

Calculation Of The Helmholtz Resonator Using COMSOL In An Acoustic Lecture

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Introduction

In the field of electronic, electro-acoustic apparatus is one of the important applications. There is an acoustic class in the department of electronic engineering, Shibaura Institute of Technology. To foster the student's comprehension on frequency characteristics of a cavity and on an acoustic element, the students will learn numerical analysis of a Helmholtz resonator. Generally, a resonance frequency of Helmholtz resonator is calculated by acoustic element concept. In the class, a can bottle or a glass one is employed as the Helmholtz resonator. Because of practice on the Helmholtz resonator, we expect that they would understand the validity of FEM in order to improve the precision of the calculation result of the resonator.

Schedule of the lecture

There is an acoustic class in the department of electronic engineering, Shibaura Institute of Technology. The class is "Acoustic systems" which opens in 2nd semester of 3rd year.[1] Table 1 shows the schedule of the lecture "Acoustic systems". The former part of the lectures is learned the fundamental knowledge of the acoustics. In the mid part, the students learn the electro-acoustic transducers, that is, a speaker, a microphone and so on. In the last two times after the examination, the student do the FEM practice using COMSOL.

Lecture on the Helmholtz Resonator

As well known, the Helmholtz Resonator is a lumped system treatment in an Acoustic field. Generally, the wave equation of acoustic phenomena is derived by the approximation of the fluid phenomena. Assuming harmonic time variation, the Helmholtz equation shown in Figs.1 is obtained. In both closed end tube condition or open end tube condition, sound pressure distributions in each condition are obtained. Applying the condition that wave length is much longer than tube length to the

Table 1. Schedule of the lecture "Acoustic Systems".

1. Introductions of the course, history, the relation of the electronic on the acoustics
2. Propagation of the sound in one dimensional system
3. Plane wave propagation and wave equation of the sound in three dimensional system
4. Acoustic impedance and acoustic absorption
5. Basic principle of the vibration
6. Analogies of acoustics to machineries and electronics
7. Kinds of an electro-acoustic transducer and an operation equation of the transducer
8. Design of principle of an electro-acoustic transducer (1)
9. Design of principle of an electro-acoustic transducer (2)
10. Hearing of sense
11. Digital signal processing for acoustics
12. Final exam and comments
13. Practice of sound filed calculation using FEM(1)
14. Practice of sound filed calculation using FEM(2)

sound pressure distribution, acoustic compliance and acoustic mass as acoustic element values are obtained shown in Fig.2. By connecting an acoustic compliance element to an acoustic mass element, the Helmholtz resonator is composed. Since the Helmholtz resonator can be treated in the same manner as series connection circuit of a coil and a condenser. a resonance frequency exists. The Helmholtz resonator is often employed as a noise reduction system. These days, attention to the noise reduction using the Helmholtz resonator increases.

FEM practice class

In the last two lectures, the student do the FEM practice using COMSOL. The lectures of FEM concept and COMSOL are given by KESCO (Keisoku Engineering System Co. Ltd.,) engineers. At first

lecture of FEM practice, the topics are "Short Introduction to FEM and COMSOL Multiphysics" and "Pressure acoustic simulation-a bench mark problem".

Wave equation in the three dimensions

Equation of motion : $\rho_0 \frac{\partial \mathbf{v}}{\partial t} = -\nabla p = \left(-\frac{\partial p}{\partial x}, -\frac{\partial p}{\partial y}, -\frac{\partial p}{\partial z} \right)$

Conservation of mass: $\frac{\partial \rho}{\partial t} = -\rho_0 \nabla \cdot \mathbf{v} = -\rho_0 \left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} \right)$

Gas state equation: $p = c^2 \rho$ (where $c^2 = \frac{\gamma P_0}{\rho_0}$)

Wave equation in the three dimensions

$$\nabla^2 p = \frac{1}{c^2} \frac{\partial^2 p}{\partial t^2}$$

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(a)

Helmholtz equation

In the acoustic phenomena, the time variation is expressed by the harmonic function $e^{j\omega t}$

Sound pressure $p = \rho_0 \frac{\partial \Phi}{\partial t} = j\omega \rho_0 \Phi$ Particle velocity $\mathbf{v} = -\nabla \Phi$

By substituting above equation to the wave equation, the following equation "Helmholtz equation" is obtained.

$$\nabla^2 \Phi + k^2 \Phi = 0$$

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(b)

Figure 1. Slide of the derivation of the Helmholtz equation.

Acoustic element

Closed end tube: L

acoustic compliance

$$Z_a = \frac{p}{Q} = \frac{\rho c}{j\omega S_a k S_a} = \frac{1}{j\omega C_a} \quad C_a = \frac{S_a^2}{\rho c^2}$$

Open end tube: L

acoustic mass (inertia)

$$Z_a = \frac{p}{Q} = j\omega \rho l S_a = j\omega m_a \quad m_a = \rho l S_a$$

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(a)

Helmholtz resonator

Helmholtz resonator consists of small and short open end tube and short tank.

$$f = \frac{1}{2\pi} \sqrt{\frac{1}{C_a m_a}} = \frac{c}{2\pi} \sqrt{\frac{S_a}{LV_a}}$$

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(b)

Figure 2. Slide of the derivation of the Helmholtz resonator.

At second lecture, the topics are "From eigenfrequency to frequency domain analysis" and "Modeling practice". The students do a simple experiment of the Helmholtz resonator and compare rough calculation to measurement. And also they do FEM simulation and compare the result.

In the class, a can bottle or a glass one is employed as the Helmholtz resonator. Before experiment, the students are able to calculate the resonance frequency by only measuring the dimensions of the can bottle or the glass bottle as shown in Fig.3. The thinner cylindrical part in the bottle corresponds to the acoustic mass and the thicker part in the bottle is the acoustic compliance. Each acoustic element value is able to be determined by the dimension, air density and air sound speed. In the practical measurement, we prepare a simple FFT analyzer[2]. Using the built-in microphone in a personal computer, they can measure emitted tone sound when they blow the mouth of the bottle as shown in Fig.4. As a result, they can confirm the precision of their rough calculation result. Due to the slope part in the bottle, the difference between the rough calculation and the measurement is around 30%. After the rough calculation, they analyze the frequency response of the bottle they employed using the finite element application, "COMSOL".



Figure 3. Simple experiment of the Helmholtz resonator.

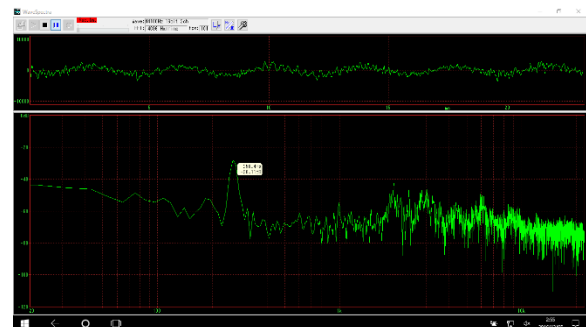


Figure 4. Measurement result.

Subjects of the FEM practice

To foster the student's comprehension, the following subject are prepared.

- 1) Select a bottle among 4 kinds prepared ones. And measure the dimensions of the bottle.
- 2) Estimate roughly eigenfrequency of a bottle using the concept of Helmholtz resonator.
- 3) Make a sound by blowing the bottle. And check the frequency using the spectrum analysis application on the notebook PC.
- 4) Compare the measured frequency with the rough estimation result.
- 5) Simulate both eigenfrequency and frequency domain analysis using COMSOL FEM application.
- 6) Compare the FEM analysis result with the measured frequency.
- 7) Create the report using COMSOL function.
- 8) Add your comment on the rough estimation result and the FEM analysis one with the report file COMSOL created.
- 9) Submit the report to the Submission folder in the ShareFolder. The file name should be changed Student_ID.docx.

Conclusions

Students created their subject report by COMSOL report function. They also have to add their comment on comparison between the measurement, rough estimation and FEM results. Our future plan is to make the students consider the results more deeply.

References

1. <http://syllabus.sic.shibaura-it.ac.jp/syllabus/2018/ko1/105105.html.en>
2. <http://efu.jp.net/soft/ws/ws.html>