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Modeling two-phase flow in strongly heterogeneous porous media

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1 Introduction

Modeling Two-phase flow through strongly heterogeneous porous media is of importance in many disciplines including petroleum industry, hydrology etc.

There are, however, still some challenges in numerical simulation of such flow problems especially the flows in fractured porous media and fractured vuggy porous media.

The aim of this report is to implement in COMSOL Multiphysics a two-phase fluid flow model in strongly heterogeneous porous media using a finite element approach.

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Existing Problems

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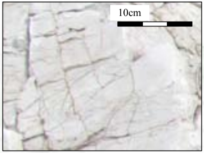
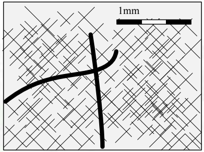
- Problem 2
- Problem 3

Problem 1 ... The existence of fractures or vugs can influence the fluid flow in porous media largely. However a reliable modeling method is difficult for fractured (or/and vuggy) porous media due to the complex geometries of fractures (or/and vugs) at multiple scales.





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Problem 2 ... The dual-porosity model has traditionally been used to simulate the flow in fractured hydrocarbon reservoirs. This approach, although very efficient, suffers from some important limitations.
(1) One limitation is that the method cannot be applied to disconnected fractured media and cannot represent the heterogeneity of such a system.
(2) Another shortcoming is the complexity in the evaluation of the transfer function between the matrix and the fractures.

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Problem 3 ... Naturally fractured vuggy porous media present multiple challenges for numerical simulations of various fluid flow problems. Such media are characterized by the presence of fractures and vugs at multiple scales. The main difficulty in numerical simulations in such media is the co-existence of porous and free flow regions especially for two-phase flow.

2 Fractured Porous Media

2.1 Discrete Fracture Model

The discrete-fracture model describes the fractures explicitly in the medium similarly to the single-porosity model. However, unlike the single-porosity model, the fractures gridcells are geometrically simplified by using $n-1$ dimensional gridcells in an n-dimensional domain. As a result, computational efficiency is improved considerably.

$$\begin{aligned}\int_{\Omega} FEQ\mathrm{d}\Omega &= \int_{\Omega_\mathrm{m}} FEQ\mathrm{d}\Omega_\mathrm{m} + \int_{\Omega_\mathrm{f}'} FEQ\mathrm{d}\Omega_\mathrm{f}' \\ &= \int_{\Omega_\mathrm{m}} FEQ\mathrm{d}\Omega_\mathrm{m} + a \times \int_{\Omega_\mathrm{f}} FEQ\mathrm{d}\Omega_\mathrm{f}\end{aligned}$$

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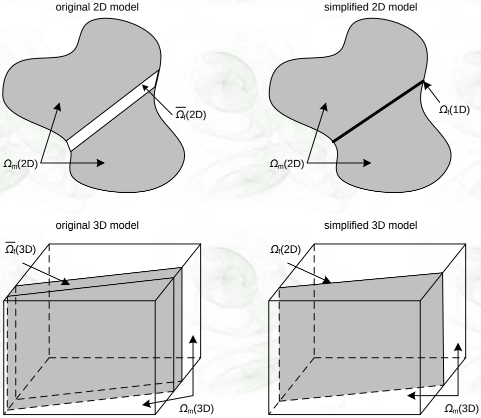

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2.2 Two-phase Flow Model

(1) Mathematical formulation

Mass continuity

$$\phi \frac{\partial S_\alpha}{\partial t} + \triangledown \cdot \boldsymbol{v}_\alpha = q_\alpha \, , \alpha = \mathsf{w}, \ \mathsf{o}$$

Darcy's law

$$\boldsymbol{v}_\alpha = -\boldsymbol{K} \frac{k_{r\alpha}}{\mu_\alpha} \left(\triangledown p_\alpha + \rho_\alpha g \triangledown z \right) , \alpha = \mathsf{w}, \ \mathsf{o}$$

Auxiliary equations

$$S_\mathsf{w} + S_\mathsf{o} = 1$$

$$p_\mathsf{c} \left(S_\mathsf{w} \right) = p_\mathsf{o} - p_\mathsf{w} \geqslant 0$$

Define the flow potential as fowllows

$$\Phi_\alpha = p_\alpha + \rho_\alpha g z, \alpha = \mathsf{w}, \ \mathsf{o}$$

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The whole mathematical model can be given by

$$\begin{bmatrix} 0 & 0 \\ 0 & \phi \end{bmatrix} \frac{\partial}{\partial t} \begin{bmatrix} \Phi_{\mathrm{w}} \\ S_{\mathrm{w}} \end{bmatrix} + \triangledown \cdot \left\{ - \begin{bmatrix} \boldsymbol{K}(\lambda_{\mathrm{w}} + \lambda_{\mathrm{w}}) & \boldsymbol{K}\lambda_{\mathrm{o}} p_{\mathrm{c}}' \\ \boldsymbol{K}\lambda_{\mathrm{w}} & 0 \end{bmatrix} \triangledown \begin{bmatrix} \Phi_{\mathrm{w}} \\ S_{\mathrm{w}} \end{bmatrix} \right\} = \begin{bmatrix} q_{\mathrm{o}} + q_{\mathrm{w}} \\ q_{\mathrm{w}} \end{bmatrix}$$

(2) The Galerkin finite element formulation

The formulation for water flow potential equation

$$\int_\Omega \triangledown \cdot \left[-\boldsymbol{K} (\lambda_{\mathrm{w}} + \lambda_{\mathrm{o}}) \triangledown \Phi_{\mathrm{w}} \right] \delta\Phi_{\mathrm{w}} \mathrm{d}\Omega + \int_\Omega \triangledown \cdot \left(-\boldsymbol{K}\lambda_{\mathrm{o}} p_{\mathrm{c}}' \triangledown S_{\mathrm{w}} \right) \delta\Phi_{\mathrm{w}} \mathrm{d}\Omega$$
$$= \int_\Omega (q_{\mathrm{o}} + q_{\mathrm{w}}) \delta\Phi_{\mathrm{w}} \mathrm{d}\Omega$$

The formulation for water saturation equation

$$\int_\Omega \phi \frac{\partial S_{\mathrm{w}}}{\partial t} \delta S_{\mathrm{w}} \mathrm{d}\Omega + \int_\Omega \triangledown \cdot (-\boldsymbol{K}\lambda_{\mathrm{w}} \triangledown \Phi_{\mathrm{w}}) \delta S_{\mathrm{w}} \mathrm{d}\Omega = \int_\Omega q_{\mathrm{w}} \delta S_{\mathrm{w}} \mathrm{d}\Omega$$

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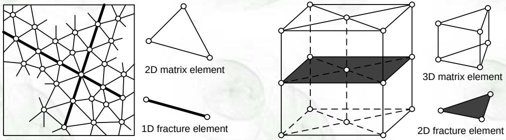
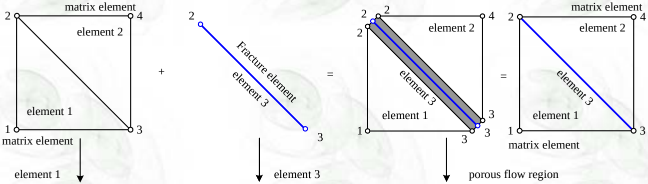


Schematic of matrix and fracture elements



$$\begin{pmatrix} a^1_{11} & a^1_{12} & a^1_{13} & 0 \\ a^1_{21} & a^1_{22}+a^2_{22} & a^1_{23}+a^2_{23} & a^2_{24} \\ a^1_{31} & a^1_{32}+a^2_{32} & a^1_{33}+a^2_{33} & a^2_{34} \\ 0 & a^2_{42} & a^2_{43} & a^2_{44} \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & a^3_{22} & a^3_{23} & 0 \\ 0 & a^3_{32} & a^3_{33} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} a^1_{11} & a^1_{12} & a^1_{13} & 0 \\ a^1_{21} & a^1_{22}+a^2_{22}+a^3_{22} & a^1_{23}+a^2_{23}+a^3_{23} & a^2_{24} \\ a^1_{31} & a^1_{32}+a^2_{32}+a^3_{32} & a^1_{33}+a^2_{33}+a^3_{33} & a^2_{34} \\ 0 & a^2_{42} & a^2_{43} & a^2_{44} \end{pmatrix}$$

element 2

FEM implementation for matrix and fracture elements

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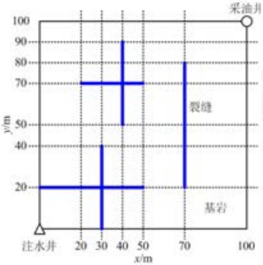
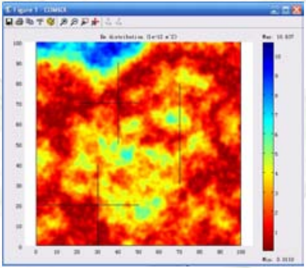
2.3 An Example

Based on the above theory, we use the Coefficient PDE Form in COMSOL Multiphysics 3.5a to implement the discrete fracture model.



(a) Waterflooding model



(b) Permeability distribution

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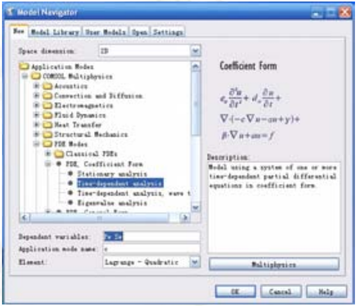
First, open the Model Navigator to set the Coefficient Form's variables.



Space dimension: 2D

Coefficient Form

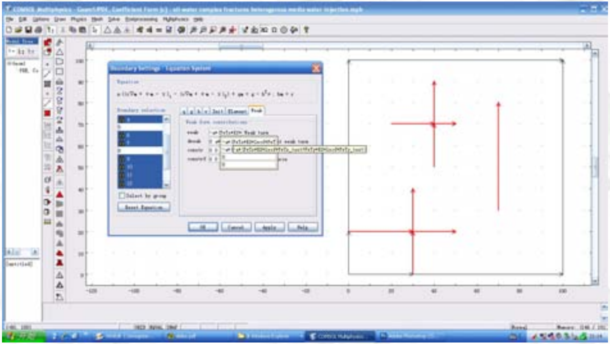
$$e_a\frac{\partial^2 u}{\partial t^2}+d_a\frac{\partial u}{\partial t}+\nabla\cdot(-c\nabla u-\alpha u+\gamma)+\beta\cdot\nabla u+au=f$$

Description: Model using a system of one or more time-dependent partial differential equations in coefficient form.

Application mode name: c

Element: Lagrange - Quadratic

Now set up the flow model for the fracture is the the key step. In this example we choose Physics $->$ Equation Systems $->$ Boundary Settings using the weak form.



Click here to run Discrete Fracture.wmv

Fluids in fractured porous media move quickly through the fractures but also migrate, albeit relatively slowly, through the tiny pores within the surrounding matrix blocks.

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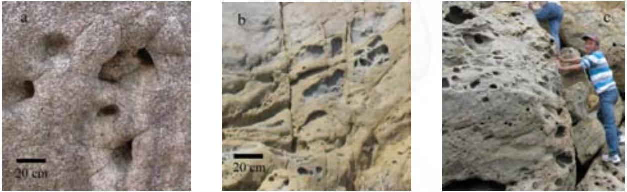
3 Fractured Vuggy Media

Fractured vuggy porous media are common in carbonate rocks, and are endemic to many of the world’s groundwater aquifers and petroleum reservoirs. Recently, fractured vuggy porous media have received much attention because a number of fractured vuggy reservoirs have been found worldwide that can significantly contribute to oil & gas reserves and production.



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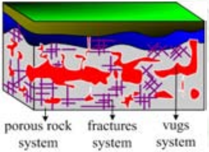
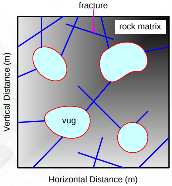
3.1 Discrete Fracture-Vug Network

The fractured vuggy porous medium is a huge discrete fracture-vug network space. With this concept, we proposed a novel model named Discrete Fracture-Vug Network Model (DFVN).





DFVN model is an extension of classic discrete fracture model for fractured vuggy porous media. The flow in matrix and fractures follows Darcy law, and the vugs are free-flow region.

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3.2 Upscaling and Equivalent Permeability

Based on homogenization upscaling technique, we can derive the equivalent permeability.

$$-\nabla_y^2 \boldsymbol{w}_{\mathsf{s}}^j + \nabla_y \pi_{\mathsf{s}}^j = \boldsymbol{e}_j, \quad \text{in } \boldsymbol{Y}_{\mathsf{s}}$$

$$\nabla_y \cdot \boldsymbol{w}_{\mathsf{s}}^j = 0, \quad \text{in } \boldsymbol{Y}_{\mathsf{s}}$$

$$\boldsymbol{K}^{-1} \boldsymbol{w}_{\mathsf{d}}^j + \nabla_y \pi_{\mathsf{d}}^j = \boldsymbol{e}_j, \quad \text{in } \boldsymbol{Y}_{\mathsf{d}}$$

$$\nabla_y \cdot \boldsymbol{w}_{\mathsf{d}}^j = 0, \quad \text{in } \boldsymbol{Y}_{\mathsf{d}}$$

$$\boldsymbol{w}_{\mathsf{s}}^j \cdot \boldsymbol{n}_{\mathsf{s}} = \boldsymbol{w}_{\mathsf{d}}^j \cdot \boldsymbol{n}_{\mathsf{s}}, \quad \text{on } \Sigma$$

$$2\boldsymbol{n}_{\mathsf{s}} \cdot \boldsymbol{S}\left(\boldsymbol{w}_{\mathsf{s}}^j\right) \cdot \boldsymbol{n}_{\mathsf{s}} = \pi_{\mathsf{s}}^j - \pi_{\mathsf{d}}^j, \quad \text{on } \Sigma$$

$$\boldsymbol{w}_{\mathsf{s}}^j \cdot \boldsymbol{\tau}_{\mathsf{s}} = -2\frac{\sqrt{\boldsymbol{\tau}_{\mathsf{s}} \cdot \boldsymbol{K} \cdot \boldsymbol{\tau}_{\mathsf{s}}}}{\alpha} \boldsymbol{n}_{\mathsf{s}} \cdot \boldsymbol{S}(\boldsymbol{w}_{\mathsf{s}}^j) \cdot \boldsymbol{\tau}_{\mathsf{s}}, \quad \text{on } \Sigma$$

where $\boldsymbol{w}_l^j$ and π_l^j (l= s, d) are the $\boldsymbol{Y}$-periodic vector fields. The macroscopic equivalent permeability $\boldsymbol{\kappa}$ is then computed by averaging the fine-scale velocities

$$\boldsymbol{\kappa}_{ij} = \frac{1}{|\mathbf{Y}|}\left(\int_{\mathbf{Y}_\mathrm{s}} (\boldsymbol{w}_\mathrm{s}^j)_i \mathrm{d}\boldsymbol{y} + \int_{\mathbf{Y}_\mathrm{d}} \left(\boldsymbol{w}_\mathrm{d}^j\right)_i \mathrm{d}\boldsymbol{y}\right)$$

$$\int_{\mathbf{Y}_\mathrm{d}} \left(\boldsymbol{w}_\mathrm{d}^j\right)_i \mathrm{d}\boldsymbol{y} = \int_{\mathbf{Y}_\mathrm{m}} (\boldsymbol{w}_\mathrm{m}^j)_i \mathrm{d}\boldsymbol{y} + e \times \int_{\mathbf{Y}_\mathrm{f}} \left(\boldsymbol{w}_\mathrm{f}^j\right)_i \mathrm{d}\boldsymbol{y} \,, \qquad \text{m=matrix, f=fracture}$$

The macroscopic equivalent flux is given by the Darcy's law on coarse scale as $\varepsilon \to 0$

$$\mu(\boldsymbol{\kappa})^{-1}\overline{\boldsymbol{u}} + \nabla p^0 = \rho \boldsymbol{f}$$

and subject to conservation of mass

$$\nabla \cdot \overline{\boldsymbol{u}} = 0$$

Above procedure is very similar to the one employed for upscaling the Stokes-Darcy Eqs. in a vuggy porous media.

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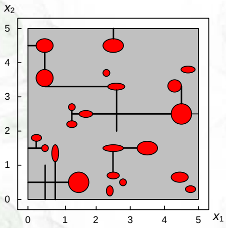
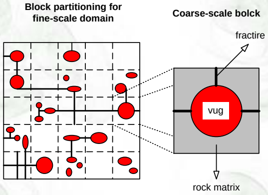
3.3 A simple computation test



a Fine-scale domain



b 5×5 coarse-scale block partitioning

In practice, for full field-scale problems, we need to solve the Darcy macro-model on a coarse grid, using equivalent permeability upscaling from the fine scale Darcy-Stokes cell problem. At first we should compare a fine-scale reference solution with the coarse-scale model to verify the validity of the proposed upscaling method.

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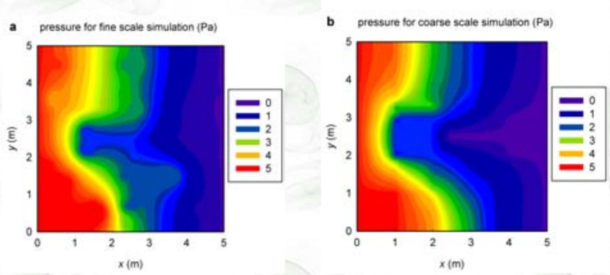

Then we can easily model the two-phase flow in such media on the field-scale using the similar procedure above.

Click here to run Field-Scale Model.avi

4 Conclusions

(1) Two-phase flow has been implemented with COMSOL Multiphysics 3.5a successfully based on Pressure-Saturation model, which can be conducted conveniently using Coefficient PDE Form.

(2) We coupled discrete fracture model and Pressure-Saturation model to simulate the fluid flow in fractured media. The results demonstrate that the fractures are the dominate-flow paths, and they can intensify the heterogeneity and anisotropy.

(3) The discrete fracture-vug network model provides a natural way of modeling fluids flow through fractured vuggy porous media.

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3. J. Yao, Z. Q. Huang, Y. J. Li et al. Discrete Fracture-Vug Network Model for Modeling Fluid Flow in Fractured Vuggy Porous Media[C]. Paper SPE 130287, prepared for presentation at the CPS/SPE International Oil & Gas Conference and Exhibition in China held in Beijing, China, 8-10 June 2010. （SPE会议论文）

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Thanks for your attention!

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