. It is a sector model with anti-symmetric side boundaries like the latter, but it has a constant prescribed rotation like the former.)

- The fundamental torque ripple frequency N_L matches those obtained from measurements and FE analysis.
  - COMSOL – AC/DC single quadrant model of 36 pole 28 slot machine (i.e. N_P=36, N_S=28, and therefore N_L= 252) produces shown torque ripple.
  - The ~25.2 cycles occur over an angular displacement of about 0.63 radians, yielding the requisite 252 cycles in 2π radians.
- High frequency torque ripple is modulated by low frequency component but no provision for description in standard formulation
- In this plot, modulation component seems aperiodic

Low Frequency Modulation
Clearly Periodic in B-Field Energy Plots





- Corresponding $W_g(\alpha)$ shows periodic cogging torque modulation
- Modulation component essentially single frequency
- This case - modulation amplitude roughly equal to that of high frequency ripple

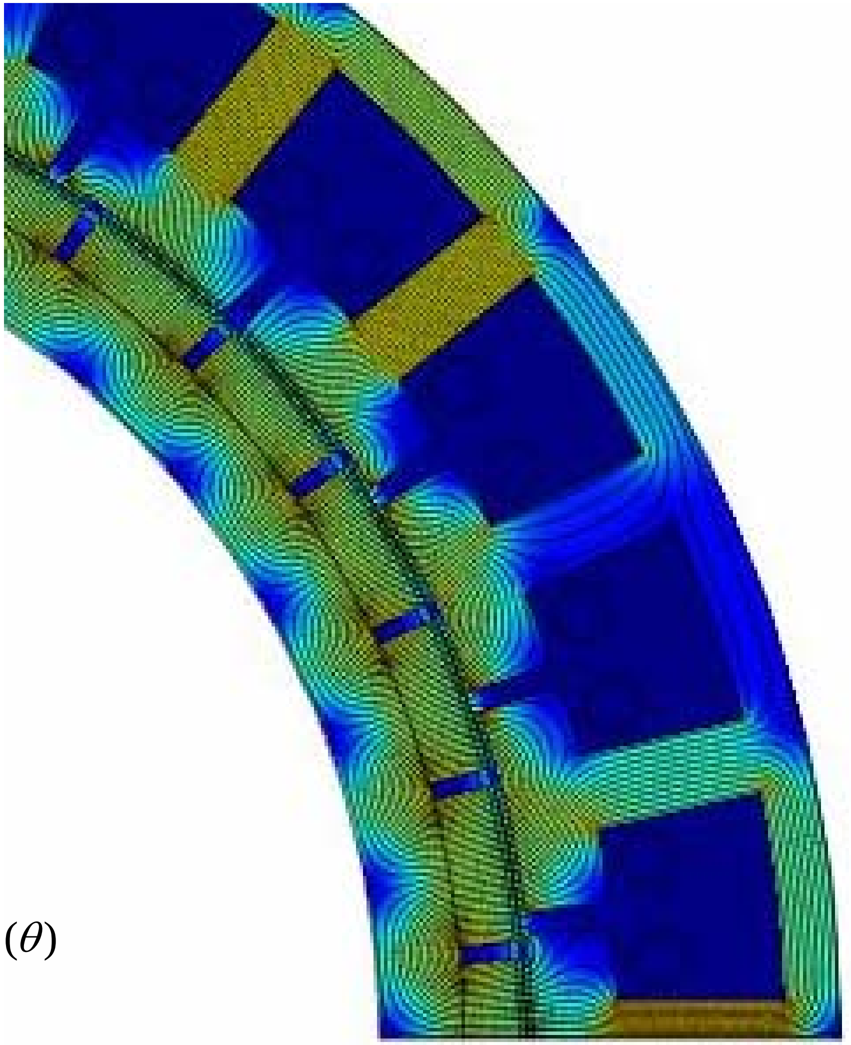
- $W_g(\alpha)$ for a version of the Fig. 1 machine with slightly convex shoes
- This case - modulation amplitude greater than that of the high frequency ripple

Extended Formulation

From magnetic circuit analysis:

$$B_g = \frac{B_r}{1 + \frac{P_m}{P_g} + 4\frac{P_{ml}}{P_g}}$$

P_m = PM permeance
P_{ml} = magnet leakage permeance between PM's
= $P_{ml0} + \Delta P_{ml}$
$\Delta P_{ml} = \Delta P_{ml}(\theta)$ because variation caused by stator
P_g = airgap permeance



Via power series expansion, the square of the above equation for B_g can be approximated as:

$$B_g^2(\theta,\alpha) = B_r^2(\theta-\alpha)G_0^2(\theta) - B_r^2(\theta-\alpha)G_1^2(\theta)$$

$$G_0 = \left(1 + \frac{P_m + 4P_{ml0}}{P_g}\right)^{-1} \qquad G_1^2(\theta) = \frac{8\Delta P_{ml}(\theta)}{P_g}G_0^3(\theta)$$

Extended Formulation

$$W_g = \frac{1}{2\mu_0}\int B_g^2 dV$$

$$W_g(\alpha) = W_{g0}(\alpha) - \left[\begin{array}{l}\frac{1}{4\mu_0}L_A(R_2^2 - R_1^2) \\ \cdot\int_0^{2\pi} G_1^2(\theta) B_r^{'2}(\theta-\alpha)d\theta\end{array}\right]$$

or,

$$W_g(\alpha) = W_{g0}(\alpha) - W_{g1}(\alpha)$$

$$T(\alpha) = -\frac{\partial W_g(\alpha)}{\partial \alpha}$$

$$T(\alpha) = T_0(\alpha) + T_1(\alpha)$$

Similar to the standard formulation, we Fourier - expand G_1^2:

$$G_1^2(\theta) = \sum_{n=0}^{\infty}\left[G'_{anNc}\cos nN_c\theta + G'_{bnNc}\sin nN_c\theta\right]$$

where, N_C = GCF{N_P ,N_S} = number of primary cells (in this case 4) – i.e. generator arc with smallest number of matched slots and poles.

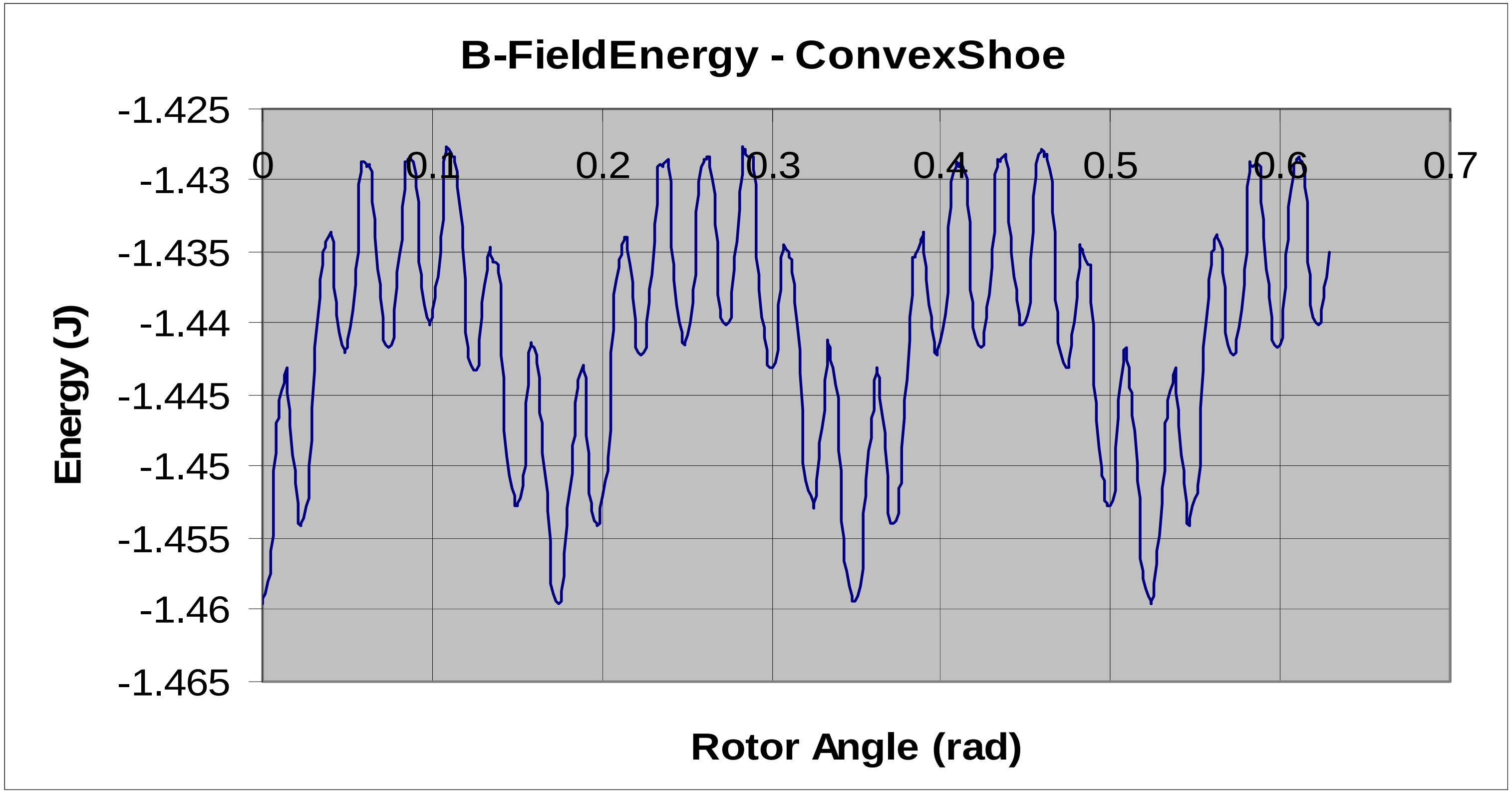
Substitution into $W_g(\alpha)$ Eqn. yields:

$$W_{g1}(\alpha) = \frac{\pi L_A}{4\mu_0}(R_2^2 - R_1^2)\sum_{n=0}^{\infty} G'_{anN_P} B_{anN_P}\cos nN_P\alpha$$

By definition, N_P = LCM{N_C, N_P}

Comparison With FE Results

Fundamental Modulation Frequency = N_P = 36 = 252/7



Correct frequency calculation validates extended formulation

Comparison With FE Results

Modulation Amplitude vs. Ripple Amplitude

For linearity of ΔP_{ml} w.r.t. P_{ml}, and W_g w.r.t. G, the extended formulation predicts that:

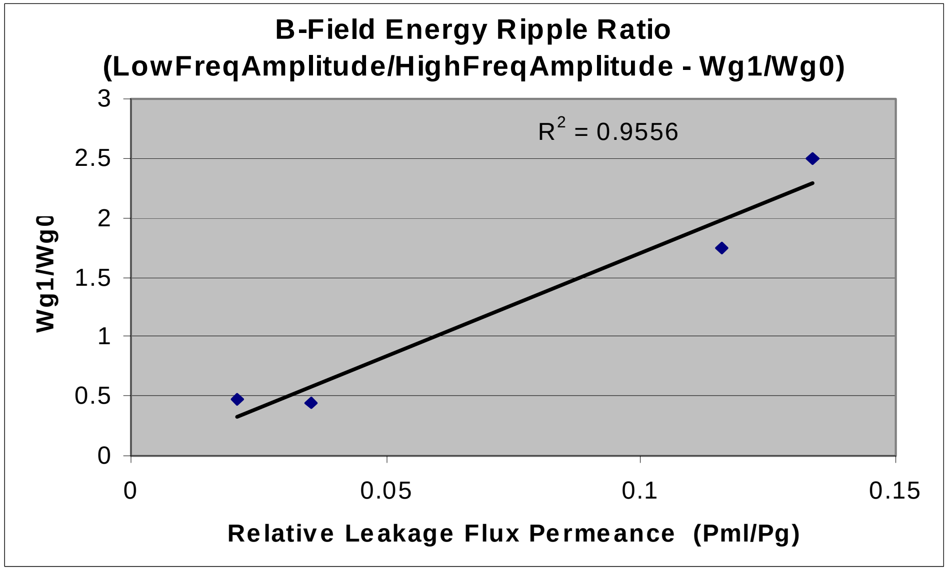
$$\frac{W_{g1}}{W_{g0}} \propto \frac{P_{ml}}{P_g}$$

Which can be expressed as:

$$\frac{P_{ml}}{P_g} = \frac{g}{\pi \tau_m} \ln\left[1 + \pi \frac{g}{\tau_p - \tau_m}\right]$$

τ_m is the magnet length, τ_p is the pole pitch (i.e. distance between adjacent magnet centers), and g is the airgap thickness.

Model	g (mm)	τ_p (mm)	τ_m (mm)	P_{ml}/P_g	W_{g1}/W_{g0}
Baseline	3.18	39.90	25.40	0.02	0.47
Long Mag	3.18	39.90	35.91	0.04	0.44
Long/Thin Mag	6.99	39.90	35.91	0.12	1.75
Thin Mag	9.53	39.90	25.40	0.13	2.50
Slightly Convex Shoe	3.18	39.90	25.40	0.02	2.80



Strong correlation validates extended formulation and linearity approximation.

Conclusion

The work accomplished the following:

- Used FE analysis to identify and characterize low-frequency modulation of PM machine cogging torque,
- Obtained an analytical formulation that describes and explains the modulation,
- Demonstrated good agreement between the analytical formulation and the FE analysis for modulation frequency and amplitude,
- Identified analytical relationships that provide a means of minimizing the low frequency cogging torque component.