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Analysis of Lubricant Flow through Reynolds Equation

J.S. Crompton, L.T. Gritter, S.Y. Yushanov, K.C. Koppenhoefer
AltaSim Technologies, LLC

R.O. Edwards
Cummins Fuel Systems, Inc.

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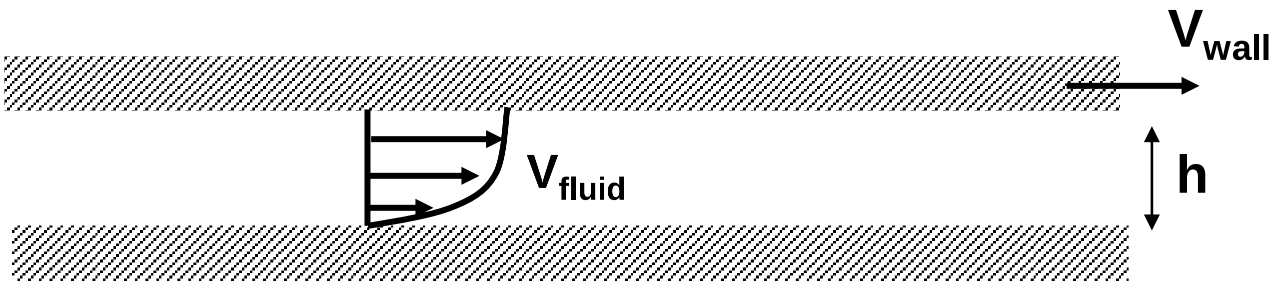
What is Reynolds Equation?



- Special case of Navier-Stokes Equations

  1) fluid thickness small compared to length/width,

  2) pressure gradients through fluid thickness are small,

  3) no external forces act on the fluid film,

  4) no slip at the bearing surfaces, and

  5) velocity gradients along the thickness dominate all other velocity gradients.

Applications

- Common solution to tribology problems
- Thin layer of fluid reduces friction/wear

Why COMSOL?

- Designed for multiphysics problems
- Designed for simple implementation of "new" physics
  - PDE mode
  - Weak boundary conditions
  - Boundary extrusion coupling variables (calculate *h*)

Procedure

- Implement Reynolds Equation in COMSOL Multiphysics
- Include thermodynamics of fluid properties
- Verify w/ existing solution for simple case

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Reynolds Equation – Weak Form

Reynolds equation: $$\frac{\partial}{\partial x}\left(\frac{\rho h^3}{\eta}\frac{\partial P}{\partial x}\right)+\frac{\partial}{\partial y}\left(\frac{\rho h^3}{\eta}\frac{\partial P}{\partial y}\right)=6\frac{\partial}{\partial x}\left(\rho U_0 h\right)+6\frac{\partial}{\partial y}\left(\rho V_0 h\right)+12\rho W_0$$

or $$\frac{\partial}{\partial x}\left(\frac{\rho h^3}{12\eta}\frac{\partial P}{\partial x}-\frac{1}{2}\rho U_0 h\right)+\frac{\partial}{\partial y}\left(\frac{\rho h^3}{12\eta}\frac{\partial P}{\partial y}-\rho V_0 h\right)=\rho W_0$$

where

h gap

U_0, V_0, W_0 velocity components of gap boundary

Equivalent form or Reynolds equation: $\nabla\cdot\left(-\mathbf{q}\right)=\rho W_0$

$$\mathbf{q}=\left\{\rho\left(\frac{U_0 h}{2}-\frac{h^3}{12\eta}\frac{\partial P}{\partial x}\right),\rho\left(\frac{V_0 h}{2}-\frac{h^3}{12\eta}\frac{\partial P}{\partial y}\right)\right\}$$ mass flow rate per unit width $\left\lfloor\frac{kg}{m\cdot s}\right\rfloor$

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Reynolds Equation – Weak Form

Multiply Reynolds equation by test/shape function P_{test} and integrate over domain Ω :

$$\int_{\Omega} P_{test} \nabla \cdot (-\mathbf{q}) d\Omega = \int_{\Omega} P_{test} \rho W_0 d\Omega$$

- use identity $\nabla \cdot (P_{test}\mathbf{q}) = P_{test}\nabla \cdot \mathbf{q} + \mathbf{q} \cdot \nabla P_{test}$
- use Gauss theorem: $\int_{\Omega} \nabla \cdot (P_{test}\mathbf{q}) d\Omega = \int_{\partial\Omega} P_{test}\mathbf{q} \cdot d\mathbf{n}$

$$\Rightarrow \int_{\partial\Omega} P_{test}\mathbf{q} \cdot d\mathbf{n} - \int_{\Omega} \mathbf{q} \cdot \nabla P_{test} d\Omega = \int_{\Omega} P_{test} \rho W_0 d\Omega$$

n: unit normal to boundary

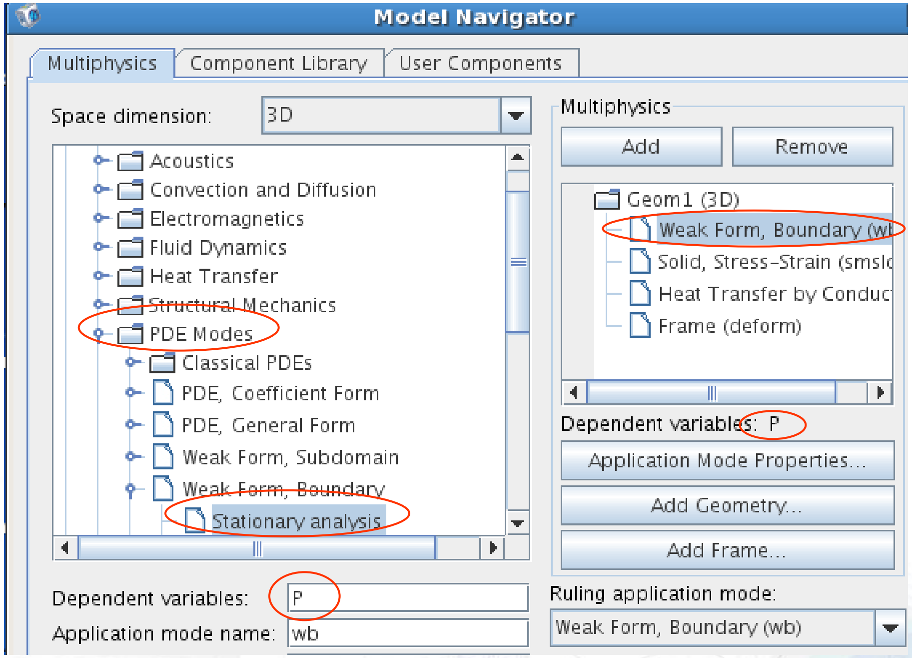
- introduce Lagrange multiplier μ
- assume Neumann BC: $-\mathbf{n} \cdot \mathbf{q}|_{\partial\Omega} = G + \mu H$

Complete weak form formulation of Reynolds equation:

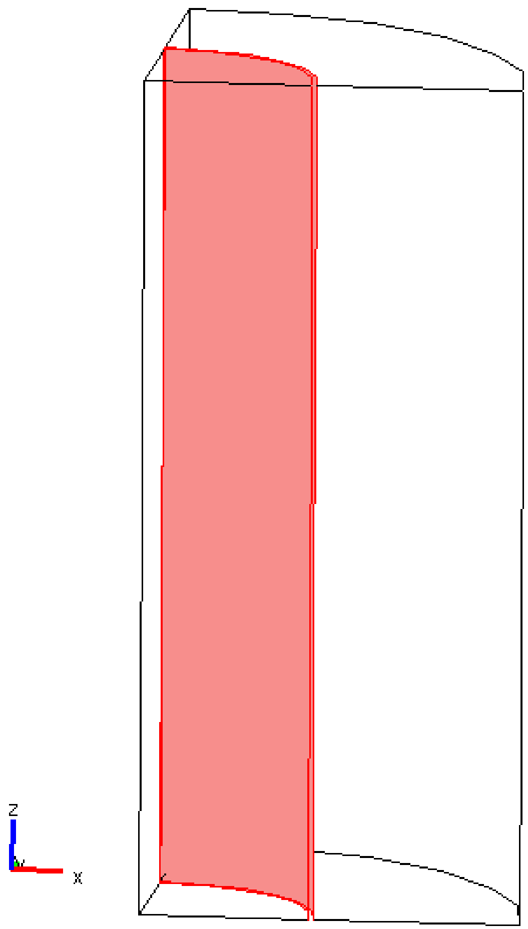
$$\begin{cases} 0 = \int_{\Omega} (\nabla P_{test} \cdot \mathbf{q} + P_{test} \rho W_0) d\Omega + \int_{\partial\Omega} P_{test} (G + \mu H) dn \\ 0 = (P_0 - P)|_{\partial\Omega_1} \quad \text{Dirichlet BC} \end{cases}$$

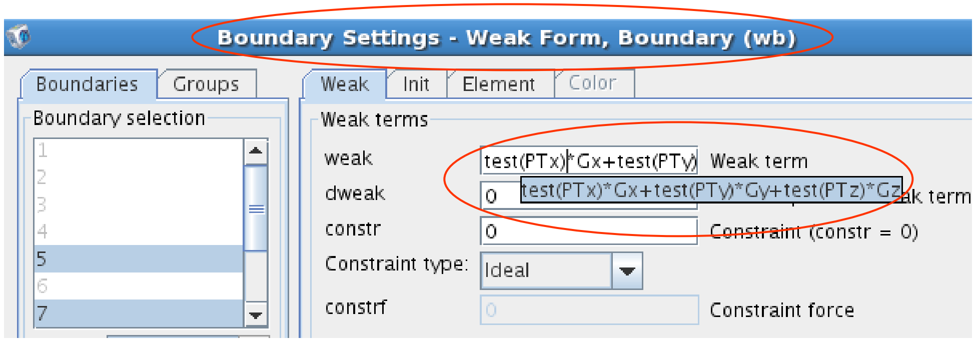
Reynolds Equation – COMSOL v3.5a

- Weak Form Boundary Mode :



Reynolds Equation - COMSOL





- Type in Reynolds equation in weak form for gap boundaries:

$$\frac{\partial P}{\partial x} = PTx$$

$$\frac{\partial P_{test}}{\partial x} = test(PTx)$$

G_x, G_y is mass flow rate (defined as global variables)

PTx is tangential derivative

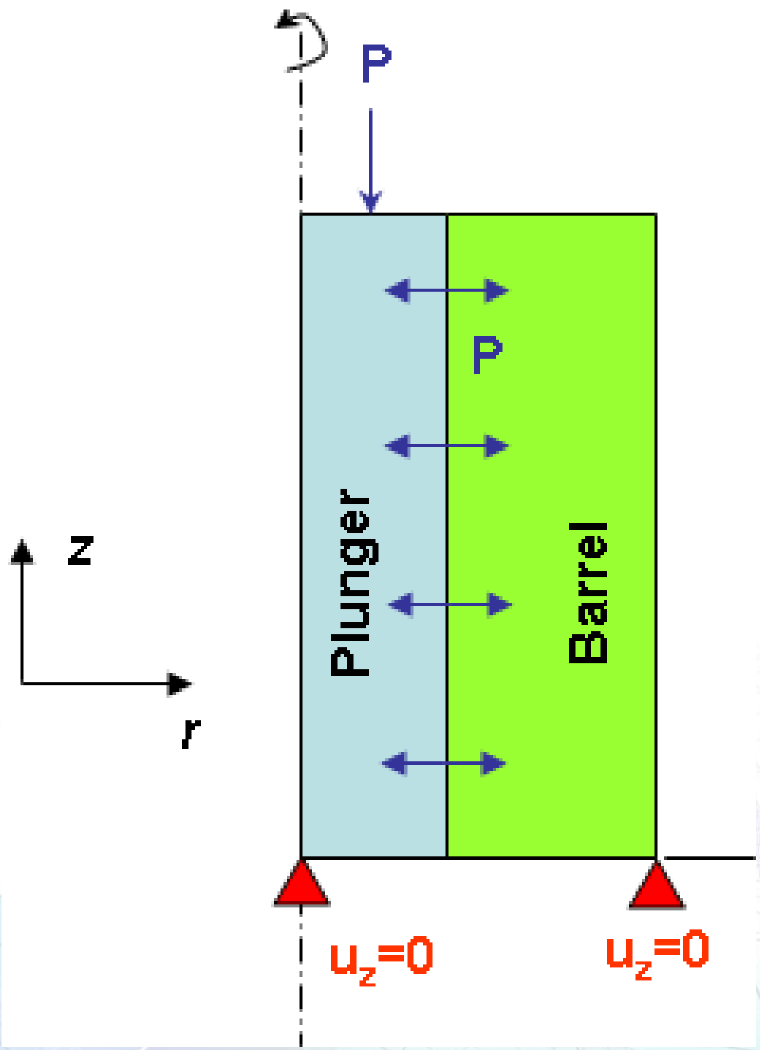
Reynolds Equation - COMSOL

•Define expressions G_x, G_y, G_z for mass flow rate per unit width :

Global Expressions

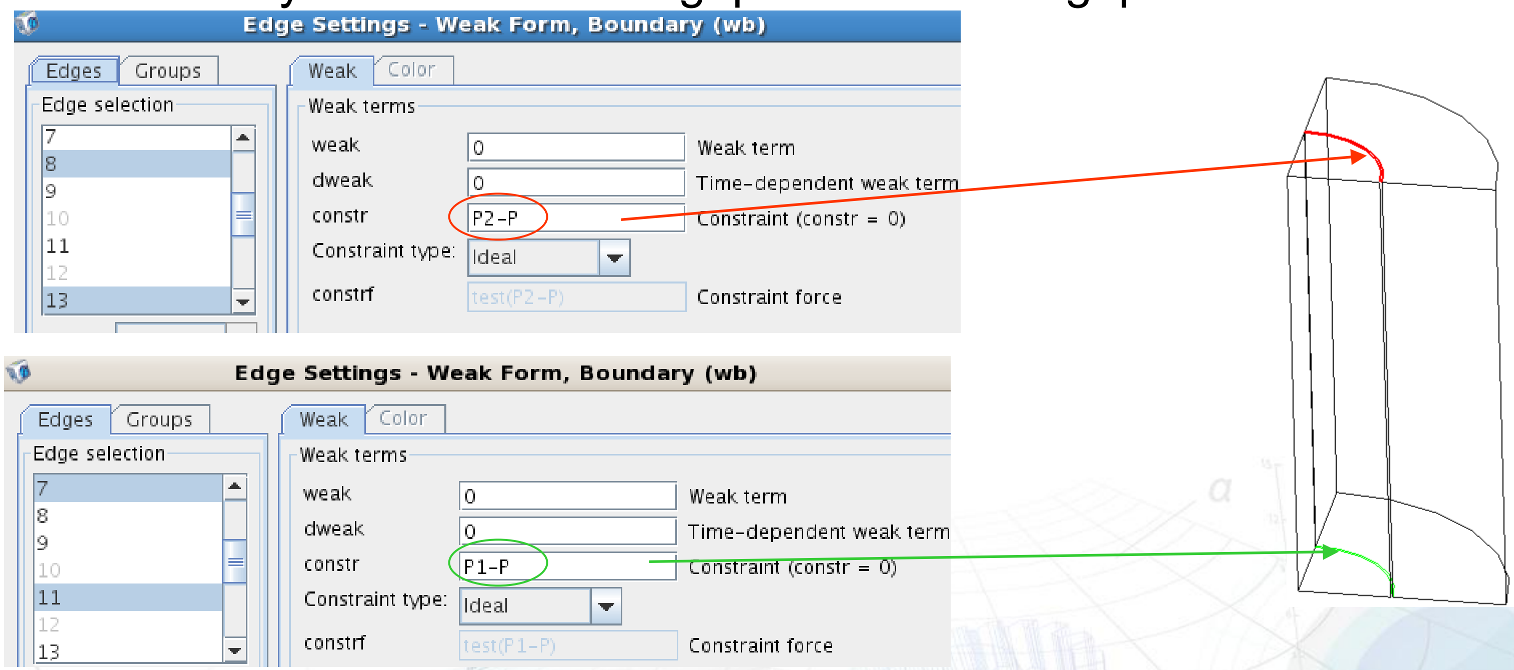
Name	Expression	Description
eta	visc(P,T)	viscosity P-T dependence
rho	rho_fluid(T,P/bar)	density P-T dependence
Gx	rho*g^3/(12*eta)*PTx-rho*g*U0/2	x-dir. mass flow rate per unit width, [kg/(m*s)]
Gy	rho*g^3/(12*eta)*PTy-rho*g*V0/2	y-dir. mass flow rate per unit width, [kg/(m*s)]
Gz	rho*g^3/(12*eta)*PTz-rho*g*W0/2	z-dir. mass flow rate per unit width, [kg/(m*s)]
mdot_x	-Gx*2*pi*R	mass flow rate in x-dir. [kg/s]
mdot_y	-Gy*2*pi*R	mass flow rate in y-dir. [kg/s]
mdot_z	-Gz*2*pi*R	mass flow rate in z-dir. [kg/s]

Axisymmetric Problem Setup



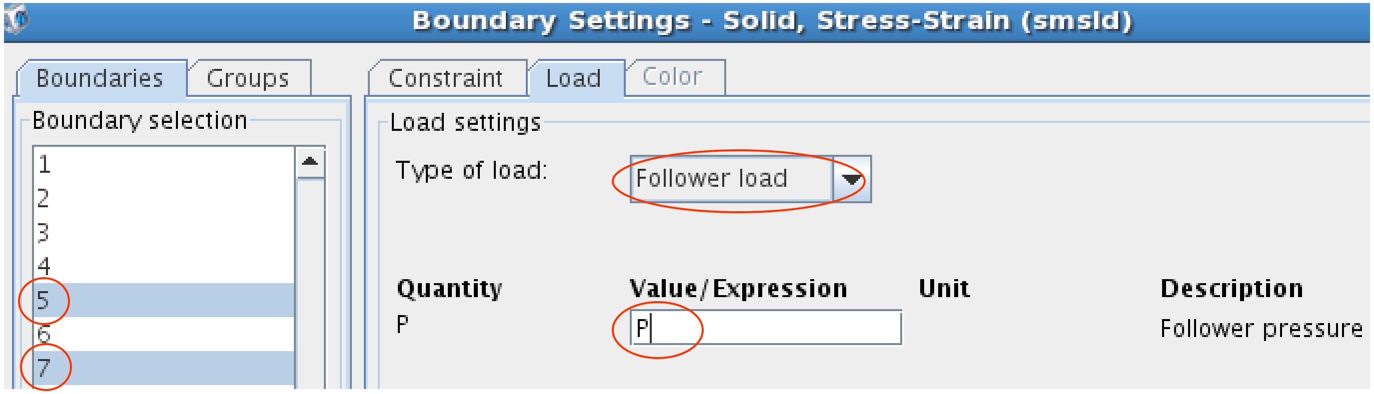
Reynolds Equation - COMSOL

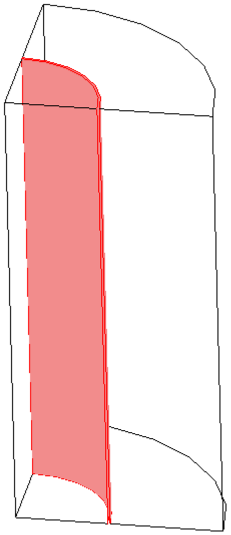
- Define boundary conditions at the gap entrance and gap exit :



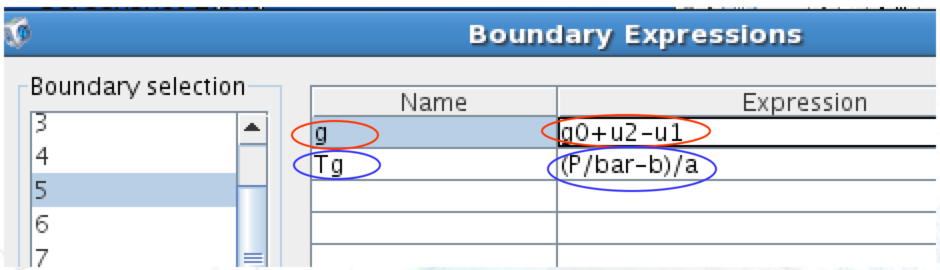
Reynolds Equation /Structural Analysis Coupling

•Apply Reynolds pressure to both surfaces of gap





•Define gap using initial gap u_0, displacement u_2 of gap OD, and displacement u_1 of gap ID



•Define temperature Tg along gap using relation P=P(T,enthalpy)

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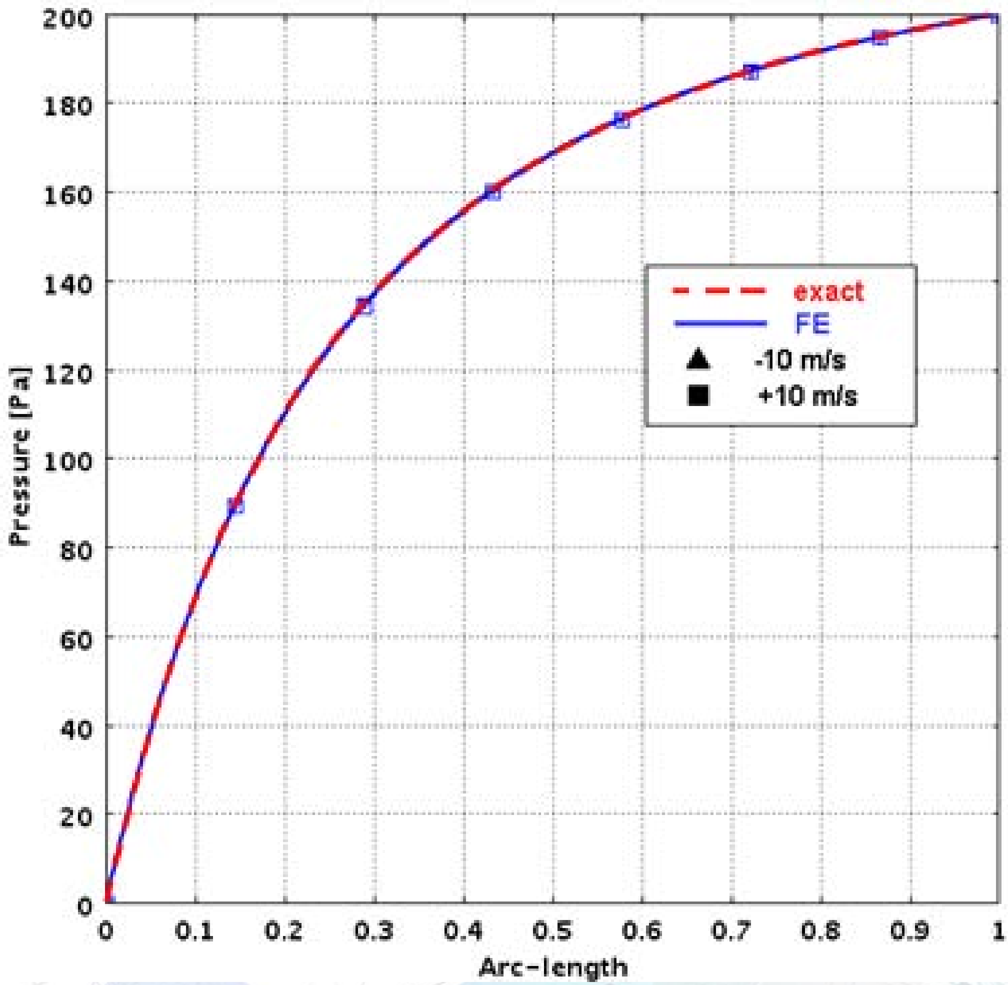
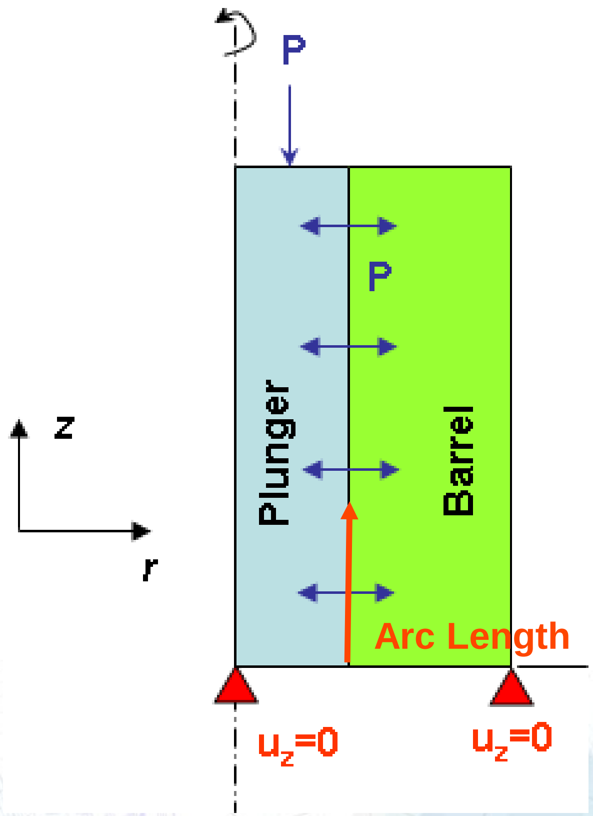
Variation of Pressure along Path





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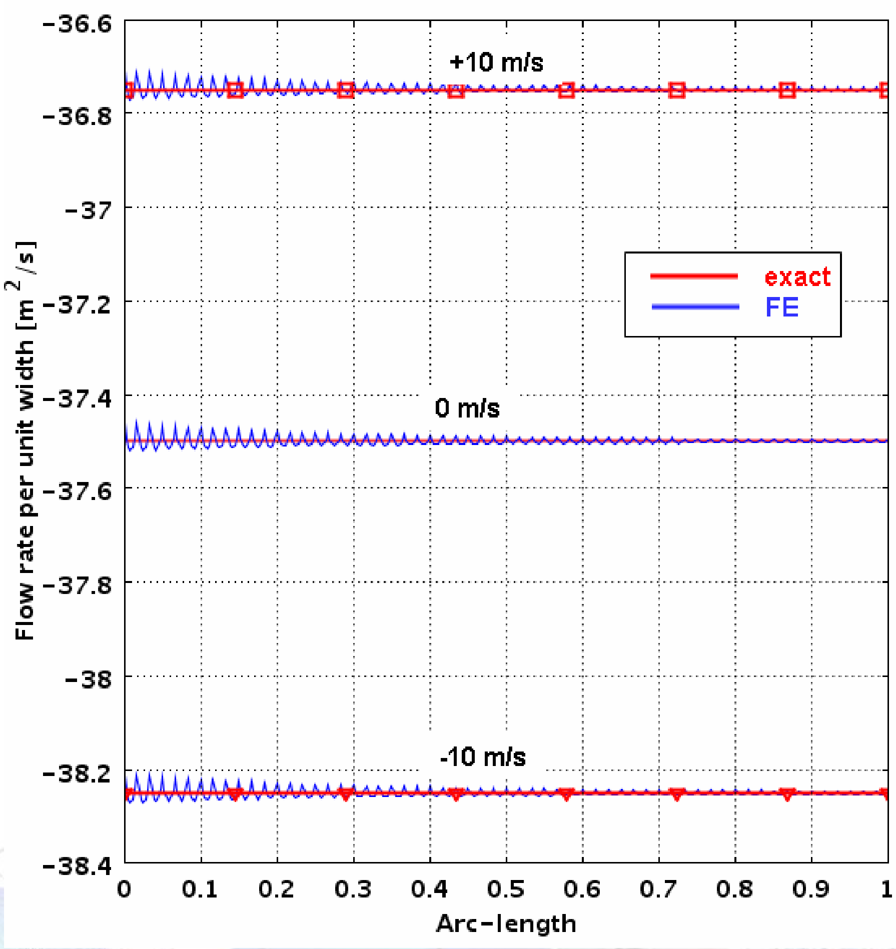
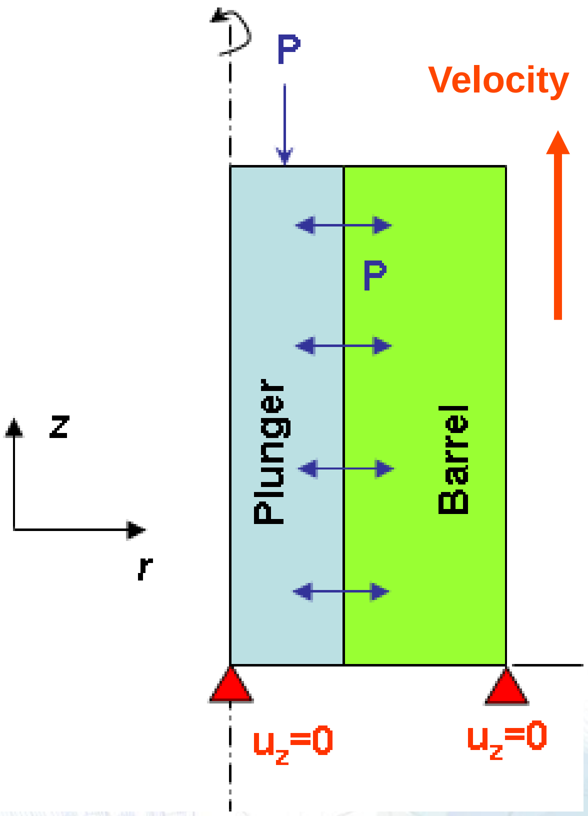
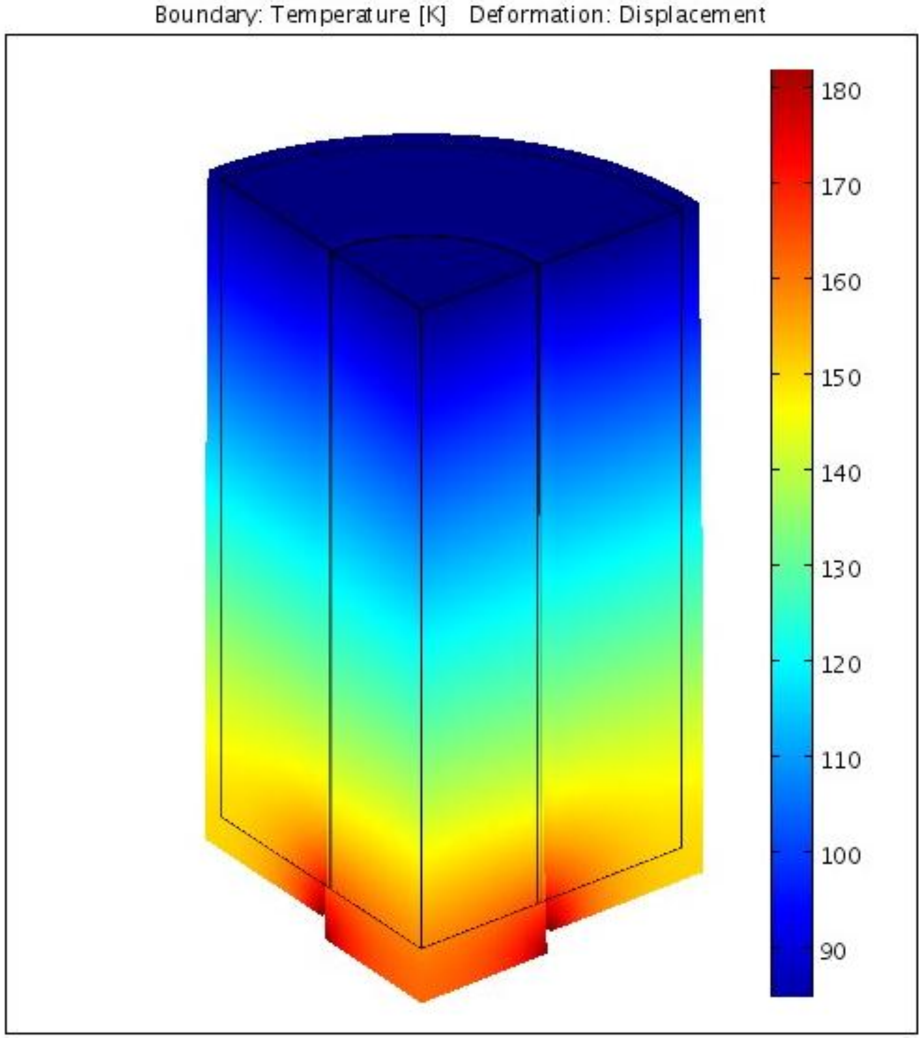
Effect of Velocity on Flow Rate





Temperature Distribution in Plunger/Barrel



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Summary

- Analysis method developed that fully couples lubricant fluid flow with heat transfer and stress analysis
- Solution implemented in v3.5a
- Reynolds equations available in CFD Module in v4.0a
- Weak form methodology available in COMSOL Multiphysics without need for CFD module