A Numerical Model for Transient Heat Conduction in Semi-Infinite Solids Irradiated by a Moving Heat Source

N.Bianco[1], O.Manca*[2], S.Nardini[2] and S.Tamburrino[2]
[1]Dipartimento di Energetica, Termofluidodinamica applicata e Condizionamenti ambientali, Università degli Studi Federico II, Piazzale Tecchio 80, 80125 Napoli, Italia
[2]Dipartimento di Ingegneria Aerospaziale e Meccanica, Seconda Università degli Studi di Napoli, via Roma 29, 81031 Aversa (CE), Italia
*Oronzio Manca: DIAM, Seconda Università degli Studi Napoli, via Roma, 29 – 81031 Aversa, Italy, oronzio.manca@unina2.it

Abstract: A numerical analysis on transient three-dimensional temperature distribution in a semi-infinite solid, irradiated by a moving Gaussian laser beam, is carried out numerically by means of the code COMSOL Multiphysics 3.3.. The investigated work-piece is simply a solid. A laser source is considered moving with constant velocity along the motion direction. The convective heat transfer on the upper surface of the solid is taken into account to simulate an impinging jet. The results are presented in terms of temperature profiles and thermal fields are given for some Biot numbers.

Keywords: Transient Heat Conduction, Laser Source, Manufacturing, Moving Sources, Jet impingement.

1. Introduction

Moving and stationary heat sources are frequently employed in many manufacturing processes and contact surfaces. In recent years applications of localized heat sources have been related to the development of laser and electron beams in material processing, such as welding, cutting, heat treatment of metals and manufacturing of electronic components [1-2]. In some laser beam applications, such as surface heat treatment, the contribution of convective heat transfer must also be taken into account [3].

The impinging jet has a increasingly use in industry to cool or heat a surface in some applications such as the surface hardening. In fact, it produces high heat transfer coefficients. Quasi-steady state thermal fields induced by moving localized heat sources have been widely investigated [3,4], whereas further attention seems to be devoted to the analysis of temperature distribution in transient heat conduction. The one-dimensional unsteady state temperature distribution in a moving semi-infinite solid subject to a pulsed Gaussian laser irradiation was investigated analytically by Modest and Abakians [5]. Shankar and Gnamamuthu [6] obtained a finite difference numerical solution to the three-dimensional transient heat conduction for a moving elliptical Gaussian heat source on a finite dimension solid.

Rozzi et al. [7] carried out the experimental validation for a transient three-dimensional numerical model of the process by which a rotating silicon nitride work-piece is heated with a translating CO_2 laser beam, without material removal. In a companion paper Rozzi et al. [8] used the aforementioned transient three-dimensional numerical model to elucidate the effect of operating parameters on thermal conditions within the work-piece. Rozzi et al. [9, 10] extended the above referred numerical and experimental investigation to the transient three-dimensional heat transfer in a laser assisted machining of a rotating silicon nitride work-piece heated by a translating CO_2 laser and material removing by a cutting tool.

Transient and steady state analytical solutions in a solid due to both stationary and moving plane heat sources of different shapes and heat intensity distributions were derived in [11], by using the Jaeger's heat source method. Yilbas et al. [12] presented a numerical study for the transient heating of a titanium work-piece irradiated by a pulsed laser beam, with an impinging turbulent nitrogen jet. Gutierrez and Araya [13] carried out the numerical simulation of the temperature distribution generated by a moving laser heat source, by the control volume approach. Radiation and convection effects were accounted for. Bianco et al. [14,15] proposed two numerical models for two and three dimensional models to evaluate transient conductive fields due to moving laser sources. Transient numerical models were accomplished in [16,17] in order to extend the analysis given in [14,15] also to a semi-infinite solid.

In this paper a three dimensional transient conductive model is investigated. The convective heat transfer on the upper surface of the solid is taken into account to simulate also an impinging jet. The numerical analysis is accomplished by COMSOL Multiphysics 3.3 code.

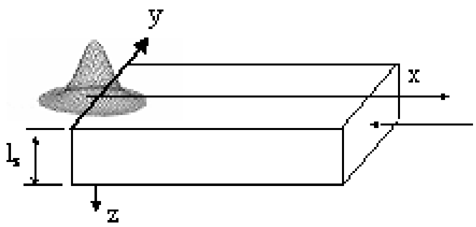


Figure 1. Sketch of the semi-infinite work-piece

2. Mathematical Description

The mathematical formulation for the proposed model is reported in the following. A brick-type solid irradiated by a moving heat source is considered. The solid dimension along the motion direction is assumed to be semi-infinite, while finite thickness and width are assumed. A 3-D model is presented. The thermophysical properties of the material are assumed to be temperature dependent, except the density. The conductive model is considered to be transient.

A sketch of the investigated configuration is reported in Fig. 1. If a coordinate system fixed to the heat source is chosen, according to the moving heat source theory [18,19], a mathematical statement of the three dimensional thermal conductive problem is:

$$\frac{\partial}{\partial x}\left(k(T)\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(k(T)\frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial z}\left(k(T)\frac{\partial T}{\partial z}\right)=\rho c\left(\frac{\partial T}{\partial \theta}-v\frac{\partial T}{\partial x}\right) \quad (1)$$

$v\theta \le x; 0 \le |y| \le l_y/2; 0 \le z \le l_z; \theta > 0$

The boundary and initial conditions are reported in the following:

$$-k\frac{\partial T(v\theta, y, z)}{\partial x}=h\left[T(v\theta, y, z)-T_f\right] \quad (1a)$$

$$T(x \to +\infty, y, z, \theta)=T_{in} \quad (1b)$$

for $0 \le |y| \le l_y/2;\ 0 \le z \le l_z;\ \theta > 0$

$$\frac{\partial T(x, 0, z, \theta)}{\partial y}=0 \quad (1c)$$

$v\theta \le x; 0 \le z \le l_z; \theta > 0$

$$\frac{\partial T(x, \pm l_y/2, z, \theta)}{\partial y}=0 \quad (1d)$$

$v\theta \le x; 0 \le z \le l_z; \theta > 0$

$$-k\frac{\partial T(x, y, 0, \theta)}{\partial z}=q(x, y)+h_u \quad (1e)$$

$v\theta \le x; 0 \le |y| \le l_y/2; \theta > 0$

$$\frac{\partial T(x, y, l_z, \theta)}{\partial z}=0 \quad (1f)$$

$v\theta \le x; 0 \le |y| \le l_y/2; \theta > 0$

$$T(x, y, z, 0)=T_{in} \quad (1g)$$

$v\theta \le x; 0 \le |y| \le l_y/2; 0 \le z \le l_z; \theta > 0$

where the absorbed heat flux q(x,y) is:

$$q(x,y)=q_0\exp\left[-\left(\frac{x^2+y^2}{r_G^2}\right)\right] \quad (2)$$

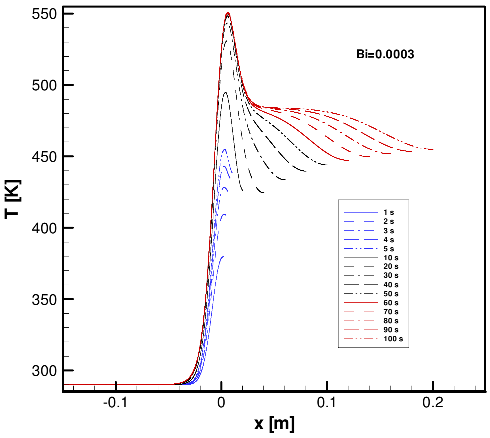
The solid is assumed to be semi-infinite along the motion direction and the problem is considered geometrically and thermally symmetric along the *y* direction. Convective heat losses on the lateral and bottom surfaces are neglected and radiative ones are neglected on all the surfaces. On the upper surface a convective heat transfer due to an impinging jet is considered. The coefficient h_u is evaluated by the correlations reported in Appendix.

The 3-D conductive model is solved by means of the COMSOL Multiphysics 3.3 code. For the thermal model *"Heat Transfer Module"* and "*Transient analysis"* in *"General Heat Transfer"* window have been chosen in order to solve the heat conduction equation.

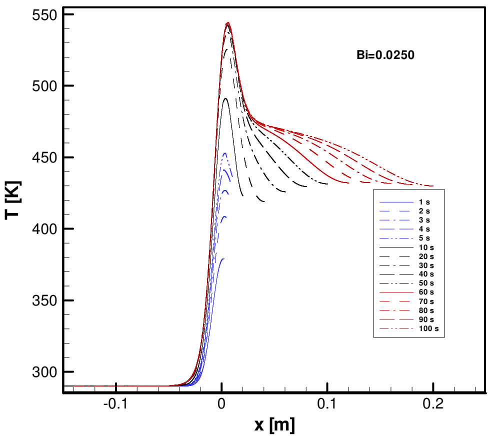
Several different grid distributions have been tested to ensure that the calculated results are grid independent. Maximum temperature differences of the fields is less than 0.1 precent by doubling the mesh nodes. The grid mesh is unstructured.

3. Results and Discussion

Results are presented for two cases: a) a semi-infinite workpiece along the motion direction with constant heat transfer coefficients on the upper (h_u) surfaces, for several Biot number, b) a semi-infinite workpiece along the motion direction with an impinging jet on upper surface, for several Reynolds jet number . The spot radius r_G, the width and the height of the workpiece are equal to 0.0125 m. Temperature dependent thermophysical properties are, from [20], for a 10-18 steel material: k=53.7-0.03714(T-273.15) W/mK, ρ=7806 kg/m^3 and c_p=500.0 + 0.40(T-273.15) J/kg K



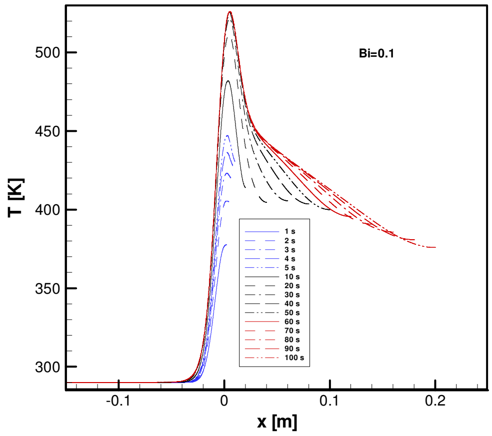
a



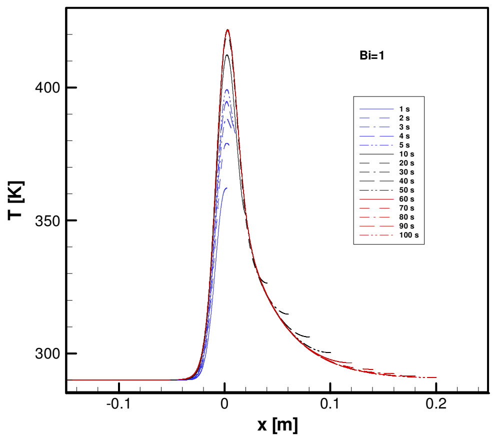
b

Figure 2. Temperature profiles along x coordinate on the upper surface for y=0 and different Bi: a) Bi=0.0003, b) Bi=0.0250

The absorbed laser heat flux is equal to 120 W/cm^2. The workpiece velocity is in the range from 2.11x10^{-3} m/s to 2.11x10^{-2} m/s. The ambient temperature is assumed equal to 290 K. For the case with constant h_u, Biot is defined as Bi=$h_u L_y$/k and the results are obtained for its values in the range from 0.0003 to 1. The Bi considered values are corresponding to h_u values equal to 1, 100 400 and 4000 W/m^{2}K.



a



b

Figure 3. Temperature profiles along x coordinate on the upper surface for y=0 and different Bi: a) Bi=0.1, b) Bi= 1

In Fig. 2 and 3, temperature profiles, along the motion direction, x, for several times are given and they show the thermal development along the heated surface. At the first considered times, t=1 s, it is observed that the temperature values, along x, increase at increasing the time. It can be observed that the temperature profiles are nearly symmetrical with reference to x=0 at the beginning of the heating. Fig. 2b, related to the upper surface, points out that a decreasing temperature profile along the motion direction is obtained. Due to the heat transfer coefficient imposed on the upper surface. It is worth observing that the slope of this curve is constant.

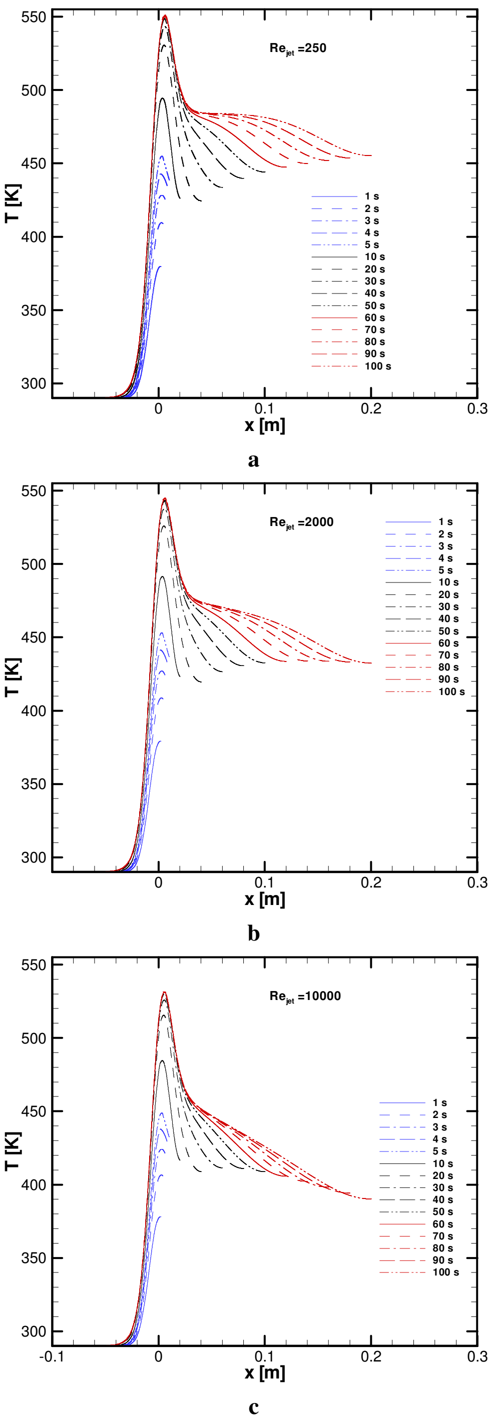


Figure 4. Temperature profiles along x on the upper surface for y=0 and different Re$_{jet}$: a) Re$_{jet}$=250, b) Re$_{jet}$=2000, c) Re$_{jet}$=10000

The slope increases at increasing the Biot number values. Moreover, the temperature values decrease at the heat transfer coefficients increasing.

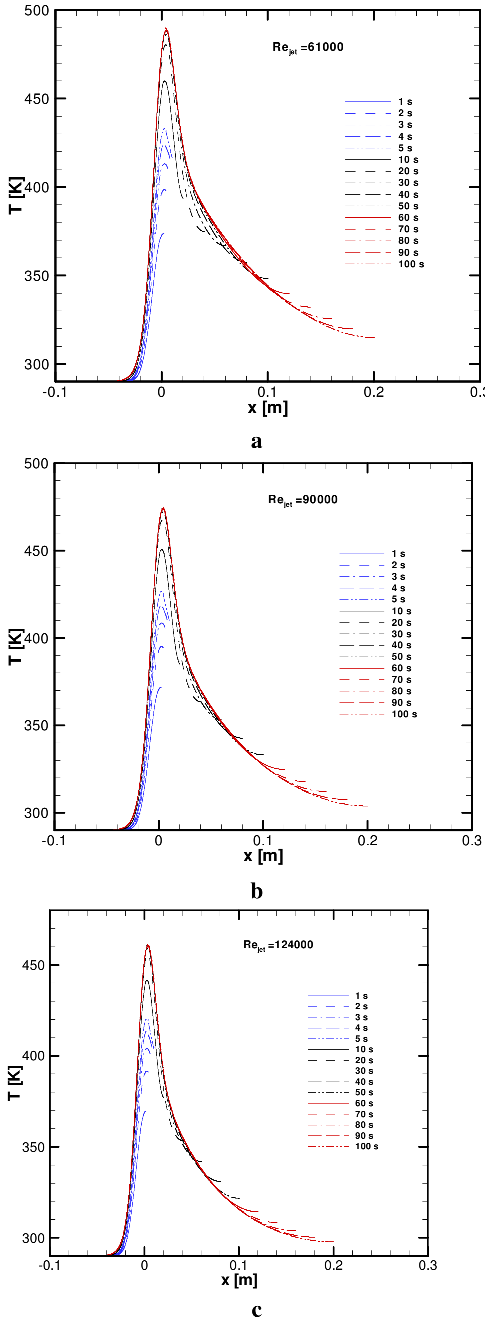


Figure 5. Temperature profiles along x coordinate on the upper surface for y=0 and different Re$_{jet}$: a) Re$_{jet}$=61000, b) Re$_{jet}$=90000, c) Re$_{jet}$=124000

In Figs. 4 and 5, are reported the temperature profiles for Re_{jet} numbers in the range from 250 to $1.24x10^5$. For this configuration, the temperature of the impinging jet, supposed to be helium, has been set equal to 290 and the plate-to-nozzle spacing, H/D_{jet}, equal 7.0, where D_{jet} is equal to 0.022 m. The asymptotic value of temperature is reached, for Re_{jet} =250, for $t \geq 70$ s, in fact, after t=70 s, all the profiles have the same concaveness (Fig 4a) and the maximum temperature is constant. It is observed that the maximum temperature value decreases at increasing the Reynolds jet number value. In fact, for $Re_{jet} \geq 61000$, the maximum temperature value is less than 500 K. Moreover, when the Re_{jet} number increases, the slope becomes steeper.

4. Conclusions

A numerical investigation was carried out in order to estimate a three dimensional transient heat conductive field in semi-infinite metallic solids due to a moving laser source. Temperature profiles along the x axis showed that a quasi steady state was reached and convective heat transfer on the upper surface was found to have a strong effect on the temperature distributions inside the work-piece. The maximum temperature value decreased at increasing the Reynolds jet number value and the slope of temperature profiles became steeper.

5. Nomenclature

Bi Bi=hLy/k
c specific heat (J kg^{-1} K^{-1})
h convective heat transfer coefficient (W m^{-2} K^{-1})
H plate-to-nozzle spacing, m
k thermal conductivity (W m^{-1} K^{-1})
l length (m)
Nu Nusselt number
Pr Prandtl number
q absorbed heat flux (W m^{-2})
r radius (m)
R radius $(x^2+y^2)^{1/2}$, m
Re Reynolds number
T temperature (K)
v velocity of the work-piece (m s^{-1})
x,y,z Cartesian coordinates (m)

5.1 Greek Letters

α thermal diffusivity (m^2 s^{-1})
θ time (s)
ρ density (kg m^{-3})

5.2 Subscripts

a ambient
b bottom surface
f fluid
G Gaussian beam
in initial for $x \rightarrow +\infty$
jet impinging jet
u upper surface
x,y,z along axes.

6. References

1. Tanasawa, I. and Lior, N., *Heat and Mass Transfer in Material Processing*, Hemisphere, Washington, D.C (1992).
2. Viskanta, R. and Bergman, T. L., *Heat Transfer in Material Processing*, in *Handbook of Heat Transfer*, Chap. 18, McGraw-Hill, New York(1998).
3. Shuja, S. Z., Yilbas, B. S., and Budair, M. O., Modeling of Laser Heating of Solid Substance Including Assisting Gas Impingement, *Numer. Heat Transfer A*, **33**, pp. 315-339 (1998).
4. Bianco, N., Manca, O. and Nardini, S., Comparison between Thermal Conductive Models for Moving Heat Sources in Material Processing, *ASME HTD*, **369-6**, pp. 11-22 (2001).
5. Modest, M. F. and Abakians, H., Heat Conduction in a Moving Semi-Infinite Solid Subjected to Pulsed Laser Beam, *J. Heat Transfer*, **108**, pp. 597-601 (1986).
6. Shankar, V. and Gnamamuthu, D., Computational Simulation of Laser Heat Processing of Materials, *J. Therm. Heat Transfer*, **1**, pp. 182-183 (1987).
7. Rozzi, J. C., Pfefferkon, F. E., Incropera, F. P. and Shin, Y. C., Transient Thermal Response of a Rotating Cylindrical Silicon Nitride Workpiece Subjected to a Translating Laser Heat Source, Part I: Comparison of Surface Temperature Measurements with Theoretical Results, *J Heat Transfer*, **120**, pp. 899-906, (1998).
8. Rozzi, J. C., Incropera, F. P. and Shin, Y. C., Transient Thermal Response of a Rotating Cylindrical Silicon Nitride Workpiece Subjected to a Translating Laser Heat Source, Part II:

Parametric Effects and Assessment of a Simplified Model, *J. Heat Transfer*, **120**, pp. 907-915 (1998).
9. Rozzi, J. C., Pfefferkon, F. E., Incropera, F. P. and Shin, Y. C., Transient, Three-Dimensional Heat Transfer Model for the Laser Assisted Machining of Silicon Nitride: I. Comparison of Predictions with Measured Surface Temperature Histories, *Int. J. Heat Mass Transfer*, **43**, pp. 1409-1424 (2000).
10. Rozzi, J. C., Incropera, F. P. and Shin, Y. C., Transient, Three-Dimensional Heat Transfer Model for the Laser Assisted Machining of Silicon Nitride: II: Assessment of Parametric Effects, *Int. J. Heat Mass Transfer*, **43**, pp. 1425-1437 (2000).
11. Hou, Z. B. and Komanduri, R., General Solutions for Stationary/Moving Plane Heat Source Problems in Manufacturing and Tribology, *Int. J. Heat Mass Transfer*, **43**, pp. 1679-1698 (2000).
12. Yilbas, B. S., Shuja, S. Z. and Hashmi, M. S. J., A Numerical Solution for Laser Heating of Titanium and Nitrogen Diffusion in Solid, *J. Mat. Proces. Tech.*, **136**, pp. 12-23 (2003).
13. Gutierrez, G. and Araya, J. G., Temperature Distribution in a Finite Solid due to a Moving Laser Beam, *Proc. IMECE '03, IMECE2003-42545, 2003 ASME Int. Mech. Eng. Congr.*, Washington, D.C., November 15-21, (2003).
14. Bianco, N., Manca, O., Naso, V., Numerical analysis of transient temperature fields in solids by a moving heat source*, HEFAT2004, 3rd Inte. Conf. on Heat Transfer, Fluid Mechanics and Thermodynamics*, Paper n. BN2, 21 – 24 June 2004, Cape Town, South Africa (2004).
15. Bianco, N., Manca, O., Nardini, S., Two dimensional transient analysis of temperature distribution in a solid irradiated by a Gaussian laser source, *Proc. ESDA04, paper n. ESDA2004-58286 7TH Biennial Conf on Eng. Systems Design and Analysis*, July 19–22, 2004, Manchester, United Kingdom (2004).
16. Bianco, N., Manca, O., Nardini, S., Tamburrino, S., Transient heat conduction in solids irradiated by a moving heat source, *COMSOL Users Conference 2006*, Milano 14 Novembre 2006.
17. Bianco, N., Manca, O., Nardini, S., Tamburrino, S.,, Transient Heat Conduction in Semi-Infinite Solids Irradiated by a Moving Heat Source, *COMSOL Users Conference 2007* Grenoble, 23-24 Ottobre
18. Rosenthal, D., The Theory of Moving Sources of Heat and its Application to Metal Treatments, *Trans. ASME*, **68**, pp. 3515-3528 (1946).
19. Manca, O., Morrone, B., Nardini S., Thermal models in moving heat sources with high Peclet number, *ASME International Mechanical Engineering Congress,*12-17 Novembre, 1995.
20. *Metals Handbook*, 9th Ed. ASM, Metal Park, OH (1981).
21. Vickers, J. M. F., Heat Transfer Coefficients between Fluids Jets, *Industrial Engineering Chemistry*, Vol. 51, pp. 967-972 (1959)
22. Martin, H., Heat and Mass Transfer between Impinging Gas Jet and Solid Surfaces, *Advances in Heat Transfer*, Vol. 13, pp. 1-60 (1977).
23. Goldstein, R., Behbahani, A. I. and Heppelmann, K. K., Streamwise Distribution of the Recovery Factor and the Local Heat Transfer Coefficient to an Impinging Circular Jet Flow, *International Journal of Heat and Mass Transfer*, Vol. 29, No. , pp.1227-1235 (1986).

7. Acknowledgements

This research is supported by Regione Campania with a Legge n. 5/2001 grant for the year 2005.

8. Appendix

The Nusselt number on the upper surface of the work-piece have been evaluated by means of following correlation. In this paper the results have been presented for six value of the Reynolds jet number:

for $250 < Re_{jet} < 950$ in [21]

$$Nu_{jet} = \left(1.25 - 0.20\frac{R}{D_{jet}}\right)\cdot\left(Re_{jet} - Re''_{jet}\right)10^{-4} \quad (A1)$$

where

$$Re''_{jet} = 0 \text{ for } \frac{H}{D_{jet}} \leq 7.7$$

for $1900 < Re_{jet} < 6.1 \cdot 10^4$ with $R/D_{jet} < 2.5$, Nu_{jet} has been evaluated by value given by Martin ([22], pp.16-17), whereas for $R/D_{jet} < 2.5$ we have been made use of the following correlation in [22]:

$$Nu_{jet} = \frac{D_{jet}}{R} \frac{1 - 1.1 \frac{D_{jet}}{R}}{1 + 0.1 \left(\frac{H}{D_{jet}} - 6 \right) \frac{D_{jet}}{R}} g(\mathrm{Re}_{jet}) \mathrm{Pr}^{0.42} \quad \text{(A2)}$$

where

$$g(\mathrm{Re}_{jet}) = 2\,\mathrm{Re}_{jet}^{1/2} \left(1 + \frac{\mathrm{Re}_{jet}^{0.55}}{200} \right)^{0.5}$$

for $6.1 \cdot 10^4 < \mathrm{Re}_{jet} < 1.24 \cdot 10^5$, with $R/D_{jet} < 0.5$,we use of the correlation suggested in [23]

$$Nu_{jet} = \frac{24 - \left| \frac{H}{D_{jet}} - 7.75 \right|}{533 + 44 \left(\frac{R}{Djet} \right)^{1.285}} \mathrm{Re}_{jet}^{0.76} \quad \text{(A3)}$$

for $R/D_{jet} < 0.5$ the Nusselt numbers have been evaluated in [22].