# Magnetic manipulation of lateral migration behavior of a ferrofluid droplet in a plane Poiseuille flow

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**Abstract**

This study investigates the effect of a uniform magnetic field on the lateral migration behavior of a ferrofluid droplet in a plane Poiseuille flow at arbitrary directions by means of numerical simulation. A numerical scheme called level set method in combination with laminar two phase flow under fluid flow module is used to solve the flow field both inside and outside of the droplet, while level set method is required to track the dynamic motion of the droplet interface which is suspended in another immiscible medium. A parabolic flow is generated by means of a pressure gradient. Magnetic field both inside and outside of the droplet is simulated using the AC/DC module, and it is coupled to the flow domain using the volume force feature under laminar flow module. We found that at a low droplet Reynolds number ($Re_d \leq 0.05$), the magnetic field direction can effectively manipulate the final equilibrium position of the droplet along a channel. Keeping the viscosity ratio fixed (i.e., $\lambda = 1$), at $\alpha = 0^{\circ}$, the droplet finds its equilibrium position closer to the bottom wall, while at $\alpha = 45^{\circ}$ and $90^{\circ}$ the droplet settles closer towards the channel center. Also, at a steady state the droplet is found to align itself towards the direction of the magnetic field.

## 1. Introduction

Dispersion of droplets in another immiscible fluid is important in a number of industrial applications that deal with natural and synthetic products, including food products, drugs, and milk[1], [2]. Dispersion is also important in a variety of technological processes that involve liquid-liquid extraction[3], [4] where phase separation is crucial to the purification of the product, such as separation of water from crude oil and separation of glycerol from bio-diesel[5].

A single droplet in a pressure-driven flow serves as an excellent model problem to investigate the lateral migration behavior of droplets and can provide fundamental insights to more complex phenomenon that involves suspension of multiple droplets, e.g., transport of emulsions through porous media[6], [7]. In the existing literature, numerous theoretical[8]–[10], experimental[11], and numerical studies[12] have been carried out to investigate the migration behavior of droplets in shear flows.

In addition to using viscous shear force, phase separation can be enhanced by applying external force fields, such as magnetic fields which provide an additional way of manipulating the shape of a ferrofluid droplet[13], [14]. In order to use magnetic manipulation, either the droplet or the suspending medium needs to be a ferrofluid – a dispersion of magnetic nanoparticles (diameter typically around 10 nm and volume fraction about 5%). Multiphase ferrofluid droplets have notable biomedical applications, such as treatment of retinal detachment[15], due to their ability to be delivered to a specific site with the help of proper manipulation of a magnetic field. A thorough investigation on the deformation and orientation of a ferrofluid droplet under uniform magnetic fields has been carried out in our recent work[16].

Until now, only a few have studied the lateral migration behavior of a ferrofluid droplet in a Poiseuille flow under the influence of a uniform magnetic field. Recently, Zhang et al.[17] experimentally investigated the effects of magnetic field strength, direction, and interfacial tension on the lateral migration mechanism of a ferrofluid droplet. One significant advantage of using the magnetic fields in compared to electric fields is that magnetic field can be applied at arbitrary directions with ease, while the direction of electric fields is often limited by the placement of electrodes.

By using two-dimensional numerical simulations, this paper investigates the lateral migration behavior of a ferrofluid droplet in a plane Poiseuille flow under a uniform magnetic field at arbitrary directions. For computational efficiency, we have chosen to use 2D simulations in order to study a wide range of parameter space i.e. capillary number, magnetic bond number, and magnetic field direction. Our numerical simulation, built with commercial FEM solver, models the dynamic deformation of droplet interface by using



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the level-set method and coupling the magnetic and flow fields.

The remainder of the paper is organized as follows: in section 2, the mathematical model and numerical method with COMSOL settings are described. In section 3, we present the lateral migration behavior of the droplet in a plane Poiseuille flow without the presence of any external forces and validate our results against the existing theories in the literature. We then examine the effect of magnetic field directions on the trajectory of lateral migration of the droplet in a plane Poiseuille flow. Finally, we conclude our major findings in section 4.

## 2. Numerical Simulation Method

### 2.1 Level set method

In our model, we have used the conservative level set method to track the dynamic evolution of the interface between the droplet and suspending medium. The level set function ϕ, an auxiliary scalar function, represents different fluid phases which has a value of zero in one domain and 1 in another domain. The value of ϕ varies smoothly between 0 and 1 across the interface, and ϕ = 0.5 defines the position of the interface. The level set function ϕ is advected by the velocity field[18]:

$$\frac{\mathrm{d}\phi}{\mathrm{dt}} + \nabla\cdot(\mathbf{u}\phi) = \gamma\,\nabla\cdot\left(\varepsilon\nabla\phi - \phi(1-\phi)\frac{\nabla\phi}{|\nabla\phi|}\right) \quad (1)$$

where u, γ, and ε determine the velocity field, amount of re-initialization, and thickness of the interface, respectively. The terms on the left hand side of the equation represent the motion of the interface, while the terms on the right hand side are required for numerical stability. The level set function ϕ can also be used to find the unit normal to the interface n which is given by:

$$\boldsymbol{n} = \frac{\nabla\phi}{|\nabla\phi|} \quad (2)$$

With the level set method, the two immiscible fluids are treated as a single phase flow but the material properties vary according to the level set value. Here, a linear average is used to calculate the density (ρ), dynamic viscosity (η), magnetic permeability (μ), and magnetic susceptibility (χ) which are related to ϕ through the following equations:

$$\rho = \rho_c + (\rho_d - \rho_c)\phi, \qquad \eta = \eta_c + (\eta_d - \eta_c)\phi$$
$$\mu = \mu_c + (\mu_d - \mu_c)\phi, \qquad \chi = \chi_c + (\chi_d - \chi_c)\phi$$

where subscripts c and d represent the continuous and droplet phase respectively.

### 2.2 Governing Equations

The motion of an incompressible, immiscible ferrofluid droplet in another incompressible, immiscible medium under the effect of a uniform magnetic field is governed by the following continuity and momentum equations:

$$\nabla\cdot\mathbf{u} = 0 \quad (3)$$

$$\rho\frac{\mathrm{D}\mathbf{u}}{\mathrm{Dt}} = -\nabla p + \nabla\cdot\tau + \mathrm{F}_\sigma + \mathrm{F}_m \quad (4)$$

where, $\frac{D\mathbf{u}}{Dt}$ represents the total derivative of the velocity field **u**. The right hand side of the equation (4) represents the force terms due to pressure, viscosity, surface tension ($\mathrm{F}_\sigma$), and magnetic field ($\mathrm{F}_m$), respectively. The viscous stress tensor $\tau$ can be expressed as: $\tau = [\eta(\nabla\mathbf{u} + (\nabla\mathbf{u})^T)]$. The surface tension force $\mathrm{F}_\sigma$ can be defined by:

$$\mathbf{F}_\sigma = \nabla\cdot[\sigma\{\mathbf{I} + (-\mathbf{n}\mathbf{n}^T)\}\delta] \quad (5)$$

where, σ is the surface tension coefficient, I is identity matrix, δ is the Dirac delta function, and n is the unit normal to the interface which can be calculated using equation (2). The Dirac delta function $\delta$ can also be approximated using the level set function as:

$$\delta = 6|\phi(1-\phi)||\nabla\phi| \quad (6)$$

Assuming linear and homogeneous material properties, the different magnetic properties i.e., magnetic induction **B**, magnetization **M,** and magnetic field **H** can be related using Maxwell magneto-static relationship through the following equations:

$$\nabla\cdot\mathbf{B} = 0, \nabla\times\mathbf{H} = 0, \mathbf{M} = \chi\mathbf{H} \quad (7)$$

$$\mathbf{B} = \mu_0(\mathbf{H} + \mathbf{M}) = \mu_0(1+\chi)\mathbf{H} \quad (8)$$

where $\mu_0$ is the permeability of vacuum which is equal to $4\pi \times 10^{-7}\ N/A^2$. A scalar potential φ can be defined to satisfy the curl-free **H** i.e., $H = -\nabla\varphi$ which can be written as:

$$\nabla\cdot(\mu\nabla\varphi) = 0 \quad (9)$$

Additionally, the total magnetic force can be calculated using the magnetic stress tensors as:

$$\mathbf{F}_\mathrm{m} = \nabla\cdot\boldsymbol{\tau}_\mathrm{m} = \nabla\cdot\left(\mu\mathbf{H}\mathbf{H}^\mathrm{T} - \frac{\mu}{2}H^2\mathbf{I}\right) \quad (10)$$

where, $\boldsymbol{\tau}_\mathrm{m}$ is the magnetic stress tensor for the applied magnetic field, $H$ = |**H**| is the magnitude of the magnetic field, and **I** is the second order identity tensor. The magnetic insulation on both the left and right walls are satisfied through the following equation:

$$\mathbf{n}\cdot\mathbf{B} = 0 \quad (11)$$

We introduced some dimensionless groups to reduce the number of variables and observe which dimensionless groups affect the droplet dynamics most. The dimensionless groups are defined as:

$$Re_d = \frac{\rho_c R_0^2\dot{\gamma}_a}{\eta_c} \quad (12)$$

$$\mathrm{Ca} = \frac{\eta_c R_0\dot{\gamma}_a}{\sigma} \quad (13)$$

$$\mathrm{Bo_m} = \frac{R_0\mu_0 H_0^2}{2\sigma} \quad (14)$$

where, $Re_d$, Ca, and $\mathrm{Bo_m}$ represent droplet Reynolds number, average Capillary number, and Magnetic bond number, respectively. The parameter $\dot{\gamma}_a$ can be defined as $\frac{2u_a}{H_d}$ where $u_a$ is the average flow velocity in the domain.

### 2.3 Schematic of numerical model

Fig. 1 demonstrates the schematic illustration of a ferrofluid droplet suspended in another fluid medium in a Poiseuille flow under the application of a uniform magnetic field, $\mathbf{H}_0$. In this case, the magnetic susceptibility of the ferrofluid droplet is considered as 0.25 (i.e. $\chi_d$ = 0.25) while it is considered zero (i.e. $\chi_c = 0$) for the suspending non-magnetic fluid. The subscripts c and d represent the droplet and continuous phases, respectively. The viscosity and density of both

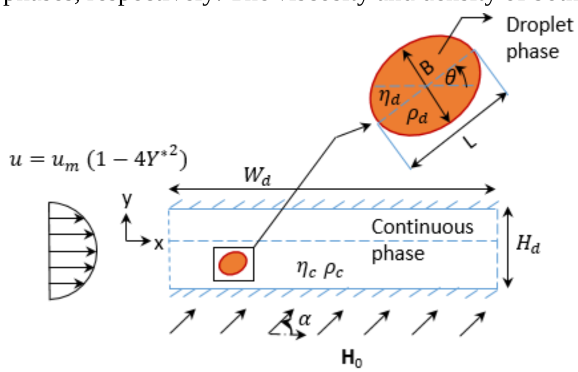


**Figure 1**. Schematic illustration of a ferrofluid droplet suspended in another medium in a Poiseuille flow under the application of a uniform magnetic field, $\mathbf{H}_0$.

the phases are considered equal to each other (i.e., $\eta_c = \eta_d$ and $\rho_c = \rho_d$). Initially, the droplet with a radius of 75 µm was placed 80 µm below the center of the domain and far away from the inlet to ignore the entrance effect. The velocity profile at the inlet can be defined by $u = u_m(1 - 4Y^{*^2})$ where $u_m$ is the maximum velocity in the domain. $Y^*$ is a non-dimensional variable which denotes the relative position of the droplet along y-axis of the channel, and this is scaled by the height of the domain $H_d$ (i.e., $Y^* = \frac{y}{H_d}$). The average velocity at the inlet is taken as 50 mm/s. A no-slip boundary condition is applied to both the top and bottom walls. A uniform magnetic field, $\mathbf{H}_0$ was applied at arbitrary directions which is denoted by angle, α. The deformation of the droplet is found out using dimensions L and B which are the lengths along the major and minor axes of the droplet, respectively. Also, the orientation angle θ is defined as the angle between the positive x-axis and major axis of the droplet when the droplet undergoes deformation under the effect of both flow and magnetic fields.

### 2.4 COMSOL Settings

Two phase laminar flow level set method in combination of transient with phase initialization feature is used to solve the flow domain and to track the deformable interface of the droplet. A parabolic velocity profile at the inlet is generated using the inlet feature with an average velocity of 50 mm/s. No-slip boundary condition is applied to both the top and bottom walls using the wall feature. The value of level set function ϕ is assigned as 1 and 0 for the droplet phase and continuous phases, respectively. The interface of the droplet is defined using initial interface condition. The re-initialization parameter $\gamma$ is equal to the maximum magnitude of the velocity in the flow domain and interface thickness $\varepsilon$ is of the order as the same size of the mesh elements. Additionally, a magnetic field is applied to the flow domain and solved simultaneously using Magnetic fields, no currents interface from AC/DC module. Keeping the magnetic field strength fixed (i.e., $\mathbf{H}_0$ = 50000 A/m), the magnetic field is applied along different arbitrary directions using the parametric sweep feature to investigate the trajectory of lateral migration of the ferrofluid droplet. For creating the mesh, we used free triangular elements in the computational domain. PARDISO solver with nested dissection multithread algorithm is used to solve our computational model.

## 3. Results and Discussions

### 3.1 Droplet migration in a Poiseuille flow

Before moving on to our intended study, we validated our model by comparing our results against the existing theories in the literature. The most thorough theoretical analysis on droplet migration behavior in a two-dimensional Poiseuille flow is given by Chan and Leal[19] who considered the effect of the deformed shape of the droplet as a critical factor on the droplet trajectory motion in a uni-directional shear flow. Two different hydrodynamic interactions are mainly responsible for droplet migration. First is the interaction between the deformed drop and the bottom wall which gradually decreases as the distance from the bottom wall increases, and this interaction causes the droplet to migrate away from the bottom wall. Second is the interaction between the deformed drop and the flow field, which vanishes in a simple shear flow but plays an important role in the quadratic flow field[20]. Chan and Leal[19] also mentioned that the migration behavior due to the presence of the second type of interaction is essentially dependent on the viscosity ratios. When $\lambda$ < 0.5 and $\lambda$ > 10, the droplet migrates towards the centerline to the channel since in this case, both types of interactions take place in the same direction. Conversely, for intermediate values of viscosity ratios (i.e., 0.5< $\lambda$ < 10), the interactions act in opposite directions resulting in an equilibrium position at a position somewhere between the bottom wall and center of the channel due to the combined effect of these forces. The equilibrium position where the two interaction forces become equal to each other also depends on the relative size of the droplet[20].

Figure 2 represents the lateral migration of a droplet at a viscosity ratio equal to 1 (i.e., $\lambda$ = 1). It can be seen that the droplet settles down approximately 19 μm below the center of the channel which also takes considerable amount of time to reach the equilibrium position. This numerical result qualitatively agrees with the theory given by Chan and Leal.

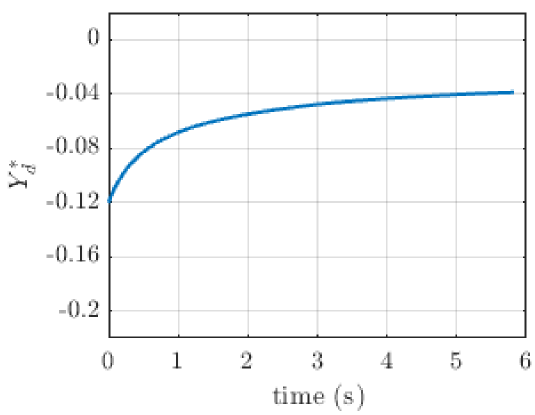


**Figure 2**. Lateral migration of a ferrofluid droplet at $\lambda = 1$ and $Bo_m = 0$.

### 3.2 Effect of magnetic field direction on the lateral migration behavior of droplet

When a magnetic field is applied to a ferrofluid droplet suspended in Poiseuille flow, the droplet deforms even more due to the additional effect of magnetic field strength on the droplet interface[16]. Here, we investigate the effect of magnetic field strength on the lateral migration behavior of the droplet at arbitrary directions. In this study, the droplet Reynolds number is considered as $\boldsymbol{Re_d} = 0.03$.

Figure 3 shows the effect of different magnetic field directions on the migration behavior of the droplet. In this case, some representative $\boldsymbol{\alpha}$ are chosen for better illustration of the results. From Fig. 3 we can see that applying a uniform magnetic field from different arbitrary directions results in different equilibrium positions along the channel relatively in a shorter period of time. Also, the droplet follows different trajectories before reaching the final equilibrium position.

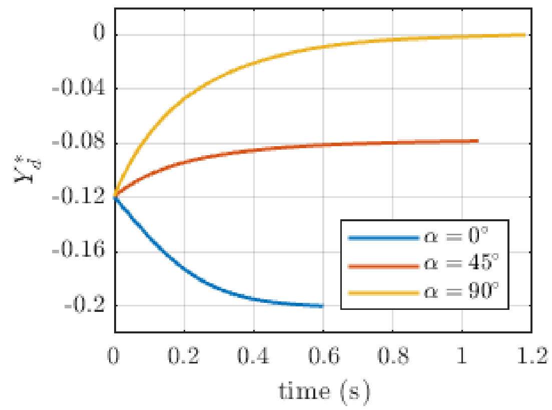


**Figure 3**. Effect of different magnetic field directions on the migration behavior of the droplet at $Re_d = 0.03, \lambda = 1,$ and $Bo_m = 8.72$.

Figure 4 shows the steady state velocity, magnetic field profiles, and equilibrium droplet shapes at $Re_d = 0.03, \lambda = 1,$ and $Bo_m = 8.72$. It can be seen that the droplet undergoes deformation and orients itself along the direction of the magnetic field. At $\alpha = 90^{\circ}$, the droplet shape is found to be symmetric with respect to the x-axis which in turn aids the droplet to settle at the center of the channel. On the other hand, at $\alpha = 0^{\circ}$ and $45^{\circ}$, due to asymmetry in the shape of the droplet, the droplet experiences different hydrodynamic interactions along the interface of the droplet, which force the droplet to settle at a position somewhere between the center and bottom wall of the channel. The flow field also becomes more distorted as the droplet tends to further align itself in the vertical direction to conform to the droplet shape. From the magnetic field profiles, it can be seen that the droplet experiences maximum magnetic field strength along the direction the magnetic field is applied, while the strength is least in magnitude in the other orthogonal direction. The magnetic field is also uniform both inside and far outside the droplet. Additionally, the magnetic field lines are parallel to each other; however, they are slightly deflected at the interface of the droplet due to the change in the magnetic susceptibility at the interface. Therefore, it can be concluded that the different droplet shapes and their alignment with the flow field along with hydrodynamic interactions play a crucial role in the trajectory of the lateral migration and the final equilibrium position.

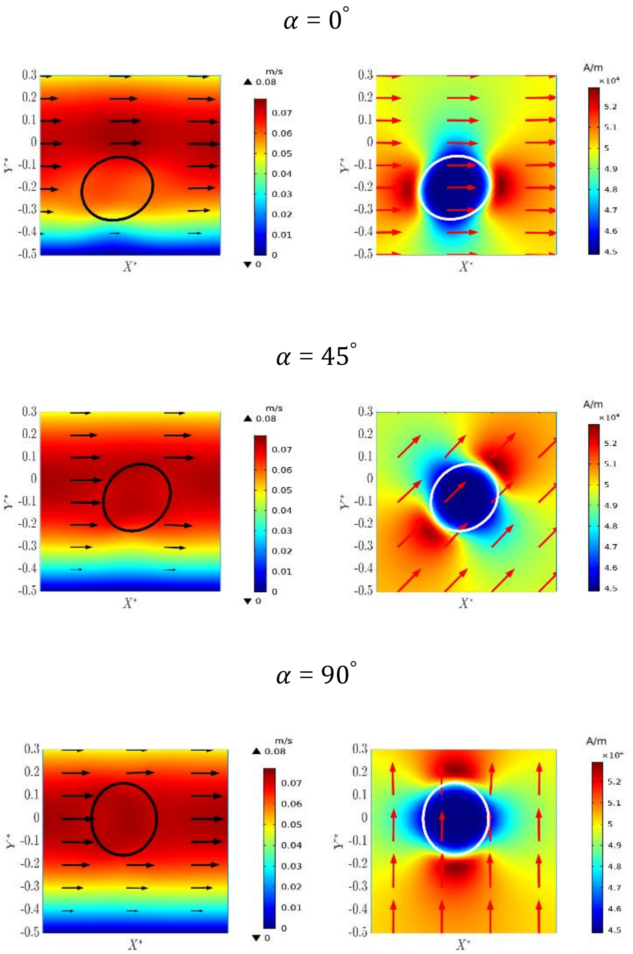


**Figure 4**. Steady state velocity, magnetic field profiles, and equilibrium droplet shapes at $Re_d = 0.03$, $\lambda = 1$, and $Bo_m = 8.72$.

## 4. Conclusions

The influence of a uniform magnetic field on the lateral migration behavior of a ferrofluid droplet in a plane Poiseuille flow is systematically studied in this paper. In the absence of any external forces, at $\boldsymbol{\lambda = 1}$, the droplet finds its equilibrium position at a location approximately 19 μm below the center of the channel. Applying a magnetic field along arbitrary directions results in different equilibrium positions along the channel due to disparate alignments of the droplet with the flow field. At $\alpha = 0^\circ$, the droplet is found to be closer to the bottom wall, while at $\alpha = 45^\circ$ the droplet settles closer to the center, and at $\alpha = 90^\circ$, the droplet finds its equilibrium position exactly at the center of the channel.

## References

[1] T. F. Tadros, "Industrial applications of dispersions," *Adv. Colloid Interface Sci.*, vol. 46, pp. 1–47, 1993.

[2] O. Skurtys and J. M. Aguilera, "Applications of microfluidic devices in food engineering," *Food Biophys.*, vol. 3, no. 1, pp. 1–15, 2008.

[3] P. J. Hailing and P. Walstra, "Protein-stabilized foams and emulsions," *Crit. Rev. Food Sci. Nutr.*, vol. 15, no. 2, pp. 155–203, 1981.

[4] L. L. Schramm and others, "Fundamentals and applications in the petroleum Industry," *Adv. Chem*, vol. 231, pp. 3–24, 1992.

[5] A. Abeynaike, A. J. Sederman, Y. Khan, M. L. Johns, J. F. Davidson, and M. R. Mackley, "The experimental measurement and modelling of sedimentation and creaming for glycerol/biodiesel droplet dispersions," *Chem. Eng. Sci.*, vol. 79, pp. 125–137, 2012.

[6] K. J. Lissant, "Emulsions and emulsion technology," *Soil Sci.*, vol. 120, no. 2, p. 160, 1975.

[7] R. Skalak, N. Ozkaya, and T. C. Skalak, "Biofluid mechanics," *Annu. Rev. Fluid Mech.*, vol. 21, no. 1, pp. 167–200, 1989.

[8] A. Karnis and S. G. Mason, "Particle motions in sheared suspensions: XXIII. Wall migration of fluid drops," *J. Colloid Interface Sci.*, vol. 24, no. 2, pp. 164–169, 1967.

[9] C. E. Chaffey, H. Brenner, and S. G. Mason, "Particle motions in sheared suspensions," *Rheol. Acta*, vol. 4, no. 1, pp. 64–72, 1965.

[10] Y. Wang and P. Dimitrakopoulos, "Low-Reynolds-number droplet motion in a square microfluidic channel," *Theor. Comput. Fluid Dyn.*, vol. 26, no. 1–4, pp. 361–379, 2012.

[11] S. D. Hudson, "Wall migration and shear-induced diffusion of fluid droplets in emulsions," *Phys. fluids*, vol. 15, no. 5, pp. 1106–1113, 2003.

[12] C. Coulliette and C. Pozrikidis, "Motion of an array of drops through a cylindrical tube," *J. Fluid Mech.*, vol. 358, pp. 1–28, 1998.

[13] J.-C. Bacri, D. Salin, and R. Massart, "Study of the deformation of ferrofluid droplets in a magnetic field," *J. Phys. Lettres*, vol. 43, no. 6, pp. 179–184, 1982.

[14] S. Afkhami *et al.*, "Deformation of a hydrophobic ferrofluid droplet suspended in a viscous medium under uniform magnetic fields," *J. Fluid Mech.*, vol. 663, pp. 358–384, 2010.

[15] O. T. Mefford *et al.*, "Field-induced motion of ferrofluids through immiscible viscous media:

Testbed for restorative treatment of retinal detachment," *J. Magn. Magn. Mater.*, vol. 311, no. 1, pp. 347–353, 2007.

[16] M. R. Hassan, J. Zhang, and C. Wang, "Deformation of a ferrofluid droplet in simple shear flows under uniform magnetic fields," *Phys. Fluids*, vol. 30, no. 9, p. 92002, 2018.

[17] J. Zhang, M. R. Hassan, B. Rallabandi, and C. Wang, "Migration of ferrofluid droplets in shear flow under a uniform magnetic field.," *Soft Matter*, 2019.

[18] COMSOL, "CFD Module Application Library Manual." .

[19] P.-H. Chan and L. G. Leal, "The motion of a deformable drop in a second-order fluid," *J. Fluid Mech.*, vol. 92, no. 1, pp. 131–170, 1979.

[20] W. Hiller and T. A. Kowalewski, "An experimental study of the lateral migration of a droplet in a creeping flow," *Exp. Fluids*, vol. 5, no. 1, pp. 43–48, 1986.