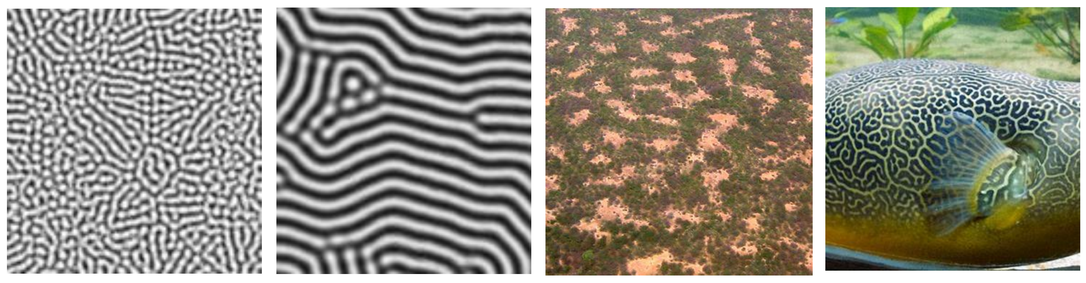
# Growth Mode Selection of Radially Growing Turing Patterns

Noah H. Somberg, Christopher Konow, Irving R. Epstein, Milos Dolnik
Department of Chemistry, Brandeis University, Waltham, MA, USA

## INTRODUCTION:

- **Turing patterns** are a model for **pattern formation in nature.**
- They appear as **stationary spots, stripes, and labyrinths** in activator-inhibitor systems.



Experimental results (CDIMA reaction) | Simulations (Lengyel-Epstein model) | Desert Vegetation | Giant Puffer Fish skin

**Figure 1**. A variety of Turing patterns

- **Pattern formation** and growth is **simulated** with the **Lengyel-Epstein** two variable model.

## METHODS:

- **Lengyel-Epstein model** (Eqns. 1 and 2) is implemented in **COMSOL Multiphysics®** using the coefficient form PDE interface.

$$\frac{\partial u}{\partial \tau} = a - u - \frac{4uv}{1+u^2} - W + \nabla^2 u \quad (1)$$

$$\frac{\partial v}{\partial \tau} = \sigma\left[b\left(u - \frac{uv}{1+u^2} + W\right) + d\nabla^2 u\right] \quad (2)$$

- The **Lengyel-Epstein model** has a **photosensitive** term to suppress pattern formation, which can be used to study Turing pattern **growth.**
- Several **methods of growth** are explored:

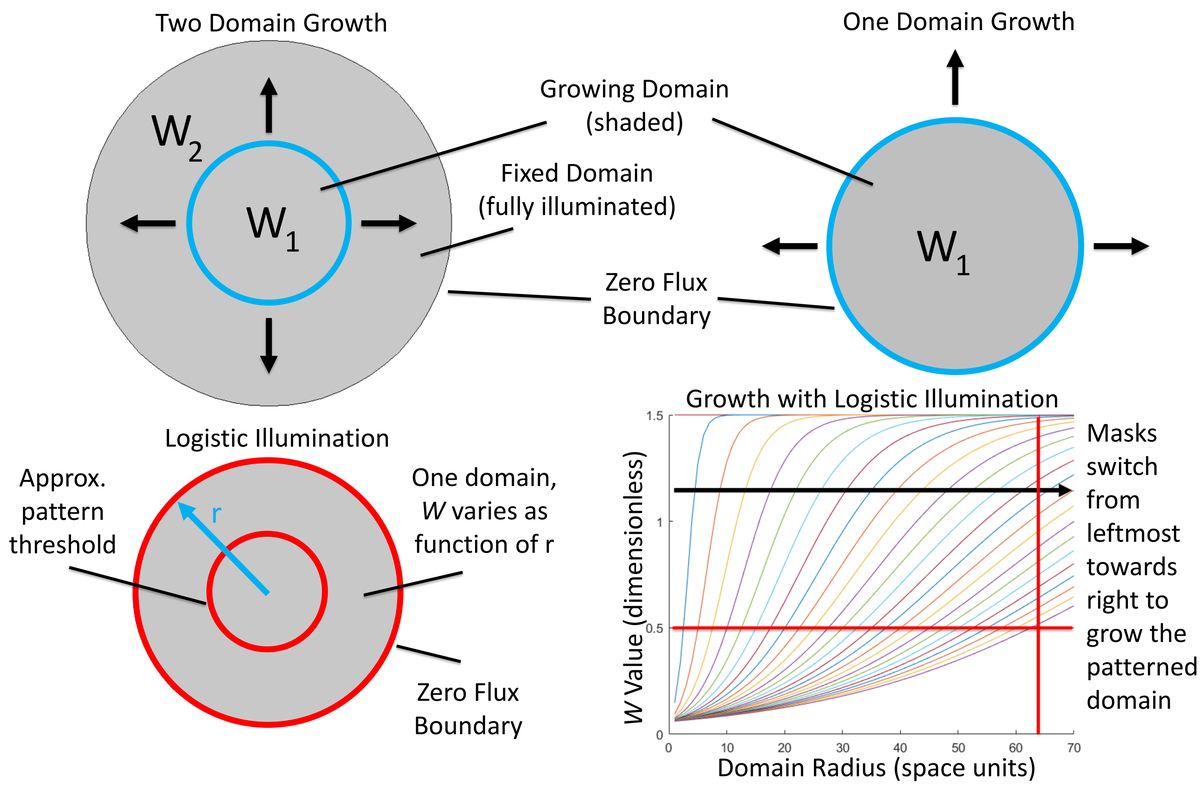



**Figure 2**. Different growth methods for Turing patterns

- **COMSOL® Application Builder** is used to automate simulation workflow, making parameter space accessible.

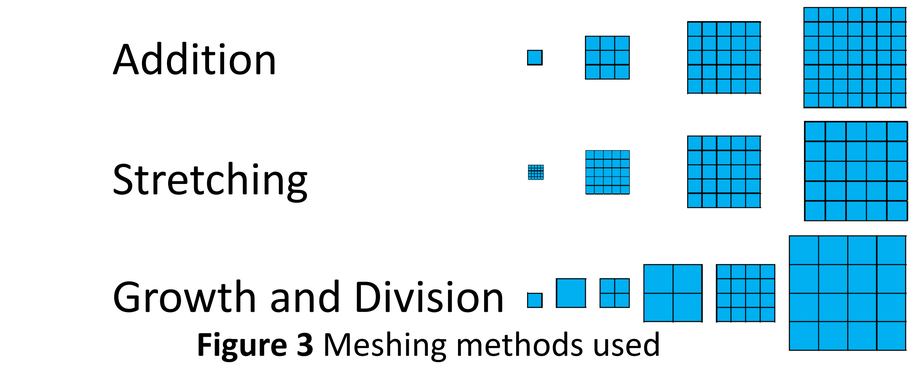



**Figure 3** Meshing methods used

## RESULTS:

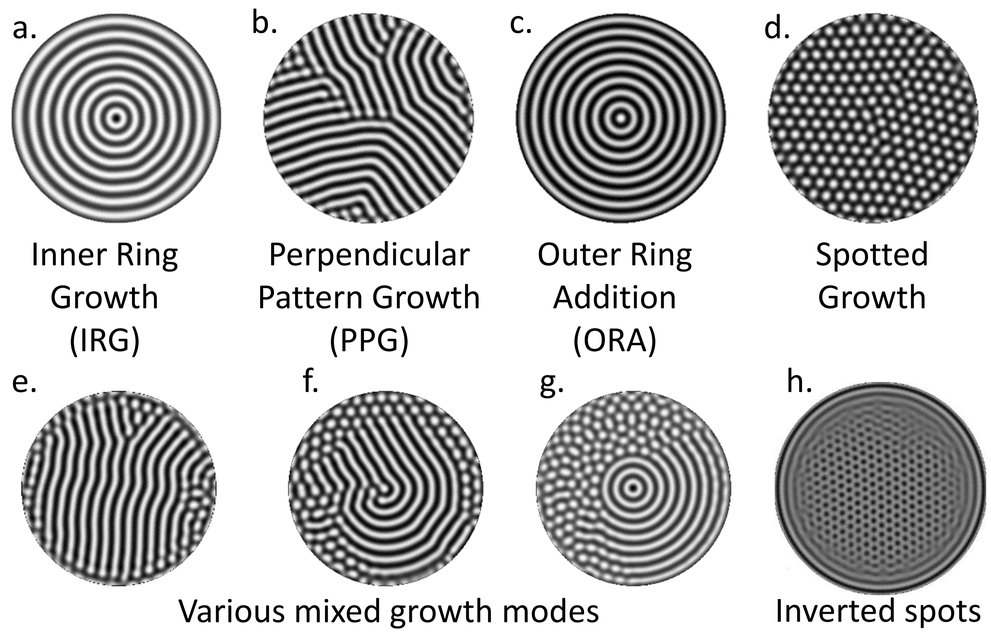



**Figure 4** Observed pattern morphologies

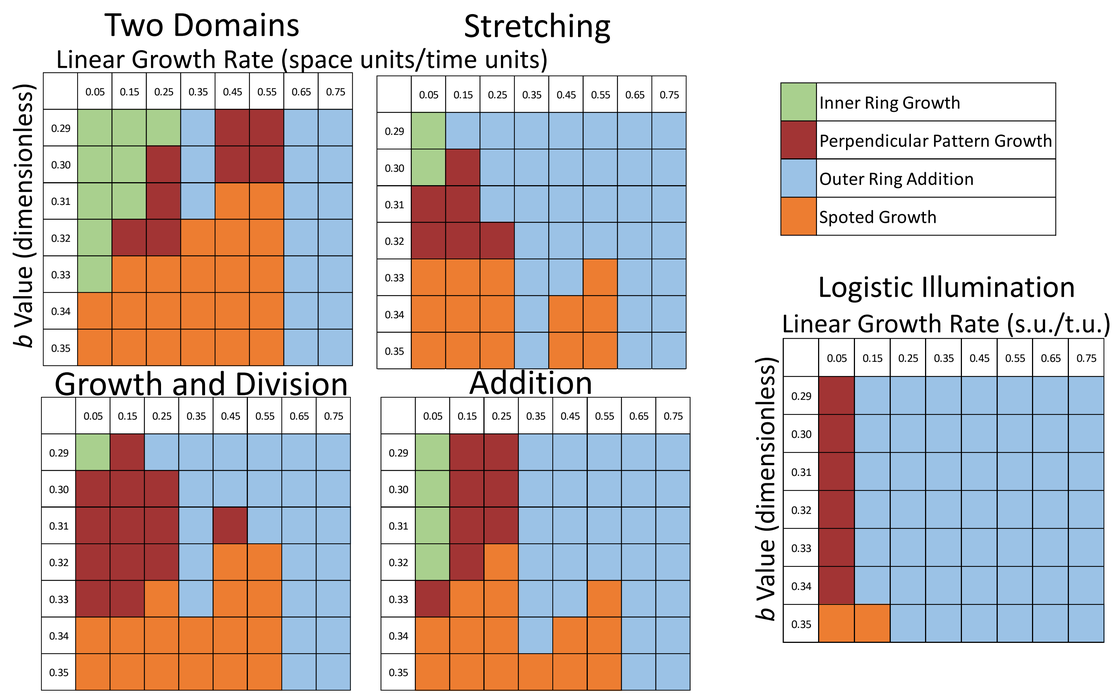



**Figure 5** Pattern morphologies trends

- Observed **morphologies** are relatively **consistent** across different growth methods.
- Logistic illumination **shifts** patterns in parametric space.
- Tendency towards ORA mode at growth rate = 3.5 s.u./t.u. may be caused by growth rate/pattern wavelength resonance.

## CONCLUSIONS:

- We observe **robust pattern selection** by growth rate across **varying growth methods**.
- Future work may include experimental investigations into logistic illumination and pattern wavelength/growth rate resonance.

References available on website, see QR code below.

Brandeis
M.R. Bauer Foundation
NSF

Scan for animations and more

Excerpt from the Proceedings of the 2019 COMSOL Conference in Boston