Visualization and Exploration of the Dynamics of Phase Slip Centers in Superconducting Wires

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INTRODUCTION: The dynamics of phase slip centers in a 1D model of superconducting wire was created based on the set of time-dependent Ginzburg-Landau equations[1,2] (TDGL). COMSOL Multiphysics® General Form PDE interface was used. The feature which distinguishes the current report here is that, for the first time, the set of TDGL equations for superconductors with finite gap[3] was used in full. The terms relevant to the presence of finite gap were included not only into the equation for the wave function of the condensate, but also into the equation for the current in the form of interference terms.

The performed thorough study of the solutions of these non-linear equations required extensive searches for multiple solutions at certain values of given parameters. We found it extremely helpful using the built-in capability of COMSOL which could be operated in tandem with MATLAB®. By developing the MATLAB code, we were able to automate findings of the relevant solutions. We obtained "branching" and "anti-branching" of these solutions at certain parameters of the problem. These properties are directly relevant to experimental results obtained with these objects.

COMPUTATIONAL METHODS: The set of TDGL equations modelled, before transformations that can be seen in the associated paper, were the order parameter, $\Delta = |\Delta|\, exp(\, i\theta)$:

$$-\frac{\pi}{8T_c}\frac{1}{\sqrt{1+(2\tau_\varepsilon|\Delta|)^2}}\left(\frac{\partial}{\partial t}+2i\phi+2\tau_\varepsilon^2\frac{\partial|\Delta|^2}{\partial t}\right)\Delta+\frac{\pi}{8T_c}[D(\nabla-2iA)^2]\Delta$$
$$+\left[\frac{T_c-T}{T_c}-\frac{7\zeta(3)|\Delta|^2}{8(\pi T_c)^2}+P(|\Delta|)\right]\Delta=0.$$

And the current density, j:

$$j=\frac{\pi\sigma_n}{4T}Q\left(|\Delta|^2-\frac{1}{\gamma}\frac{\partial|\Delta|^2}{\partial t}\right)+\sigma_n E\left\{+\frac{\sqrt{|\Delta|^2+\gamma^2}}{2T}\left[K\left(\frac{|\Delta|}{\sqrt{|\Delta|^2+\gamma^2}}\right)-E\left(\frac{|\Delta|}{\sqrt{|\Delta|^2+\gamma^2}}\right)\right]\right\}$$

Once the equation based modeling was fully implemented, with three general form PDE interfaces and Dirichlet Boundary conditions, the geometry was built as a simple 1 D wire of half-length L. The time-dependent solutions were simulated for given interval of time in seconds with time steps of 0.1.

The dynamics of time-dependent solutions were solved for by producing plots of the modulus of Ψ, $\sqrt{Re(\Psi)^2+Im(\Psi)^2} \equiv \sqrt{u^2+{u_2}^2}$, with respect to the x-coordinate. The parameters of j_0 and Δ were swept in search for the critical current, j_c, in which the first phase slip center occurs. This was accomplished through LiveLink™, which established a connection between COMSOL and MATLAB®, allowing MATLAB script via commands to control the COMSOL model. MATLAB script was created to automate sweeping through values of j_0 for select values of Δ and discovering minima of prominence ≥ 0.996 in the modulus of Ψ. In turn, when the automation code located j_c for specific values of Δ, these values were stored as well as the location of the PSC along the wire in datasets. Animations and plots of these values of energy gap versus the critical current and the location of PSC at the critical current versus delta were generated. The MATLAB® automation code and animation results are located at https://irisdorn.github.io/automatedcomsol/.

•RESULTS: Automation of searching for critical current of different deltas, Δ, generated single and double phase slippage as seen in Fig. 1 & 2. We obtained "branching" and "anti-branching" of Δ at their critical currents. The locations of the phase slippages versus Δ are plotted in Fig. 3.

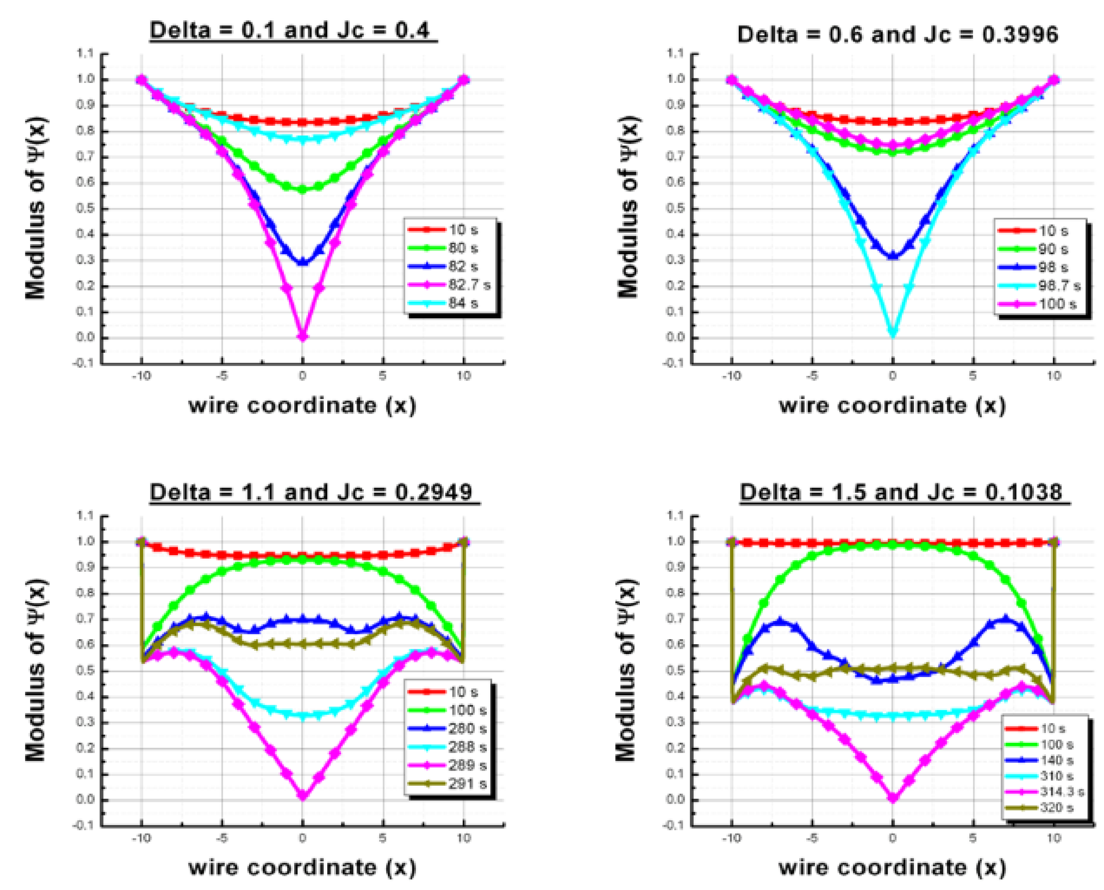

Figure 1 Set of single phase slippage along a 1D wire evolving over time for increasing Δ.

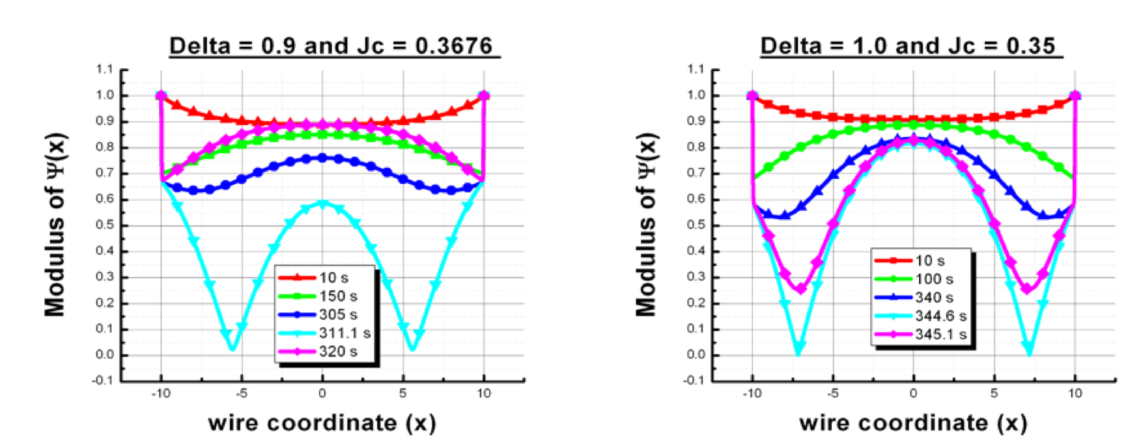

Figure 2 Set of double phase slippage evolving over time for increasing Δ.

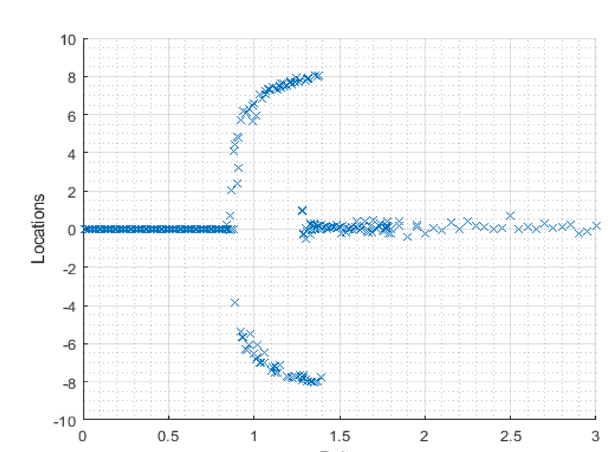

Figure 3 The locations of the phase slippages versus Δ

CONCLUSIONS: These results demonstrate only a minor part of our findings, thanks to the power of COMSOL paired with MATLAB®. These two softwares combined have provided our research the exceptional ability to go into great details in regards to visualize and exploration of microscopic phenomena fine enough to be close to experimental findings. Future work includes modeling the time evolution of the current densities (interference, normal and superconducting), sweeping through the phonon term, and modeling the current in thin 2D films and massive 3D samples.

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