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Validation of the analytical approximation for induction heating of ionic solutions using COMSOL

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Introduction

New methods of generating high-frequency magnetic fields of tens or even hundreds of millitesla at frequencies exceeding 2 MHz are allowing important new applications for induction heating. These include heating low conductivity ionic solutions in situations where electric or electromagnetic fields (e.g. microwaves) cannot be used.

Induction heating can be described by a numerical approximation. However, experimental validation is difficult at these high frequencies where the numerical approximation has remained untested. In particular, a coefficient describing the geometry of the sample being heated was questionable. Furthermore, limits for the conductivity of the heated sample and frequency response were unknown. Finally, it was unknown whether a term for the complex permittivity ought to be used with materials, such as ionic solutions, that are partially magnetically “transparent”.

The AC/DC module of COMSOL has been used to validate the numerical approximation in 3D in a frequency-transient study. A uniform magnetic field volume was generated by simulating a pair of Helmholtz coils, which was then used for heating saline samples of various concentrations in shapes of spheres, cylinders and sheets, and thus calculate accurate geometrical coefficients at various orientations for a range of frequencies for these shapes. Frequencies studied were from 2 MHz to 40 MHz. Strong temperature gradients were set up within the sample. However, setting the thermal conductivity of the saline to a very high value overcame this.

It was found that the coefficients for the induction heating numerical approximation, used up until now, were incorrect. In addition, the length to diameter ratio of cylinders orientated perpendicular to the magnetic field gave an unexpected additional term. Experiments were then conducted at 2 MHz to measure the heating rates of well-stirred saline samples and compared to the COMSOL simulation results, and good agreement was found if electrical conductivity alone, i.e. without using the complex permittivity, was used.

The study has greatly aided us in our induction heating applications, and helped drive engineering ideas. This paper gives details of the simulation used, the methodology and the results.

Theory of inductive energy transfer

If a conducting body is placed within the magnetic field driven by an alternating current of an electromagnet, then that body will have electric “eddy” currents induced within it in a direction perpendicular to the magnetic field, and thereby undergo resistive heating. The specific thermal power, *P* (in W/kg), transferred from the magnetic field to a non-magnetic material via magnetic induction heating is given analytically by Equ (1) [1, 2, 3]:

$$P = \frac{\pi^2 \sigma B_p^2 f^2 d^2}{6kD} \qquad \text{Equ (1)}$$

where
σ is the electrical conductivity of the material ($S\ m^{-1}$),
B_p is the peak magnetic field (T),
f is the frequency of the magnetic field (Hz),
d is the length perpendicular to the magnetic field (m),
k is a geometrical correction factor, and
D is the density of the material (kg/m^3).

Because there are two orthogonal planes perpendicular to the direction of the magnetic field, two degrees of freedom exist for eddy currents to move in. Thus there exists a strong geometric component to electromagnetic induction heating, represented in Equ (1) by *d* and *k*. The length dimension *d* could be the thickness of a sheet, or the diameter of a cylinder or a sphere perpendicular to the magnetic field [1]. In the case of a sphere, the value of *d* is the same and constant in all orthogonal directions, and rotating it will not make any changes to the heating rate. However, the situation is more complicated for shapes with imperfect rotational symmetry such as cuboids and cylinders, and heating rates depend upon the angle the axis lies with respect to the magnetic field direction. The constant *k* is a correction factor for geometry and, prior to this study, was thought to equal to 1 for a thin sheet and 2 for a thin cylinder.

The aim of this study was to validate Equ (1), and determine the value of *k* for a number of different geometries. Equ (1) can be re-expressed to give the rate of change of temperature of an inductively heated body:

$$\frac{\Delta T}{\Delta t} = \lambda \frac{(\pi^2 \sigma B_p^2 f^2 d^2)}{cD} \qquad \text{Equ (2)}$$

where

c is the specific heat capacity (SHC) of the material being heated, and $\lambda = \pi^2/6k$. A plot of $\frac{\Delta T}{\Delta t}$ against $\frac{(\sigma B_p^2 f^2 d^2)}{cD}$ ought to give a straight line with gradient λ, allowing the value of k to be determined for a range of situations.

Study condition

For this study, it was vital to have a well-characterised value of B_p. A Helmholtz coil, where the distance between a pair coils is equal to the radius of the coils, is known to produce a reasonably uniform magnetic field between the coils and was thus used for this study. Figure 1 shows the basic setup.

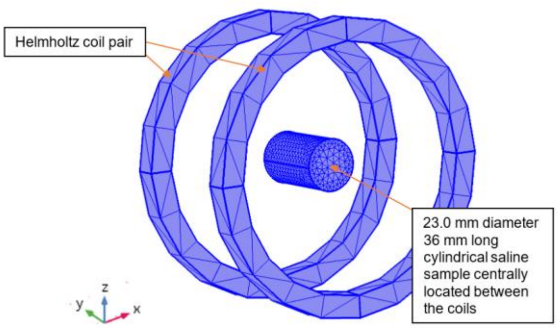


Figure 1

The AC/DC module in COMSOL was used in a 3D frequency-transient study. The coils were homogenized multi-turn with 10 turns, of radius r = 60 mm with an electrical conductivity of 6 x 10^7 S/m (i.e. copper). Heat transfer to the sample was modelled, and electromagnetic heating was coupled to the magnetic fields and heat transfer multiphysics equations. Cylinders, spheres and thin sheets of saline were set midway between the coils, varying the radius, length, orientation and saline concentration (i.e. electrical conductivity). Figure 1 also shows the mesh regime used for the modelling. To aid rapidity and yet obtain a high-fidelity for the heating of the saline, a "Normal" mesh size was used for the coils, and an "Extremely-fine free tetrahedral" mesh (the most detailed available) was use for the saline. The mesh at the interfaces adjusts automatically. The entire set-up was in air at STP but, because the rate of change of temperature was reasonably low, heat loss to the air was not included in the simulation, again to save computational effort. The uniformity of the magnetic field in the volume of the saline is clear from Figure 2, and has a variance of less than ±0.12%. The simulations were run for a heating time of 10 s in 500 ms time-steps. Each simulation took a minimum of 120 s and increased to over 3 hrs as the size of the saline sample decreased (due to the increased number of very fine cells in the magnetic field surrounding the saline).

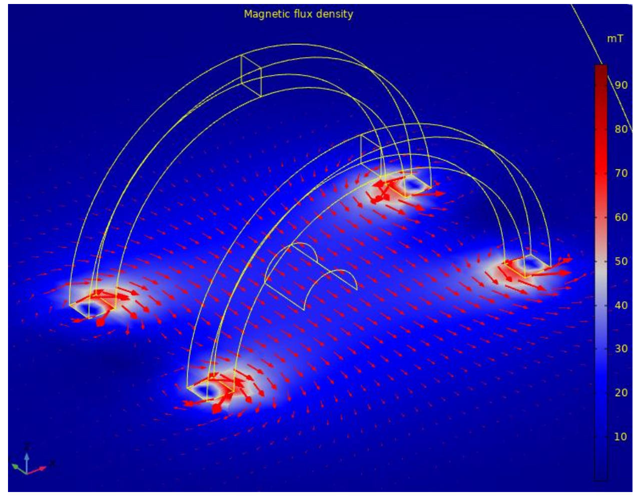


Figure 2

An assumption of Equ (2) is that the samples are homogenously heated. The saline was vigorously stirred to assure homogenous heating by allowing it a large thermal conductivity of 10^8 W/m.K.

Twelve distinct parameter sets were used for each geometry studied, ranging over six frequencies, four currents (i.e. four magnetic flux densities), three sample diameters and three sample electrical conductivities (i.e. three saline concentrations). Datapoints were carefully chosen to get a good spread of results.

The specific heat capacity and density for the product cD in Equ (2) were kept constant for the three concentrations. The percentage error this produced was negligible because the SHC slightly reduces with increased concentration, and the density slightly increases almost in direct proportion. Table 1 shows the % difference in cD was less than 2% over the range of interest.

Conc. of saline (% mass)	SHC of saline (J/g.K)	Density of saline (g/cm^3)	SHC x density	% difference wrt 5% concentration
5	3.90	1.034	4.03	0
10	3.70	1.0707	3.96	2
30	3.28	1.2238	4.01	1

Table 1

These values also change within the temperature range, but for simplicity this was also neglected due to the minimal errors this introduced. (The isobaric SHC of water changes by - 0.1%, and the density changes by - 4% from freezing to boiling.)

Results

Several studies were conducted to calculate the values of *k* under various geometrical arrangements, and each of these is now considered.

1. Cylinder (axis parallel to the coil axis)

The first study explored saline cylinders, 36 mm long and 23 mm diameter, with their long axis parallel to both the magnetic field and the axis of the Helmholtz coils as shown in Figure 1 and Figure 2. As mentioned above, the important dimension for induction heating, *d*, is the length perpendicular to the magnetic field. Here, this quantity is the cylinder diameter for both orthogonal directions. The length of the cylinder ought not make a difference to the heating rate, but this was checked anyway.

Table 2 shows the twelve distinct parameter sets used for this analysis for the cylinder. The colours in the table relate to those in the graph in Figure 4 below, and are simply to help indicate which variables were changed in each run of COMSOL.

Figure 3 shows an example of inductively heating the cylinder over a 10 s period, with a current of 210 A producing a magnetic flux density of 0.02764 T, at a frequency of 10 MHz, and a concentration of saline of 0.6% by mass with an electrical conductivity = 1 S/m (third line in Table 2). The rate of change of temperature is given by the gradient (i.e. 6.12 °C/s in this instance). The gradient was highly linear, as in all cases in these studies.

Figure 4 shows the plot of $\frac{\Delta T}{\Delta t}$ against $\frac{(\sigma B_p^2 f^2 d^2)}{cD}$ from the data from Table 2. Generally, as Table 2 shows, only one parameter was uniquely changed in each run apart from the last line in the table where the current and the concentration of saline were altered in order to fill in a large gap in the dataset. All such graphs were perfectly linear, corroborating the proportionality in Equ (2). For a 23 mm diameter cylinder, $\lambda = \pi^2/6k$ = 0.619, which gives a value *k* = 2.66 (a factor of 1.33 higher than *k* = 2 as previously thought for thin cylinders).

Figure 5 shows the value of *k* for a range of cylinders with diameters ranging from 2 mm to 40 mm with their long axes parallel to the direction of the magnetic field (see inset). The value is constant at *k* = 2.66 for all diameters.

The cylinder length ought not make any difference here because it lies parallel to the magnetic field. Nonetheless, three lengths were tested for a 40 mm diameter cylinder: 5 mm, 20 mm and 36 mm. For each length, the value *k* = 2.66, showing the cylinder length does not affect the heating rates in this configuration either.

f (Hz)	I (A)	B_p (T)	d (m)	σ (S/m)	c (J/kg.°C)	D (kg/m³)	$f^2B^2d^2\sigma/cD$	ΔT/Δt (°C/s)
2.2E+06	210	0.02764	2.3E-02	1.0	4000	1025	4.80E-01	0.30
5.0E+06	210	0.02764	2.3E-02	1.0	4000	1025	2.48E+00	1.53
1.0E+07	210	0.02764	2.3E-02	1.0	4000	1025	9.92E+00	6.12
2.0E+07	210	0.02764	2.3E-02	1.0	4000	1025	3.97E+01	24.48
3.0E+07	210	0.02764	2.3E-02	1.0	4000	1025	8.93E+01	55.10
4.0E+07	210	0.02764	2.3E-02	1.0	4000	1025	1.59E+02	98.00
2.0E+07	150	0.01982	2.3E-02	1.0	4000	1025	2.03E+01	12.5
2.0E+07	300	0.03963	2.3E-02	1.0	4000	1025	8.11E+01	49.9
4.0E+07	150	0.01983	1.5E-02	1.0	4000	1025	3.45E+01	21
4.0E+07	300	0.03970	6.0E-03	1.0	4000	1025	2.21E+01	14
5.0E+06	210	0.02764	2.3E-02	22.0	4000	1025	5.46E+01	33.6
5.0E+06	250	0.03301	2.3E-02	35.0	4000	1025	1.23E+02	75.8

Table 2

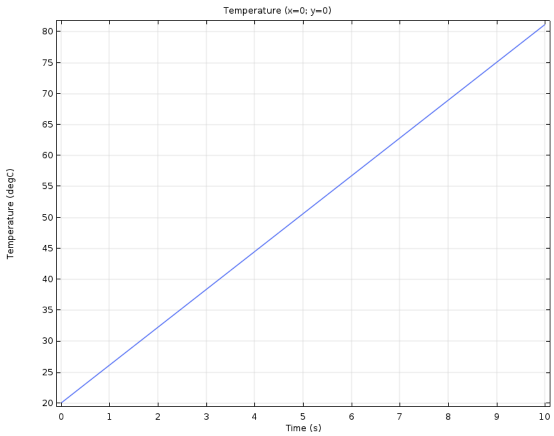


Figure 3

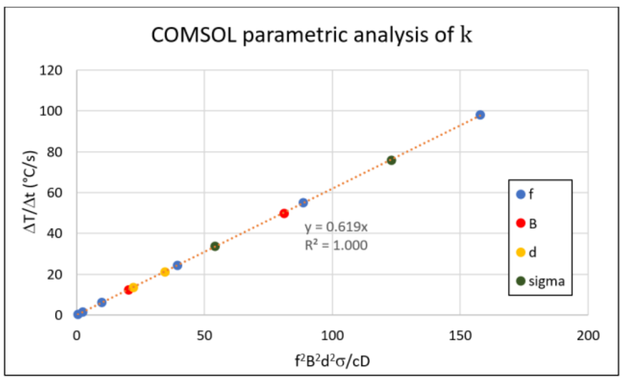


Figure 4

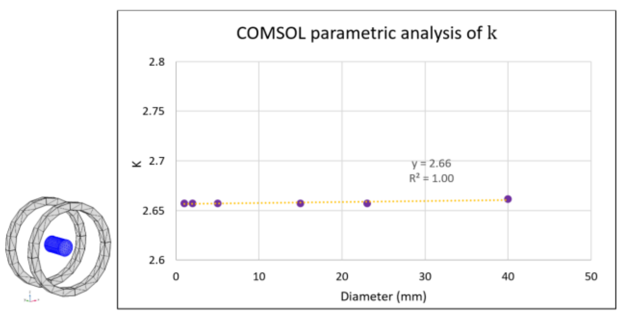


Figure 5

2. Cylinder (axis perpendicular to the coil axis)

The second study explored saline cylinders with their long axis perpendicular to the axis of the Helmholtz coils. In this geometry, both the diameter and the length are orthogonal to the magnetic field direction.

The diameter was initially kept constant at 23 mm and the length varied between 1 mm and 60 mm, keeping the cylinder centre midway within the coils. In the limit, these cylinders reduce to thin slabs, where it was thought $k = 1$. Otherwise, the value of k was generally unknown for this configuration.

Figure 6 shows the value of k for this geometry. The value decreases with decreasing length as a quadratic, tending to $k = 1$ as the length tends to zero (i.e. tending towards a thin slab).

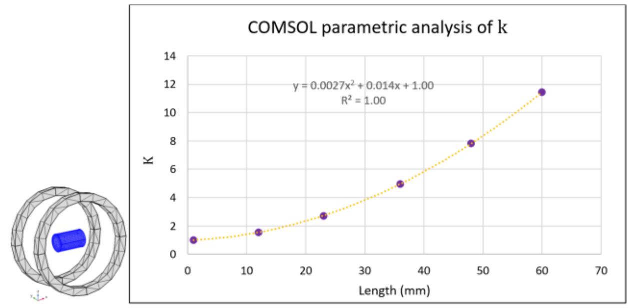


Figure 6

This study was continued by altering the diameter of the saline cylinder between 8 mm and 30 mm for the lengths 50 mm, 36 mm, 10 mm and 5 mm. Figure 7 shows k decreases with increasing radius as a power law, tending to $k = 1$ as the radius tends to infinity (i.e. again tending towards a thin slab).

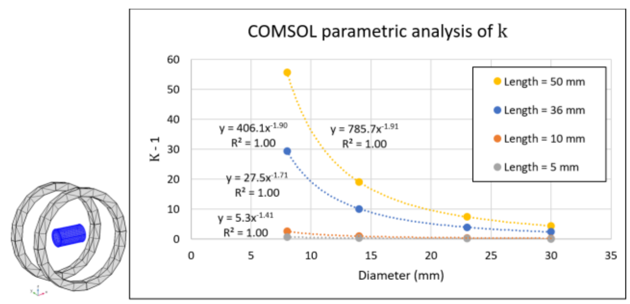


Figure 7

That k can have such a wide range of values for one given shape came as a surprise. These results mean that inductive heating will be much less efficient for long thin cylinders lying perpendicular to a magnetic field; in the limit $k = \infty$, no inductive heating will occur.

A final study for this configuration was performed for the special instance of unitary length/diameter ratio (L/D = 1). Figure 8 shows $k = 2.73$ for all cases.

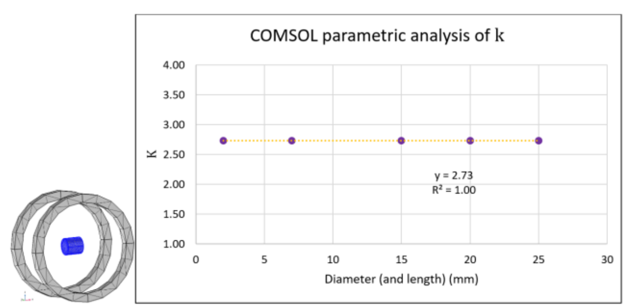


Figure 8

3. Spheres

Following the result of k = 2.66 for cylinders with L/D = 1, it was assumed that spheres would be similarly constant. Figure 9 shows that for all sphere diameters, k = 3.34.

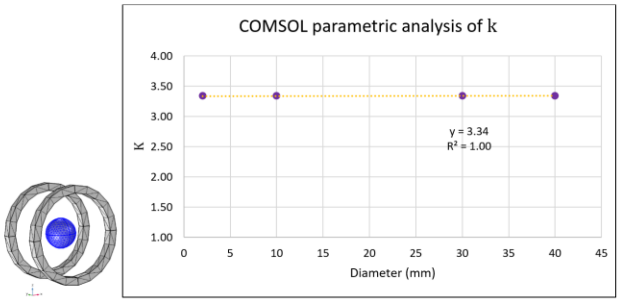


Figure 9

4. Thin slabs

Finally, thin slabs were examined. Slabs can have either one or two dimensions orthogonal to the magnetic field. Square slabs were used with dimensions 1 mm by 25 mm by 25 mm. Table 3 shows the results of this study. It was found that k = 1.00 for the two cases where only one long dimension is orthogonal to the magnetic field, or k = 2.34 for the case where both long dimensions are orthogonal to the magnetic field.

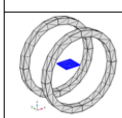
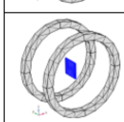
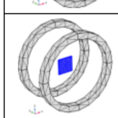
	Number of dimensions orthogonal to B-field	X (mm)	Y (mm)	Z (mm)	k
	1	25	25	1	1.00
	1	1	25	25	1.00
	2	25	1	25	2.34

Table 3

Interpretation of *k*

Electrical eddy currents are set up in the orthogonal direction to the magnetic field in conductors experiencing a change of magnetic flux. Equ (1) shows that the power transferred to an inductively heated body strongly increases with the square of the size. However, Equ (1) seems to have been derived for an infinite flat plane with only one degree of freedom for the conduction electrons. The constant k in Equ (1) is a geometrical correction where the degrees of freedom for eddy currents is greater than one. For configurations where d is the same in both directions, the value of k is a constant equal to or greater than one regardless of the magnitude of the dimensions. However, where the magnitude of d is different in the two orthogonal directions, then the value of k depends on the relative proportions of the two lengths. Table 4 is a summary of the various geometrical arrangements studied, and the values of k obtained.

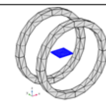
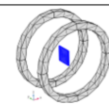
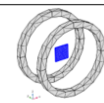
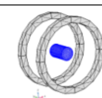
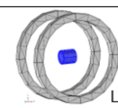
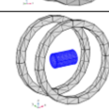
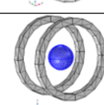
Geometry	Number of degrees of freedom	*k*
	1	1
	1	1
	2	2.34
	2	2.66
 L/D = 1	2	2.73
	2	<1 (up to ∞)
	2	3.34

Table 4

Experimental validation

In the above study, heating of the saline by the electric field was ignored. The specific thermal power, P_E (in W/kg), transferred to a body by an E-field is given by [4]:

$$P_E = \frac{2\pi D f \varepsilon_0 \varepsilon_r E_p^2 \tan\delta}{k_2 m} \qquad \text{Equ (3)}$$

where

D is the density of the material (kg/m^3),

f is the frequency of the electric field (Hz),

ε_0 is the permittivity of free space,

ε_r is the dielectric constant of saline at f,

E_p is the peak electric field strength (V/m),

$\tan\delta$ is the loss tangent of saline at f,

k_2 is a geometrical correction factor, and

m is the mass of the material (kg).

The value of *tanδ* for water at 2 MHz is very close to zero. A collection of the published values [5] are given in *Figure 10*. However, no data is available for saline, and this is required as an input value for COMSOL to calculate the complex permittivity, given by ε_r (1 – i *tanδ)*. It was assumed that the E-field heating would be negligible, but to test this assumption, experimentation had to be undertaken.

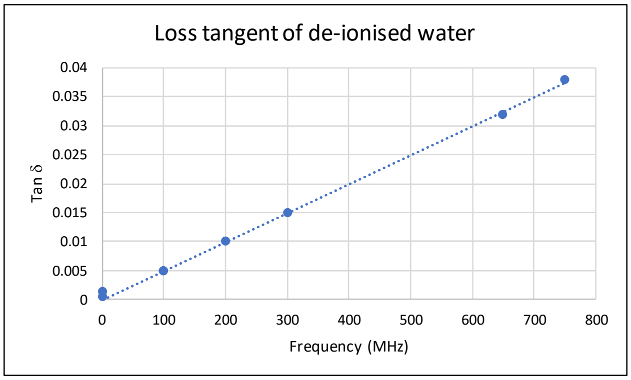


Figure 10

Inductive heating tests have been performed, courtesy of Trumpf Hüttinger GmbH in Freiburg, and then simulated assuming only magnetic induction heating. A water-cooled, 27 mm inside diameter, 4-turn coil was powered by an IG 10/2000 generator at a frequency of 2.2 MHz, and produced flux densities of up to 30 mT. A mass of 15 g of 0.5% concentration saline was poured into a 22 mm inside diameter, cylindrical glass tube and held axi-symmetrically within the coils as shown in *Figure 11*.

A 2D axi–symmetrical COMSOL model used to simulate these tests is shown on the right in *Figure 11*. As before, heat transfer to the sample was modelled, and electromagnetic heating multiphysics was coupled to the magnetic fields and heat transfer equations. An extremely fine, physics-controlled mesh was used, and the frequency-transient study ranged in 1 s time steps for 10 s. The centres of the cooling coils were modelled as a user-defined heat loss source *Mt*ht.Cp**(*Tin-T*)/(2**pi*r*Ac*), where *Mt* is the coolant water mass flow rate, *ht* is the heat transfer defined by a quadratic Lagrange function, *Cp* is the heat capacity of water, *Tin* is the coolant water inlet temperature, *r* is the coolant channel radius and *Ac* is the coolant channel cross sectional area. Heat conduction within the saline sample and to the air was modelled.

The resulting magnetic flux within the saline is shown in *Figure 12*, and the electric field is shown in *Figure 13*.

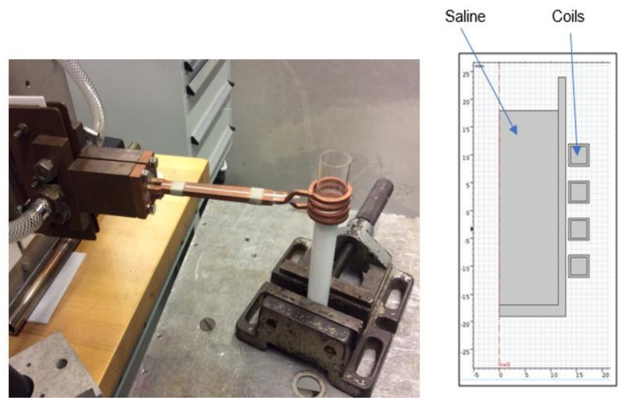


Figure 11

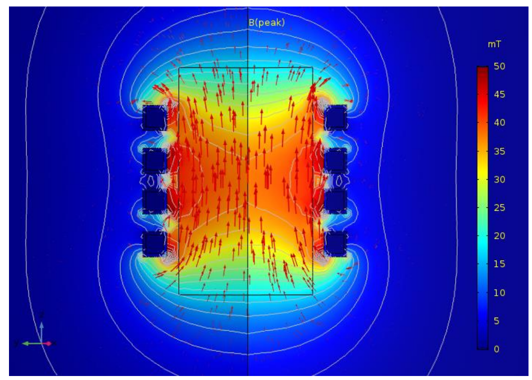


Figure 12

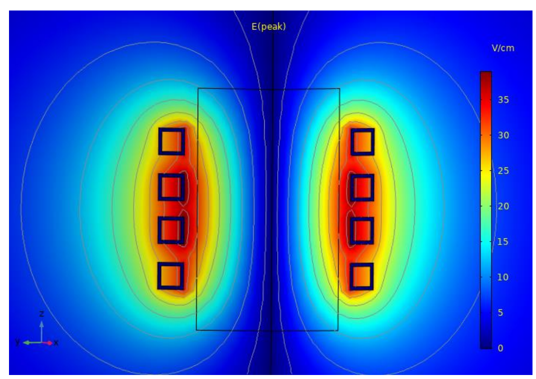


Figure 13

Clearly, the non–uniformity of the fields will significantly increase the non–uniformity of the heating, leading to heating on the outer edge, and heat is then transferred to the interior as shown in *Figure 14*. The modelling assumes heat conduction but not convection, but convection (i.e. stirring) can be artificially accomplished by setting the thermal conductivity from that of water (0.5918 W/m.K) to a very high value (10^4 W/m.K)). An average bulk heating rate *ΔT/Δt* = 0.34 °C/s is predicted, assuming only magnetic induction heating. (Note that *ΔT/Δt* was found to be highly linear, meaning that heat loss plays a negligible role.)

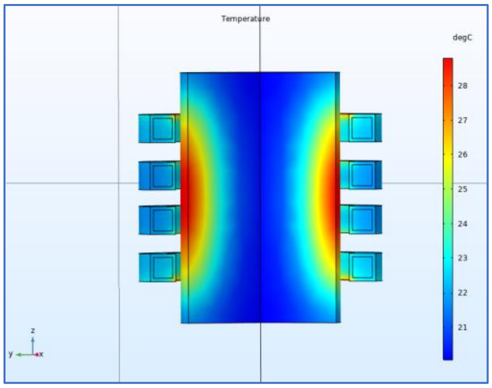


Figure 14

In the experiment, a 0.25 mm diameter k-type thermocouple was quickly inserted into the sample once the magnetic field had been turned off. Strong thermal gradients were evident, but upon gentle stirring, the temperature rapidly stabilised as the sample was mixed, and the highest temperature was recorded. The heating rate of $\Delta T/\Delta t$ = 0.33 °C/s was measured in the experiment and compares highly favourably with the value predicted by COMSOL. This means that the assumption of no electric field heating is valid at 2 MHz. However, errors might be introduced at higher frequencies. Work is in progress to measure the loss tangent of saline at frequencies up to 1 GHz.

Conclusions

An analytical approximation for solving problems in magnetic induction heating, Equ (1), has been used throughout this study. However, seemingly, nowhere in the literature has the equation been validated. Furthermore, it was thought that some applications may be using this equation outside of its parameters, for example with heating ionic salts with megahertz-frequency magnetic energy. COMSOL was thus used to simulate the heating rates of well-stirred saline samples within a uniform magnetic field at frequencies from 2 MHz to 40 MHz, changing the variables within the parameter space to explore the equation's constant of proportionality, k. k is a geometrical correction for cases where the degree of freedom for eddy currents is greater than one. Prior to this work it was held that k = 1 for thin slabs with their length perpendicular to the direction of the magnetic field, and k = 2 for thin cylinders with their diameter perpendicular to the direction of the magnetic field, so these geometries were checked. Because many of our objects being inductively heated are spheres, this geometry was also checked.

For cylinders with their diameter perpendicular to the magnetic field, it was found k = 2.66 for all diameters (thick or thin), a factor of 1.33 higher than previously thought (k = 2). To heat cylindrical samples with k = 2.66 will require a third more power, or take a third longer to heat cylindrical objects than if k = 2 as previously thought, if they lie with their longitudinal axis parallel to the magnetic field.

For cylinders with their longitudinal axis perpendicular to the magnetic field, k >1 (up to ∞). That k can have such a wide range of values for one given shape came as a surprise. These results mean that inductive heating will be much less efficient for long thin samples lying perpendicular to a magnetic field; in the limit $k = \infty$ and so no inductive heating will occur. For the special instance L/D = 1 then k = 2.73. For spheres, k = 3.34 for all diameters. For thin square slabs, k = 1.00 regardless of size for the two cases where only one long dimension is orthogonal to the magnetic field; k = 2.34 for the case where both long dimensions are orthogonal to the magnetic field.

During this study, the heating by the electric field was ignored because relevant data was unavailable. However, experimental validation has been undertaken at 2 MHz, and shows excellent agreement with the modelling, meaning that the E-field can be ignored, at least at lower frequencies. However, more work is required to validate the model at higher frequencies, and work is in progress to do this up to 1 GHz.

References

[1] Eddy currents https://en.wikipedia.org/wiki/Eddy_current

[2] F. Fiorillo, Measurement and Characterization of Magnetic Materials, Elsevier Academic Press, 2004, ISBN 0-12-257251-3, page 31

[3] Microwave Theory and Background, Chapter 3 (https://www.liverpool.ac.uk/~mimi/Chapter3.pdf) from The Microwave Palaeointensity Technique and its Application to Lava, PhD Thesis by Meirian Jane Hill, Liverpool University, 2000

[4] Radio Frequency Heating and Post-Baking, Tony Koral, Technical Director, Strayfield, Ltd, Biscuit World Issue 4 Vol 7 November 2004

[5] http://www.rfcafe.com/references/electrical/dielectric-constants-strengths.htm; http://literature.cdn.keysight.com/litweb/pdf/genesys200801/elements/substrate_tables/tablelosstan.htm; https://www.researchgate.net/figure/Permittivity-and-loss-tangent-for-water-at-different-temperatures-versus-frequency-5_fig1_260515825