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Design of Electroacoustic Absorbers Using PID Control

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Abstract: In this paper, an active electroacoustic absorber [1] is designed with the help of COMSOL Multiphysics® Acoustics Module. An "electroacoustic absorber" is a loudspeaker, used as an absorber of sound, which acoustic impedance can be varied by electrical means. This can be achieved either by plugging passive shunt electric networks at the loudspeaker terminals ("shunt loudspeakers") or by feeding back the loudspeaker with a voltage proportional to acoustic quantities, such as sound pressure and diaphragm normal velocity ("direct impedance control"). It has already been shown that COMSOL Multiphysics® could be advantageously used to simulate the acoustic performances of passive shunt loudspeakers in a recent communication [2]. The extension to an active impedance control using a PID control law on acoustic quantities is the motivation of the paper.

Keywords: electroacoustic absorber, acoustic impedance control, active noise control, PID control, electrodynamic loudspeaker.

1. Introduction

In this paper, a finite-element model (FEM) of an electroacoustic absorber is proposed in the time domain, aiming at synthesizing desired acoustic impedance at the diaphragm of a loudspeaker using a proportional-integral-derivative (PID) control law. Any conventional loudspeaker first intended to be a sound transmitter may then become a versatile electroacoustic resonator capable of absorbing (or of reflecting as much) incident sound energy in a frequency-dependent way [1]. Instead of counteracting some unwanted sound by using superposition principle, as is the case for conventional active noise control, such actuator-based strategy aims at monitoring the dynamics of a loudspeaker diaphragm so as to control the proportion of reflected sound waves on its surface. Results obtained using FEM will be compared to experimental data, measured when controlling a loudspeaker with a PID control at the end of a duct where sound waves propagate.

2. Governing equations

2.1 Loudspeaker dynamics

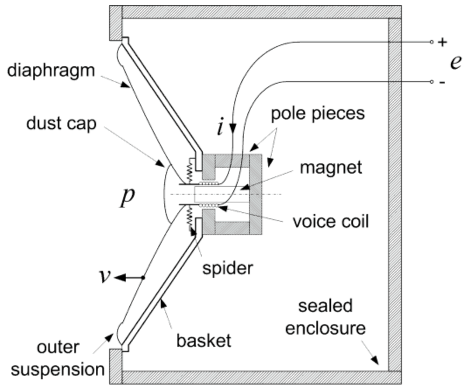

Figure 1: Schematics of a closed-box loudspeaker.

An electrodynamic loudspeaker can be easily modeled as a lumped-element system by applying Newton's second law to the moving body, and Kirchhoff's law to the electrical circuit [3]. For small displacements and below the first modal frequency of the diaphragm, a closed-box loudspeaker is commonly described using a set of differential equations

$$\begin{aligned} -Sp(t) &= M_{ms}\frac{dv}{dt} + R_{ms}v(t) + \frac{1}{C_{mc}}\xi(t) - Bli(t) \\ e(t) &= R_e i(t) + L_e\frac{di}{dt} + Bl\,v(t) \end{aligned} \qquad (1.1)$$

where p is the total sound pressure (in Pa), due to an exogenous sound source, acting on the diaphragm, v is the diaphragm velocity (in m.s^{-1}), i the electrical current flowing through the coil (in A), e the voltage applied to the electrical terminals (in V), S the surface area of the diaphragm (in m^2), M_{ms} the moving mass (in kg), C_{mc} the effective compliance (inverse of the stiffness) of the suspension and closed enclosure (in N.m^{-1}), R_{ms} the mechanical resistance (in N.s.m^{-1}), R_e the dc resistance (in Ω), L_e the coil inductance (in H), and Bl the force factor (in T.m). A schematic diagram of an electrodynamic loudspeaker is show in Fig. 1.

2.2 PID controller

Proportional-integral-derivative (PID) controllers are widely used in industrial control systems to obtain a desired control response. Basically, this closed-loop feedback mechanism attempts to minimize an error by adjusting the process control input using three control actions that depends on the present, past and future of the error [4]. The parallel form of the PID algorithm is

$$e(t) = K_p \varepsilon(t) + K_i \int_0^t \varepsilon(\tau) d\tau + K_d \frac{d}{dt} \varepsilon(t) \quad (1.2)$$

where *e(t)* is the voltage applied to the loudspeaker terminals, K_p is a proportional gain, K_i is an integral gain, and K_d is a derivative gain, and ε is the tracking error.

2.2 Tracking error

The condition for perfect sound absorption of a loudspeaker at the end of a duct [1] is expressed by the adaptation of the presented acoustic impedance with the characteristic impedance of the medium:

$$\frac{p}{v} = \rho c \quad (1.3)$$

where ρc is the characteristic impedance of air. This condition can be achieved using control engineering approach. Reformulating (1.3) as an error signal to be minimized by the controller yields

$$\frac{p(t)}{\rho c} - v(t) = \varepsilon(t) \quad (1.4)$$

where $p(t)/\rho c$ is the set-point driving signal and $v(t)$ the diaphragm velocity.

3. Finite element model

3.1 Loudspeaker model

The finite element model of the loudspeaker's drive unit is based on the COMSOL Multiphysics® tutorial [5]. For symmetry reasons, the model is realized in 2D-axi-symmetry, thus enabling some computation time reduction. In order to fit the parameters of an actual loudspeaker some adjustments are required concerning the radius of the coil, the surface area *S*, and the value of dc resistance R_e. As the force factor of the loudspeaker is supposed to be constant in case of small displacements, the magnetic force *Bli* is directly applied to the moving mass, without using the AC/DC module (as reported in a former publication [2]). The electrical current flowing through the coil is obtained using the ODEs and DAEs module that computes the differential equation from (1.1) as

$$L_e\, it + R_e\, i + Bl v_c + e = 0 \quad (1.5)$$

where the expression *it* defines in COMSOL the derivative of the current, v_c is the velocity of the voice coil, and *e* is the input voltage delivered by the controller.

The materials properties such as Young's modulus *E* and Poisson's ratio ν have also been changed so as to match the values of the actual loudspeaker. Table 1 summarizes the materials properties used within the model.

Table 1: Materials properties of the loudspeaker.

	E (Pa)	ν	ρ (kg m^{-3})	*m* (g)
dust cap	7e10	0.33	2700	1.1
diaphragm	7e10	0.33	140	0.8
spider	1e7	0.45	215	0.9
outer suspension	1e7	0.45	405	1.2
coil support	3.8e9	0.37	1500	0.6
voice coil	1.1e11	0.30	8700	7.5

The mechanical model is improved by adding a Rayleigh damping for each elements with the following Rayleigh parameter

$$\alpha_R = 0, \qquad \beta_R = \frac{0.26}{2\pi f_s} \quad (1.6)$$

where the natural resonance frequency of the moving body is f_s = 38Hz.

In order to account for the closed enclosure a new model is added at the rear face of the loudspeaker. Its boundary conditions are selected as rigid walls (sound hard wall condition).

3.2 PID model

In view of implementing the control law (1.2) in the FEM, the integral term is computed using a global equation in the ODEs and DAEs module

$$e_i\, t - \varepsilon = 0 \quad (1.7)$$

where the expression $e_i t$ corresponds, in COMSOL, to the derivative of the integral term [6]. Therefore, e_i is the sought integral term. The derivative term is defined as a global variable.

$$e_d = \frac{d\varepsilon}{dt} = \frac{1}{\rho c}\frac{dp}{dt} - \frac{dv}{dt} = \frac{1}{\rho c} pt - a \quad (1.8)$$

where the new local variables *pt* and *a* are the pressure derivative (in Pa.s^{-1}) and normal acceleration of the diaphragm (m.s^{-2}), respectively. Finally, the input voltage is defined as a global variable:

$$e(t) = K_p \varepsilon(t) + K_i e_i(t) + K_d e_d(t) \quad (1.9)$$

After Fourier transform the PID control law (1.9) can also be expressed in the frequency domain:

$$E(\omega) = K_p \varepsilon(\omega) + j\omega K_d \varepsilon(\omega) + K_i \frac{\varepsilon(\omega)}{j\omega} \quad (1.10)$$

4. Results and discussion

4.1 Specific acoustic impedance ratio

In order to assess the performance of the controlled electroacoustic absorber using a PID controller, the specific acoustic impedance ratio

$$z = \frac{p}{\rho c v} \quad (1.11)$$

is computed for various discrete frequencies in the range from 20Hz to 1000Hz. Computation of the rms value is post-processed with

$$z = \frac{1}{\rho c} \sqrt{\frac{1}{N}\sum_{i=1}^{N} p_i^2} \Bigg/ \sqrt{\frac{1}{N}\sum_{i=1}^{N} v_i^2} \quad (1.12)$$

where N is the number of points, i.e. the number of samples processed from COMSOL Multiphysics®.

4.2 Solver settings and optimization

The solver that we used is a direct fully-coupled time dependant-solver, with the generalized alpha method for computing the time step. Notice that intermediate is selected when determining the steps taken by the solver. The computation of the specific impedance ratio z is performed over n periods T with a time step of T/m. As shown in Tab. 2 and 3, both the number of periods and time step may affect the computed specific impedance ratio. In the extreme case, a too large time step may make the solver diverge.

Table 2: Influence of the number of periods on the computed specific impedance ratio.

n	f = 100Hz, K_p = 1.24 z	f = 950Hz K_p = 1.24 z
20	1.020	11.41
40	1.013	10.49
60	1.008	10.03
80	1.007	10.04
100	1.005	10.05

Table 3: Influence of time step on the computed specific impedance ratio.

m	f = 550Hz K_p = 1.54 z	f = 550Hz K_p = 1.54 K_i = 1000 K_d = 4e-3 z	f = 75Hz K_p = 1.54 K_i = 1000 K_d = 4e-3 z
20	4.8809	1.5279	1.0323
40	4.5217	1.167	1.0383
60	4.5103	1.1025	1.0435
80	4.5050	1.1047	1.0464

As shown in Tab. 2, the number of periods has a small influence on the computed z in the frequency range below 400Hz. For higher frequencies, the calculation must be done with more periods. As shown in Tab. 3, the m value determining the time step should be higher than 40 so as it has no influence on the computed z.

4.3 Acoustic performances

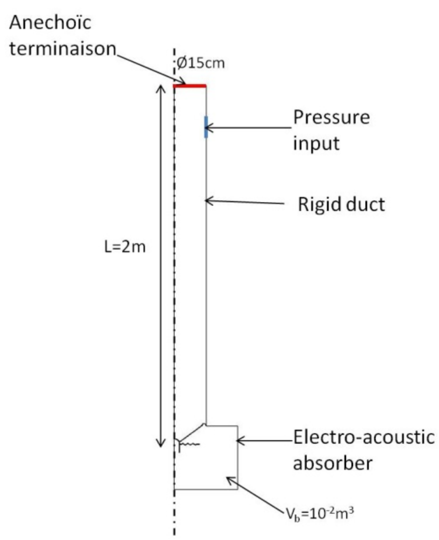

Figure 2: 2D-axisymmetry model of the electroacoustic absorber at the end of a duct.

Figures 3-5 illustrate the influence of proportional, integral and derivative feedback gains on the behavior of the electroacoustic absorber under PID control. The integral action has an influence in the frequency range below the resonance frequency of the system, while the derivative action has an influence in the frequency range above the resonance frequency.

Table 4: PID control settings.

case	*Kp*	*Ki*	*Kd*
A	-	-	-
B	1.45	0	0
C	4.35	0	0
D	1.45	0	4.14e-3
E	1.45	0	2.07e-4
F	1.45	1035	0
G	1.45	2070	0
H	1.45	1035	4.14e-3
I	1.45	1035	4.14e-3
J	1.45	1035	4.14e-3

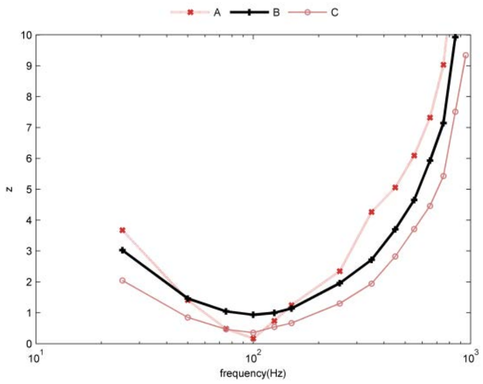

Figure 3: Computed specific acoustic impedance ratio *z* under P control. (case A: without control)

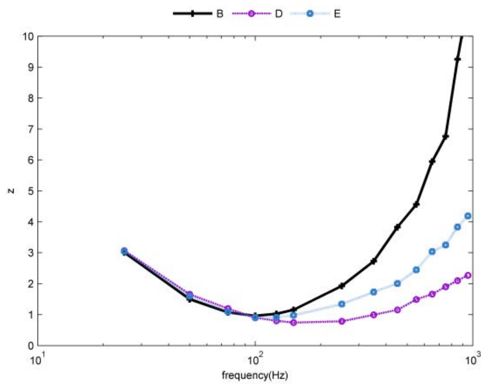

Figure 4: Computed specific acoustic impedance ratio *z* under PD control.

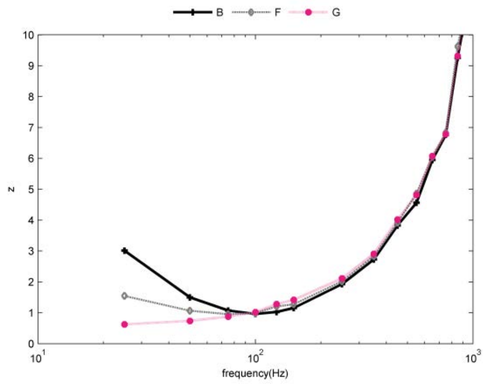

Figure 5: Computed specific acoustic impedance ratio *z* under PI control.

Figure 6 compares the computed and measured data of the electroacoustic absorber under PID control. By properly adjusting the PID feedback gain, the model clearly shows that total sound absorption, corresponding to the condition $z = 1$, can be achieved all along a large frequency range. Up to 400 Hz, computed and measured data are quite similar, meaning that the FEM is relevant to predict the future behavior of the controlled system using PID. Note also that both the time model (case H) and the frequency model (case I, where the COMSOL model is set for steady-state, with the Frequency Domain Study type) give the same results.

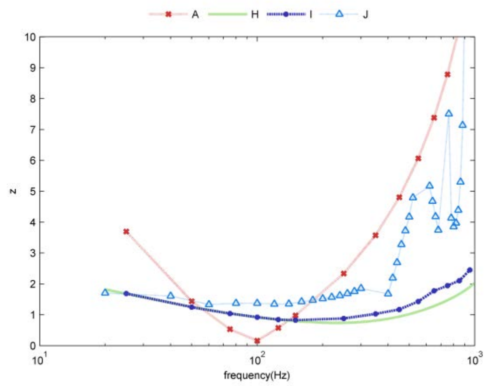

Figure 6: Computed and measured (case J) specific acoustic impedance ratio *z* under PID control. (case H with time model, case I with frequency model).

7. Conclusions

In this paper, a finite element model of a controlled electroacoustic transducer using a PID control has been presented. The PID controller has been successfully employed within the Transient Model of the electroacoustic absorber, and processed with the Time Dependant study, and behaves as expected. The obtained model provides an efficient tool in view of designing an electroacoustic absorber from a conventional loudspeaker. The time-domain PID model has been faced up to a frequency-domain PID model and both have shown similar results. Likewise, the finite-element models implemented in COMSOL Multiphysics® have been successfully compared to a lumped element model and experimental data. For higher frequencies, the FEM fails to mimic the behavior of the controlled system, due to higher modes of the diaphragm that take precedence over a purely piston motion. The next step should be to investigate the behavior of the electroacoustic absorber for higher frequencies, including higher modes of the diaphragm, and assess the behavior of such active electroacoustic absorber in 3D configurations, for example embedded in the walls of a room.

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Acknowledgements

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Appendix

Table 5: Loudspeaker Visaton AL-170 small signal parameters. The “datasheet” values come from the manufacturer and “model” reports the value processed through COMSOL.

Item	Unit	Datasheet	Model
R_e	Ω	5.6	5.6
L_e	mH	0.9	3.2
Bl	Tm	6.9	6.7
M_{ms}	g	13	12.1
R_{ms}	$\mathrm{N\ s\ m^{-1}}$	0.8	0.78
C_{ms}	$\mathrm{mm\ N^{-1}}$	1.35	1.4
f_s	Hz	38	37
S	$\mathrm{cm^2}$	133	137