CHALMERS UNIVERSITY OF TECHNOLOGY

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Simulation and Verification of Coupled Heat and Moisture Modeling

COMSOL Conference Stuttgart 2011

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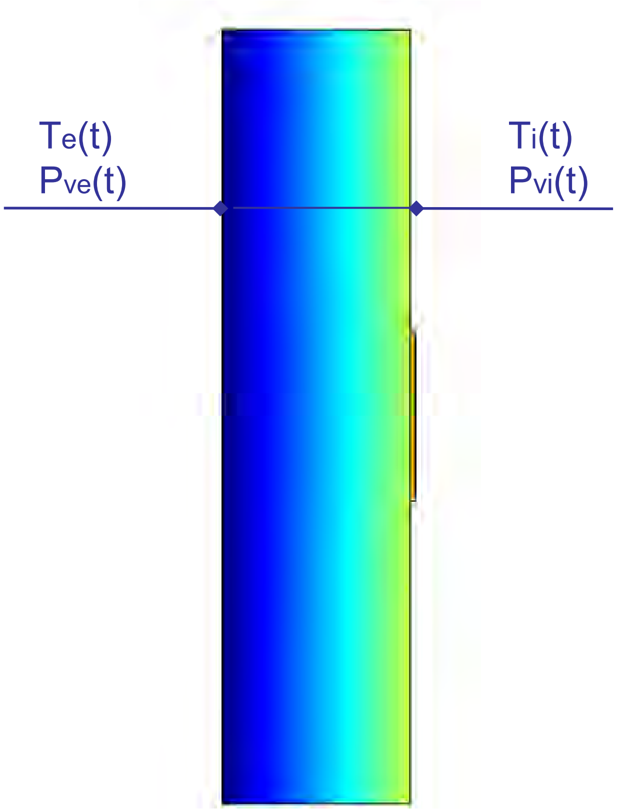
Introduction

- Modeling of coupled thermal and hygric transport in COMSOL
  - Validation and comparison of two models
  - 1- LPc and 2- Rh

- **Value:** predictive tool for possible damage-related processes in building materials and components

- **Problem:** limitations of the moisture potential



Coupled Heat and Moisture Transport

- Governing Equations:
  - Heat transfer

$$q = q_{cd} \qquad q_{cd} = -\lambda \nabla T = -\left(\lambda \frac{\partial T}{\partial x}\right)$$

  - Moisture transfer

$$g = g_v + g_\ell \begin{cases} g_v = -\delta_p \nabla p = -\left(\delta_p \dfrac{\partial p}{\partial x}\right) \\ g_\ell = -D_w \cdot \dfrac{\xi}{p_{sat}} \nabla p = -D_w \cdot \dfrac{\xi}{p_{sat}} \left(\dfrac{\partial p}{\partial x}\right) \end{cases}$$

- PDEs:
  - Energy balance

$$c_p \rho \frac{\partial T}{\partial t} = -\nabla(-\lambda \nabla T)$$

  - Moisture balance

$$\frac{\partial w}{\partial t} = -\nabla\left(\underbrace{(-\delta_p - D_w \cdot \frac{\xi}{p_{sat}}) \nabla p}_{g_v + g_\ell}\right)$$

Model 1: LPc Model

- Described using natural logarithmic of the suction pressure as moisture potential
- Described by these PDEs
- Formulated using Neumann boundary conditions

$$q_c = \alpha_c \cdot (T_s - T_a)$$

$$g_p = \beta_p \cdot (p_{vs} - p_{va}(LPc, T))$$

$$C_T \frac{\partial T}{\partial t} = \nabla \cdot (K_{11} \nabla T + K_{12} \nabla LPc)$$

$$C_{LPc} \frac{\partial LPc}{\partial t} = \nabla \cdot (K_{21} \nabla T + K_{22} \nabla LPc)$$

With:

$$LPc = {}^{10}\log(Pc)$$

$$C_T = \rho \cdot c$$

$$K_{11} = \lambda$$

$$K_{12} = -l_{lv} \cdot \delta_p \cdot \phi \cdot \frac{\partial Pc}{\partial LPc} \cdot Psat \cdot \frac{M_w}{\rho_a RT},$$

$$C_{LPc} = \frac{\partial w}{\partial Pc} \cdot \frac{\partial Pc}{\partial LPc}$$

$$K_{22} = -K \cdot \frac{\partial Pc}{\partial LPc} - \delta_p \cdot \phi \cdot \frac{\partial Pc}{\partial LPc} \cdot Psat \cdot \frac{M_w}{\rho_a RT}$$

$$K_{21} = \delta_p \cdot \phi \cdot \frac{\partial Psat}{\partial T},$$

Model 2: Rh Model

- Described using relative humidity as moisture potential
- Described by these PDEs

$$c_p \rho \frac{\partial T}{\partial t} = -\nabla(-\lambda \nabla T)$$

$$\xi \frac{\partial \varphi}{\partial t} = -\nabla\big((-\delta_p \cdot p_{sat} - D_w \cdot \xi)\nabla \varphi\big)$$

- Formulated using Neumann boundary conditions

$$q_c = \alpha_c \cdot (T_s - T_a)$$

$$g_\varphi = \beta_\varphi \cdot (\varphi_s - \varphi_a)$$

Modeling in COMSOL

- Coefficient Form PDE Interface
  - Described by simplified PDE problem

$$d_a \frac{\partial u}{\partial t} = -\nabla(-c\nabla u) \quad \text{in } \Omega$$

$$n \cdot (c\nabla u) = g - h^T \mu \quad \text{on } \partial\Omega$$

$$u = r \quad \text{on } \partial\Omega$$

- Dependent variable *u* and coefficients *da* and *c* expanded to vector form

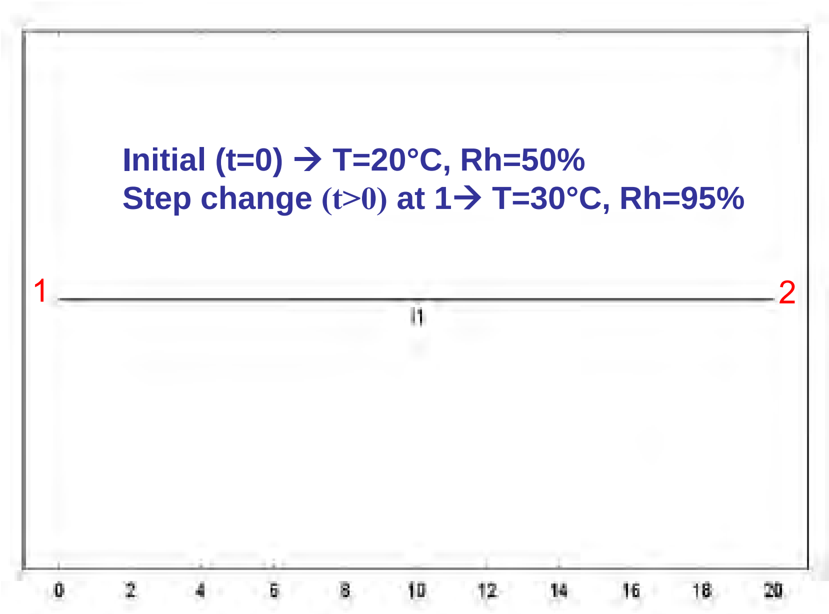
$$u = \begin{bmatrix} T \\ LPc \end{bmatrix} \text{ or } u = \begin{bmatrix} T \\ \varphi \end{bmatrix}$$

$$da \cdot \frac{\partial u}{\partial t} = \begin{bmatrix} d_{a\_T} & 0 \\ 0 & d_{a\_\varphi} \end{bmatrix} \cdot \begin{bmatrix} \frac{\partial T}{\partial t} \\ \frac{\partial \varphi}{\partial t} \end{bmatrix}$$

$$-\nabla(-c \cdot \nabla u) = \nabla \begin{bmatrix} c_T & 0 \\ 0 & c_\varphi \end{bmatrix} \cdot \begin{bmatrix} \nabla T \\ \nabla \varphi \end{bmatrix}$$

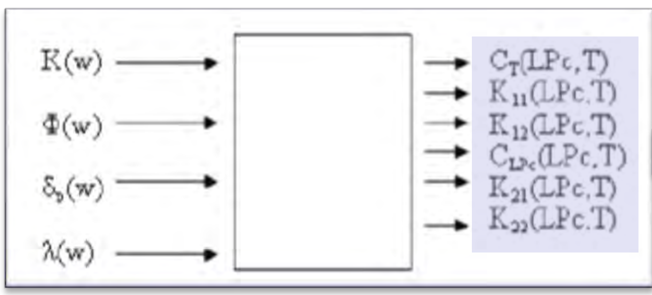
Model Verification

- Normative benchmark test of European Provisional Standard prEN 15026
  - Used to verify both LPc and Rh models
- Based on:
  - Analytical solution for 1D coupled thermal and hygric transport in a homogeneous semi-infinite domain
- Requirement:
  - Temperature and water content profiles after 7,30 and 365 days within ±2.5%

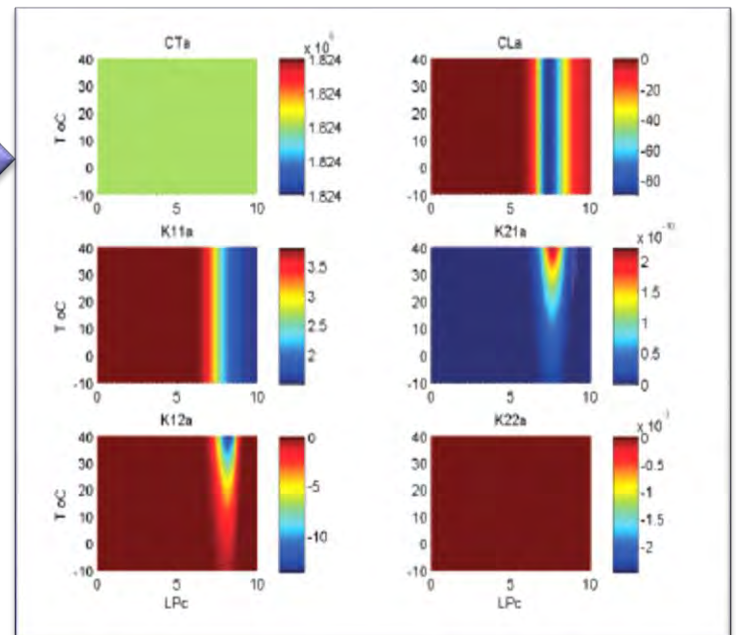


LPc Model Verification

- MatLab used for implementation of material functions
- Global definitions in COMSOL used for initial and Neumann boundary conditions

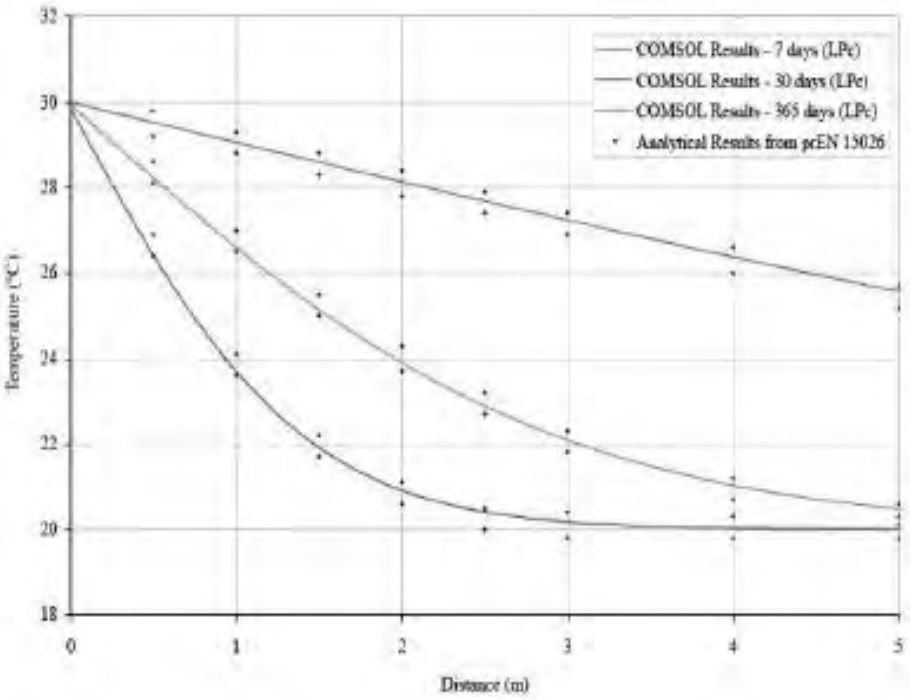


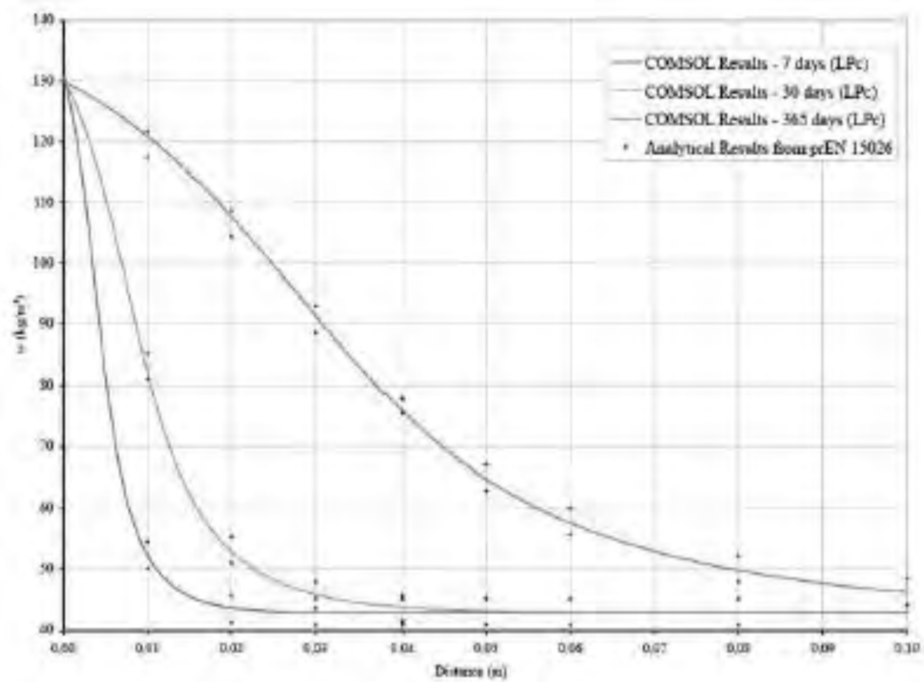
Convert to PDE coefficients



Results

LPc Model Verification



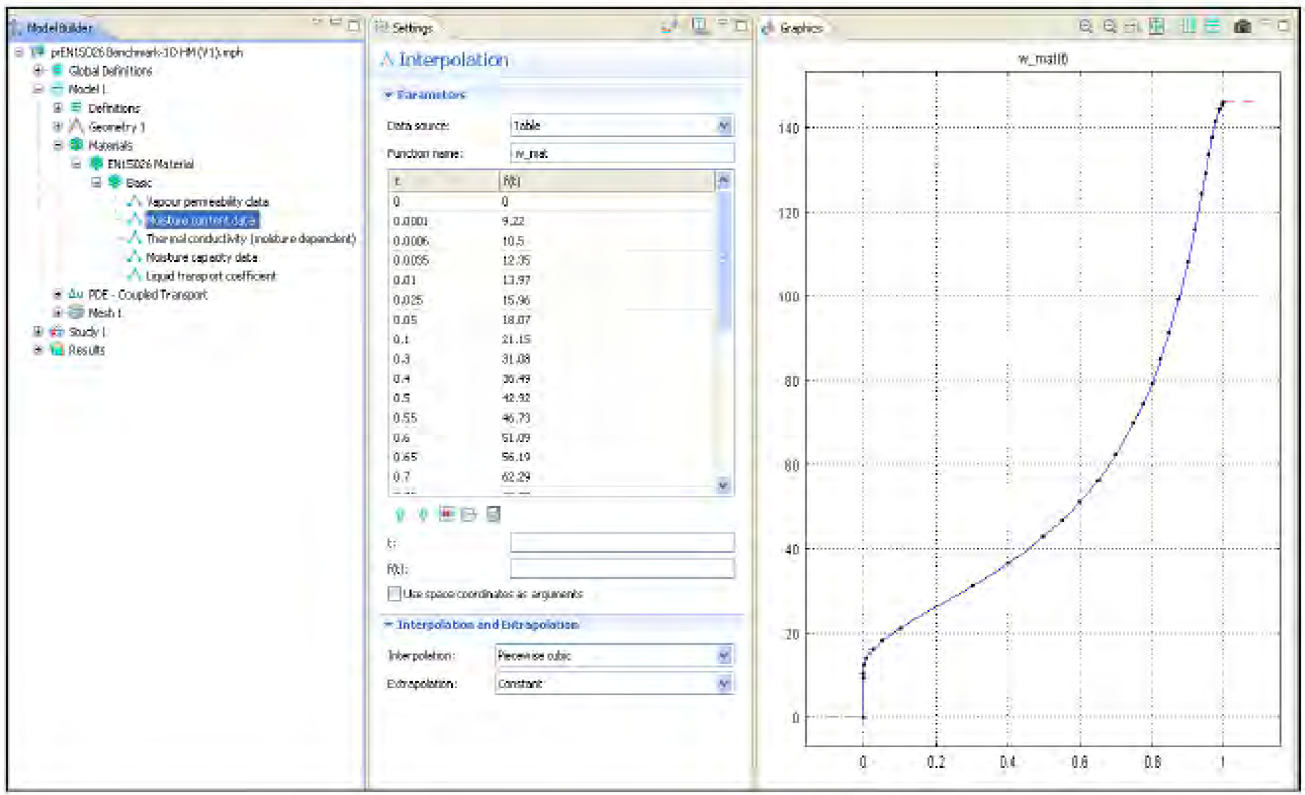


Temperature distribution

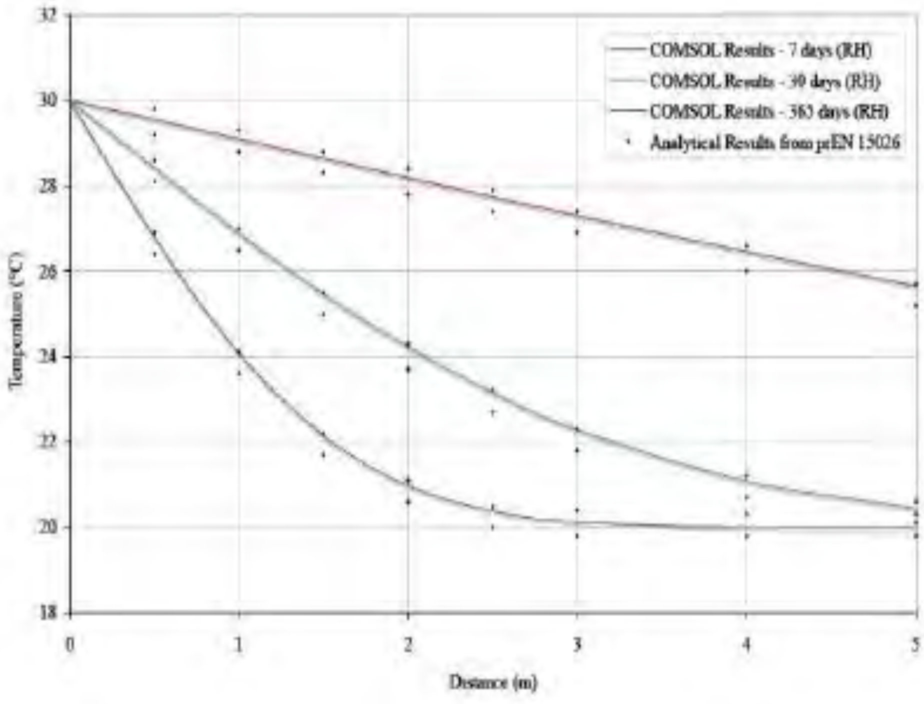
Moisture distribution

Rh Model Verification

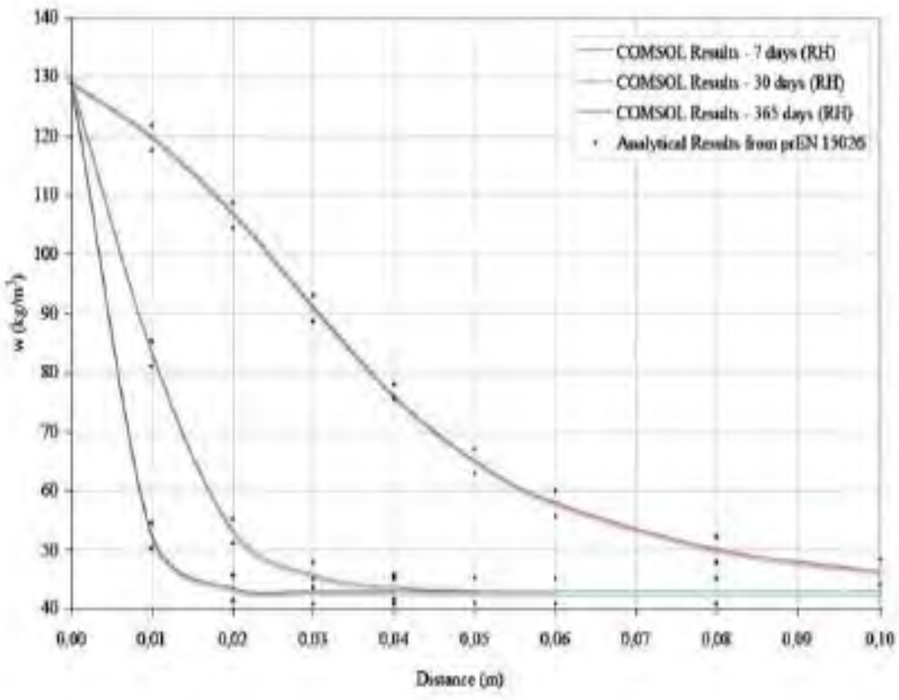
- Interpolation functions in COMSOL used for implementation of material functions
- Global definitions in COMSOL used for initial and Neumann boundary conditions



Rh Model Verification



Temperature distribution



Moisture distribution

Model Comparison

- LPc and Rh models produce similar results which agree with the benchmark
- Simulation results using COMSOL 4.2.0.228:

Model	No. Elements	Degrees of freedom	Solution time (s)
LPc	290	1742	19
Rh	1000	4002	11

Conclusions

- Both LPc and Rh models are valid predictive tools to investigate variable hygrothermal conditions in building materials

- Rh model
  - Advantage → Measured material properties directly implemented as functions in COMSOL
  - Disadvantage → Not numerically suitable for liquid water fluctuations at the boundaries

- LPc model
  - Advantage → Best suitable for extreme conditions at boundaries (i.e. liquid water fluctuations)
  - Disadvantage → PDE coefficients are calculated from measured material properties using MatLab as a pre-processor (possible source of error)

Thank you!