

Including Expert Knowledge in Finite Element Models by means of Fuzzy based Parameter Estimation

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Eutrophication of water bodies

Eutrophication:

- Toxicity
- Hygiene conditions
- Collapse of water bodies

Aim:

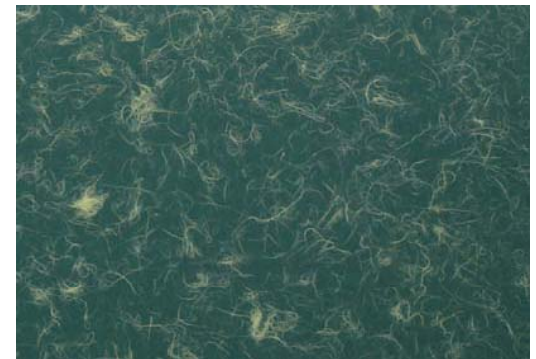
- Prediction of the development and distribution of Algae

Problem:

- Complex growth models
- Parameterization of spatially distributed models

Idea:

- Combination of FEM and Fuzzy-Methods
- Integration of expert knowledge
- Nonlinearity of biological growth



Modelling of Flow and Transport

Navier - Stokes - Equations:

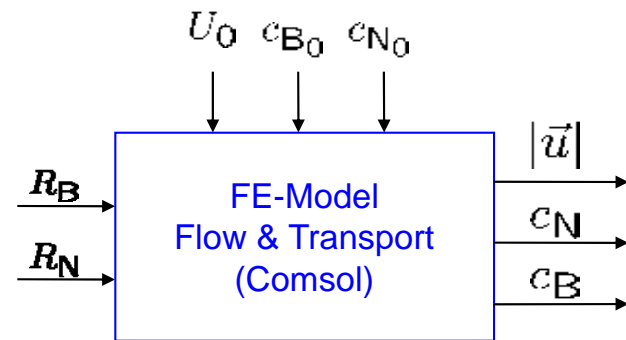
$$\frac{\partial \rho}{\partial t} + \operatorname{div}(\rho \vec{u}) = 0$$

$$\frac{\partial \vec{u}}{\partial t} + \vec{u} \cdot \operatorname{div} \vec{u} = -\frac{1}{\rho} \operatorname{grad} p + \eta \operatorname{div}(\operatorname{grad} \vec{u})$$

Transport of nutrients and biomass:

$$\frac{\partial c_N}{\partial t} = -\vec{u} \cdot \operatorname{grad} c_N + R_N(t)$$

$$\frac{\partial c_B}{\partial t} = -\vec{u} \cdot \operatorname{grad} c_B + R_B(t)$$



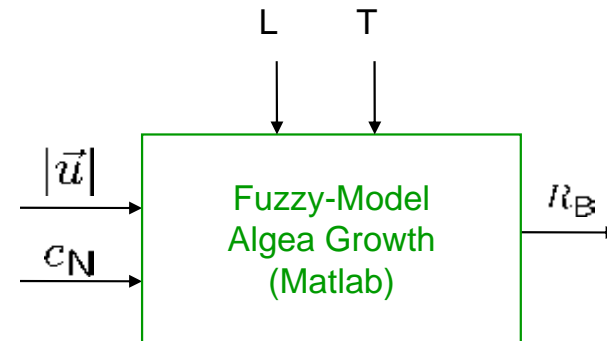
Assumptions

- Neglect of dispersion and diffusion
- Neglect of 3. dimension
- Ideal units of nutrients N

Modelling of Algea Growth

Modelling of Growth of Biomass by Fuzzy

- Flow velocity
 - Distribution of nutrients
 - Water temperature
 - Light
- } Input
- Growth rate of biomass
- } Output



Assumptions

- $u(x,t)$ and $c_N(x,t)$ from FE-model
- Temperature and sunshine duration depend on geographic location and season

Modelling of Algae Growth

Biological submodel

- Knowledge represented by *if-then*-rules
- Fuzzyfication of input data
- Evaluation of *if-then*-rules
- Derivation of crisp values for output variable(s)

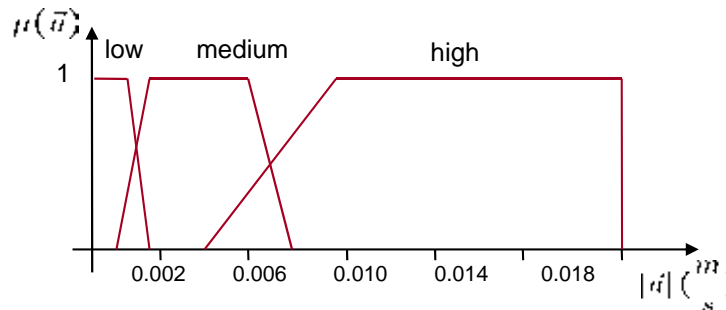
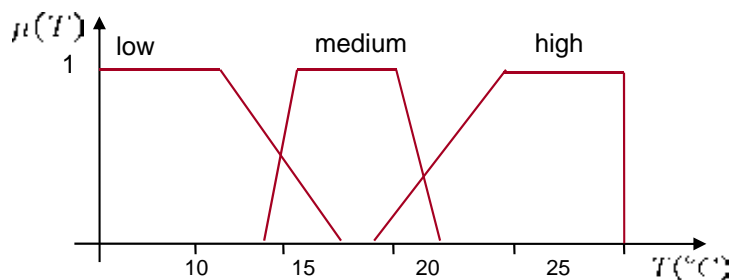


Examples

IF „Nutrients = high“ and „Temperature = high“ THEN „Growth = high“

IF „Nutrients = low“ and „Temperature = high“ THEN „Growth = low“

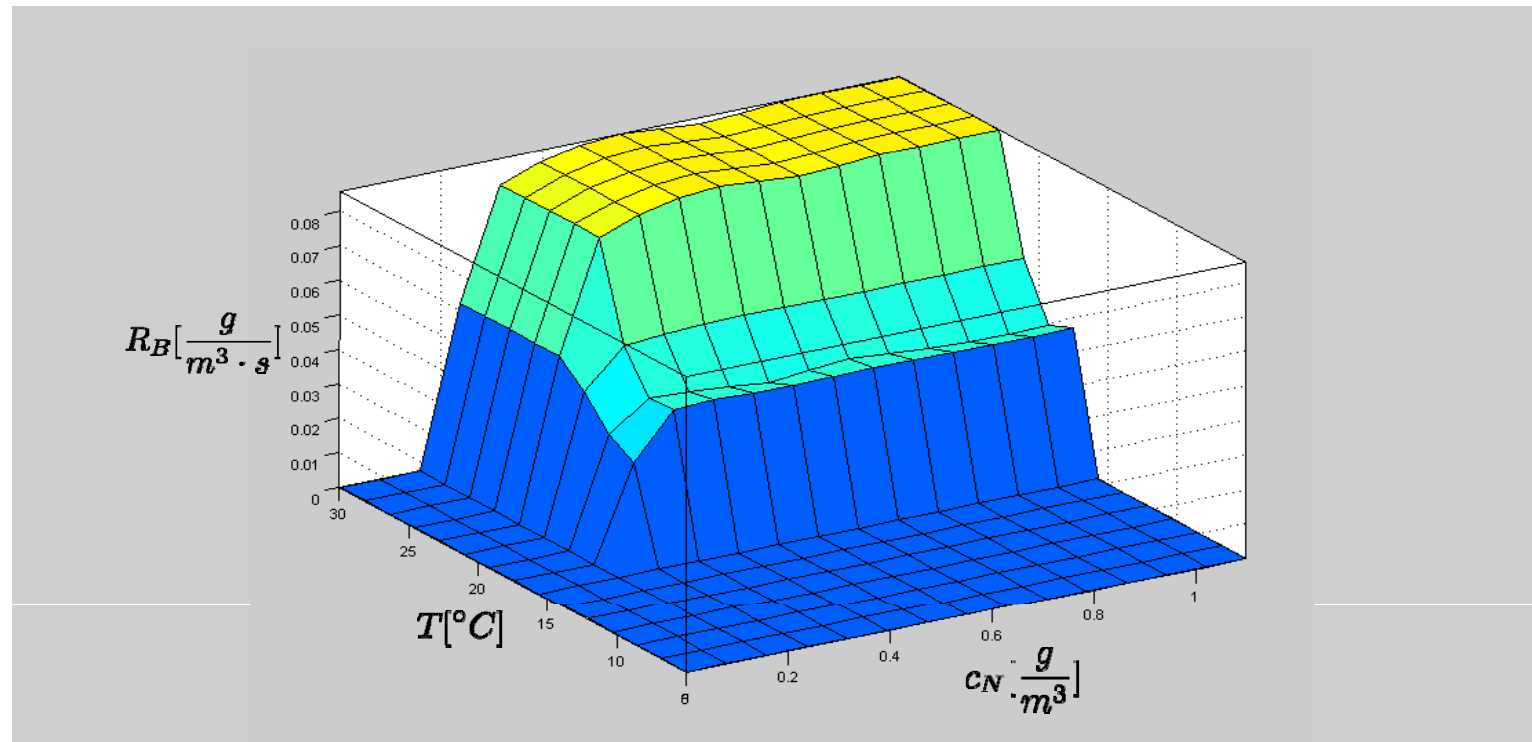
Membership-function describing fuzzy sets



Modelling of Algae Growth

Result of Fuzzy-Modelling:

Nonlinear Look-up-table for biological growth $R_B=f(T,c_N,u=\text{const})$

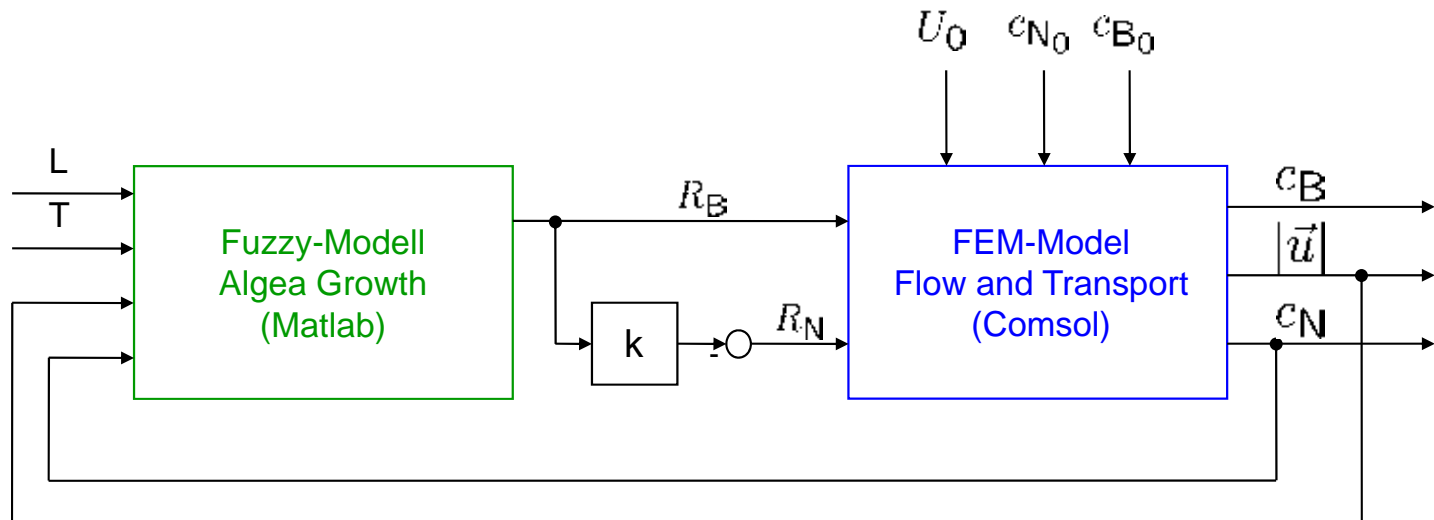


Coupling of Models

Coupling of Submodels

$$R_N = -k \cdot R_B \quad \rightarrow \text{1 Unit of biomass requires } k \text{ units of nutrients}$$

Complete Model



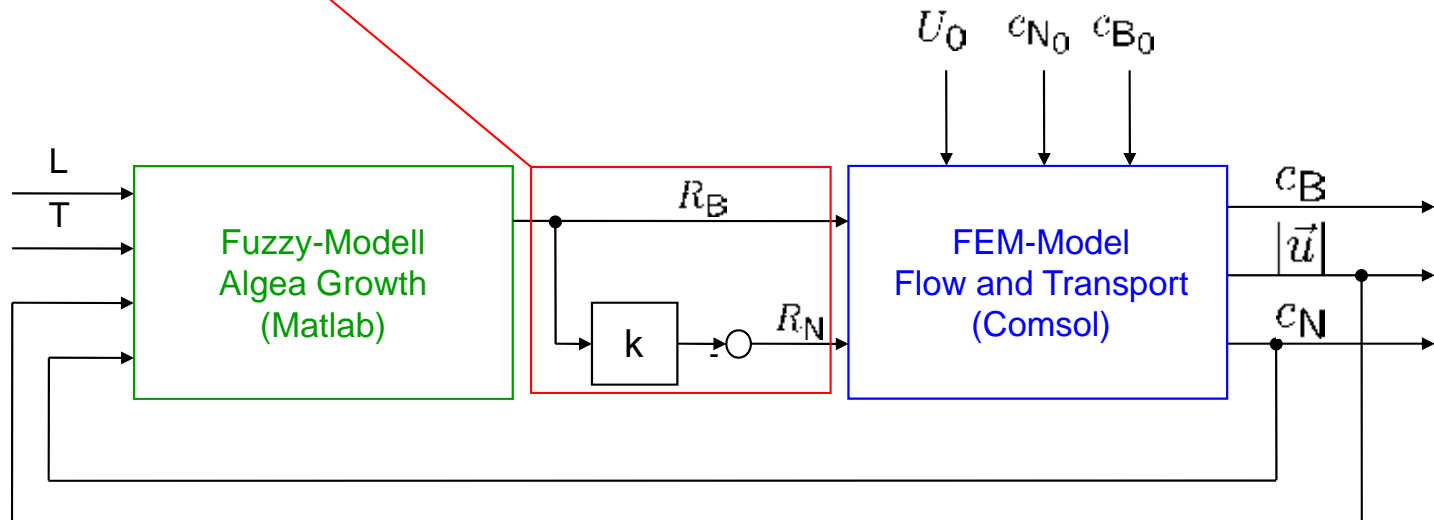
Coupling of Models

Coupling of Submodels

$$R_N = -k \cdot R_B$$

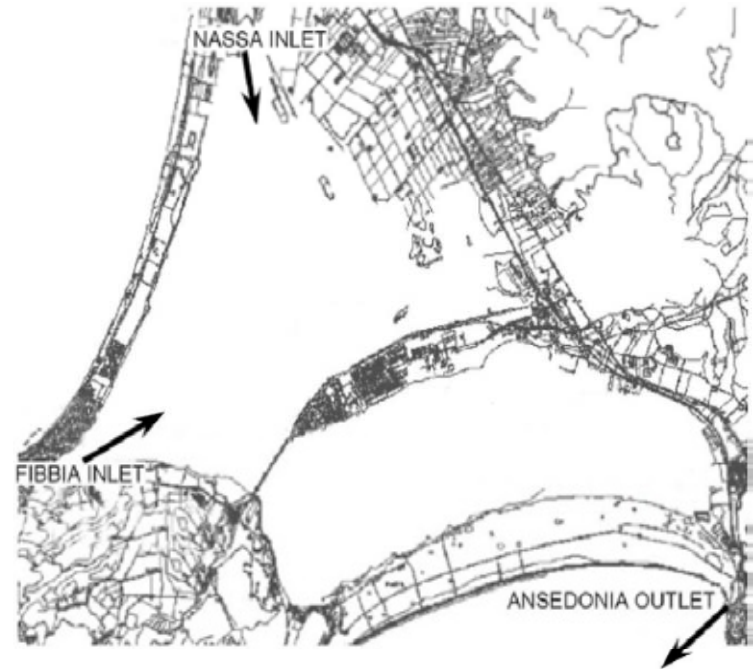
→ 1 Unit of biomass requires k units of nutrients

Complete Model



Simulation Results

Application to Orbetello Sea

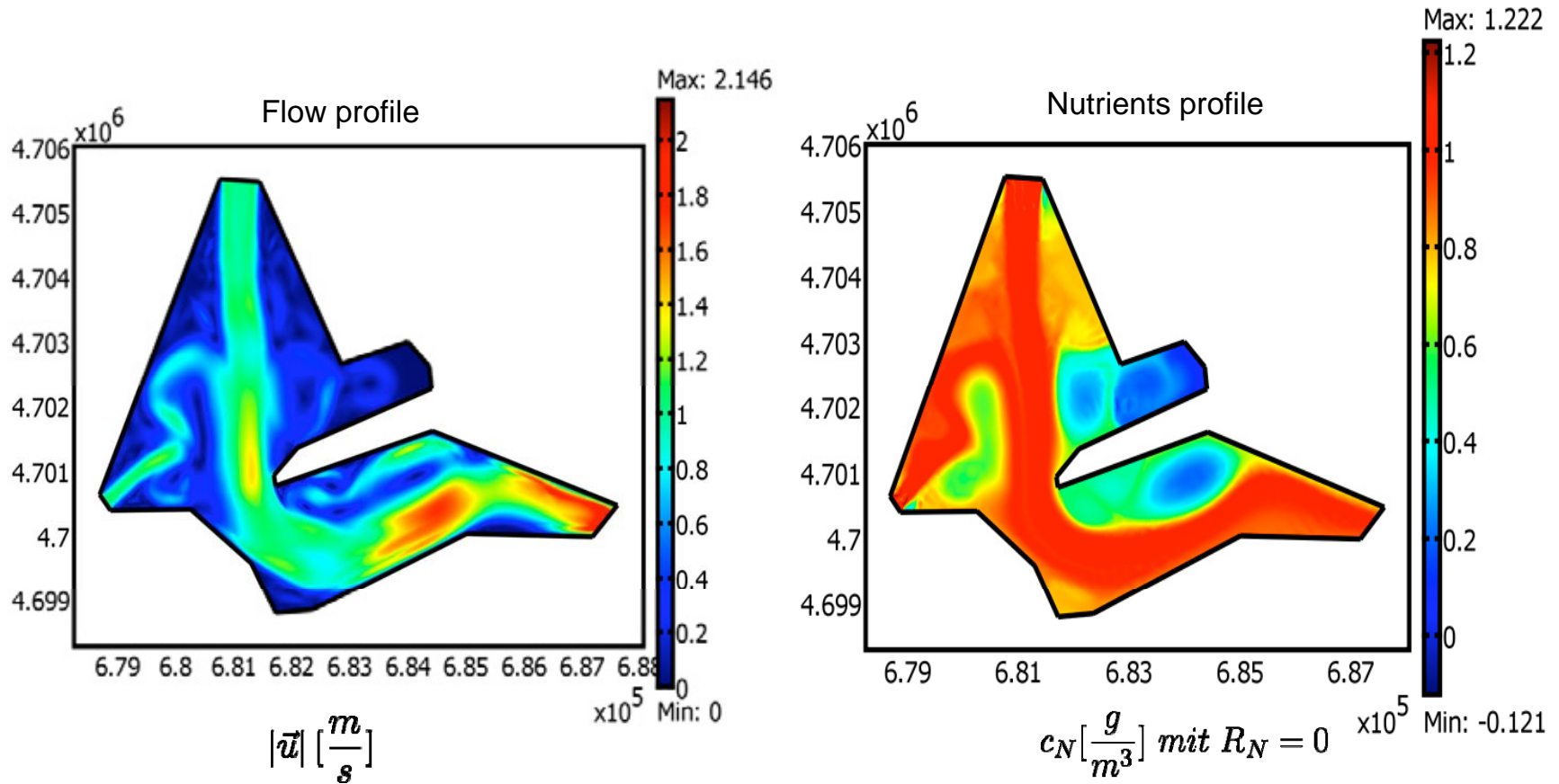


- Water body of about 27 km² at western coast of Italy
- 2 inflows and 1 outflow
- Strong Eutrophication due to intense agriculture and fish farming

Simulation Results

FE-Model: No Biological Growth

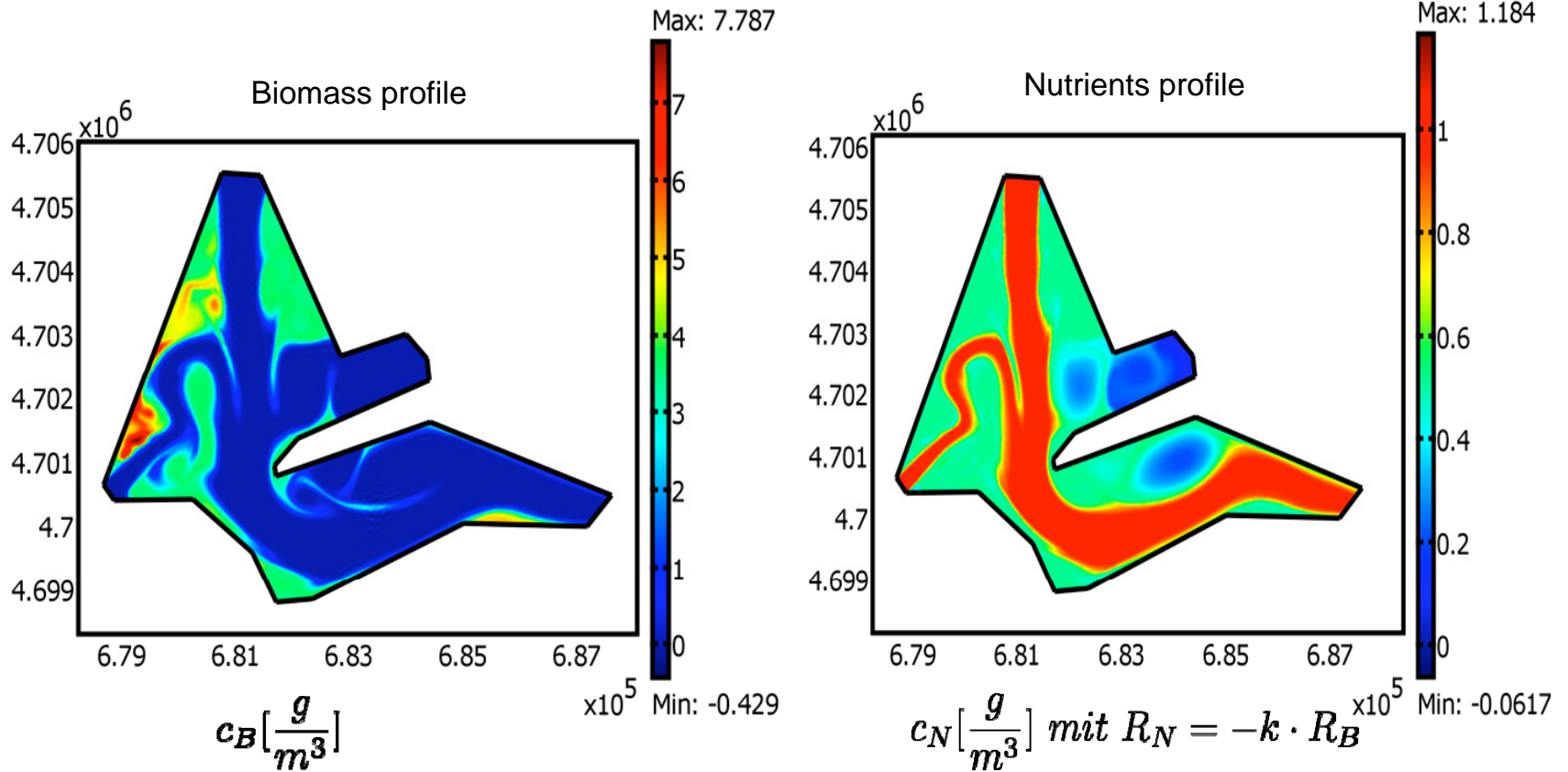
Profile of Flow and Nutrients after 1h



Simulation Results

Complete Model

Profile of Biomass and Nutrients after 1h and $k = -10$



Summary

Problem in spatially distributed transient models

- Complex models for biological growth
- Requirement of big number of parameters
- Computing time

Solution Concept

- Mathematically exact description “as far as possible”
- Qualitative uncertainties described by Fuzzy-models
- Direct incorporation of expert knowledge
- Nonlinearities

Literature

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